Ex. 2-8

(a). $a(2,-3,0)+b(0,0,1)+c(2i,i,-i)=0$

$$\Rightarrow \begin{cases} 2a+0b+2ic=0 \\ -3a+0b+ic=0 \\ 0a+b-ic=0 \end{cases}$$

$$\begin{vmatrix} 2 & 0 & 2i \\ -3 & 0 & i \\ 0 & 1 & -i \end{vmatrix} = 2(-i)+(2i)(-3) = -8i.$$

$\therefore a=b=c=0.$

$(2,-3,0)$, $(0,0,1)$ & $(2i,i,-1)$ are independent.

(b). $a(0,4,0)+b(i,-3i,i)+c(2,0,1)=0.$

$$\Rightarrow \begin{cases} 0a+ib+2c=0 \\ 4a-3ib+0c=0 \\ 0a+ib+1c=0 \end{cases}$$

$$\begin{vmatrix} 0 & i & 2 \\ 4 & -3i & 0 \\ 0 & i & 1 \end{vmatrix} = (-4)(i-2i) = 4i \neq 0$$

$\therefore a=b=c=0.$

$(0,4,0)$, $(i,-3i,i)$ & $(2,0,1)$ are independent.

(c). $a(2,-3,0)+b(0,0,1)+c(2i,i,-i)=0$

$$\Rightarrow \begin{cases} 2a+0b+2ic=0 \\ -3a+0b+ic=0 \\ 0a+b-ic=0. \end{cases}$$

$$\begin{vmatrix} 2 & 0 & 2i \\ -3 & 0 & i \\ 0 & 1 & -i \end{vmatrix} = (-1)(2i+6i) = -8i \neq 0.$$

$\therefore (2,-3,0)$, ~~$(i,-3i,i)$~~ $(0,0,1)$ & $(2i,i,-i)$ are independent.

(d) $a(i,1,2) + b(3,i,-1) + c(-i,3i+5i) = 0$

$$\Rightarrow \begin{cases} ia + 3b - ic = 0 \\ 1a + ib + 3ic = 0 \\ 2a - b + 5ic = 0. \end{cases}$$

$$\begin{vmatrix} i & 3 & -1 \\ 1 & i & 3i \\ 2 & -1 & 5i \end{vmatrix} = i(-5+3i) - 3(5i-6i) - i(-1-2i).$$

$$= -5i - 3 - 15i + 18i + i - 2$$

$$= -2i - 5$$

$$\neq 0.$$

$\therefore (i,1,2), (3,i,-1)$ & $(-i, 3i+5i)$ are linearly independent.