Ex 2-24.

Given: $[A, A^{+}] = 1 \quad \therefore [A^{+}, A] = -1.$

(a). $[A^{+}A, A] = [A^{+}, A]A + A^{+}[A, A]$

$= (-1)A + 0$

$= \underline{\underline{-A}}.$

$[A^{+}A, A^{+}] = [A^{+}, A^{+}]A + A^{+}[A, A^{+}]$

$= 0 + (A^{+})(1)$

$= \underline{\underline{A^{+}}}$

(b). $\langle a|\hat{A}|a+1\rangle = \langle a|(\sqrt{a+1}\,|a+1-1\rangle$

$= \sqrt{a+1}\,\langle a|a\rangle$

$= \sqrt{a+1}.$

$\langle a+1|\hat{A}^{+}|a\rangle = \langle a+1|(\sqrt{a+1}\,|a+1\rangle)$

$= \sqrt{a+1}\,\langle a+1|a+1\rangle$

$= \sqrt{a+1}.$

$\langle a|A^{+}A|a\rangle = \langle a|A^{+}(\sqrt{a}\,|a-1\rangle$

$= \sqrt{a}\,\langle a|A^{+}|a-1\rangle.$

$= \sqrt{a}\,\langle a|(\sqrt{a}\,|a\rangle)$

$= a\langle a|a\rangle$

$= a.$

$$\langle a|\cancel{a}AA^{\dagger}|a\rangle = \langle a|A\left(\sqrt{a+1}\,|a+1\rangle\right)$$

$$= \sqrt{a+1}\,\langle a|\sqrt{a+1}\,|a\rangle$$

$$= a+1\,\langle a|a\rangle$$

$$= \underline{\underline{a+1}}$$

(c). $$\langle a|(A+A^{\dagger})^2|a\rangle = \langle a|A^2+AA^{\dagger}+A^{\dagger}A+A^{\dagger 2}|a\rangle$$

$$= \langle a|AA^{\dagger}|a\rangle + \langle a|A^{\dagger}A|a\rangle$$

$$= a+a+1$$

$$= \underline{\underline{2a+1}}$$

$$\langle a|(A-A^{\dagger})^2|a\rangle = \langle a|A^2-AA^{\dagger}-A^{\dagger}A+A^{\dagger 2}|a\rangle$$

$$= -\langle a|AA^{\dagger}|a\rangle - \langle a|A^{\dagger}A|a\rangle$$

$$= \underline{\underline{-(2a+1)}}$$