

Ex 2-24.

Given: $[A, A^\dagger] = 1 \quad \therefore [A^\dagger, A] = -1.$

$$\begin{aligned} \text{(a). } [A^\dagger A, A] &= [A^\dagger, A]A + A^\dagger[A, A] \\ &= (-1)A + 0 \\ &= \underline{\underline{-A}}. \end{aligned}$$

$$\begin{aligned} \text{(b). } [A^\dagger A, A^\dagger] &= [A^\dagger, A^\dagger]A + A^\dagger[A, A^\dagger] \\ &= 0 + (A^\dagger)(1) \end{aligned}$$

$$\text{(c). } \langle a | (A + A^\dagger) | a \rangle = \langle a | A + A^\dagger | a \rangle$$
$$= \underline{\underline{A^\dagger}}$$

$$\begin{aligned} \text{(b). } \langle a | \hat{A} | a+1 \rangle &= \langle a | (\sqrt{a+1} | a+1 \rangle) \\ &= \sqrt{a+1} \langle a | a \rangle \\ &= \sqrt{a+1}. \end{aligned}$$

$$\begin{aligned} \langle a+1 | \hat{A}^\dagger | a \rangle &= \langle a+1 | (\sqrt{a+1} | a+1 \rangle) \\ &= \sqrt{a+1} \langle a+1 | a+1 \rangle \\ &= \sqrt{a+1}. \end{aligned}$$

$$\begin{aligned} \langle a | A^\dagger A | a \rangle &= \langle a | A^\dagger (\sqrt{a} | a-1 \rangle) \\ &= \sqrt{a} \langle a | A^\dagger | a-1 \rangle \\ &= \sqrt{a} \langle a | (\sqrt{a} | a \rangle) \\ &= a \langle a | a \rangle \\ &= a. \end{aligned}$$

$$\begin{aligned}
\langle a | AA^\dagger | a \rangle &= \langle a | A (\sqrt{a+1} | a+1 \rangle) \\
&= \sqrt{a+1} \langle a | \sqrt{a+1} | a \rangle \\
&= a+1 \langle a | a \rangle \\
&= \underline{\underline{a+1}}
\end{aligned}$$

$$\begin{aligned}
(c). \quad \langle a | (A + A^\dagger)^2 | a \rangle &= \langle a | A^2 + AA^\dagger + A^\dagger A + A^{\dagger 2} | a \rangle \\
&= \langle a | AA^\dagger | a \rangle + \langle a | A^\dagger A | a \rangle \\
&= a + a+1 \\
&= \underline{\underline{2a+1}}
\end{aligned}$$

$$\begin{aligned}
\langle a | (A - A^\dagger)^2 | a \rangle &= \langle a | A^2 - AA^\dagger - A^\dagger A + A^{\dagger 2} | a \rangle \\
&= -\langle a | AA^\dagger | a \rangle - \langle a | A^\dagger A | a \rangle \\
&= \underline{\underline{-(2a+1)}}
\end{aligned}$$