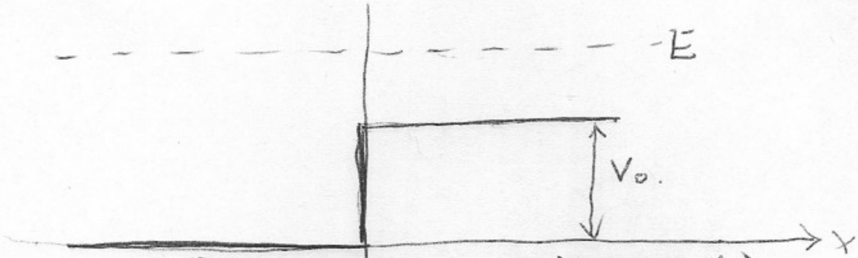
Ex 4.8

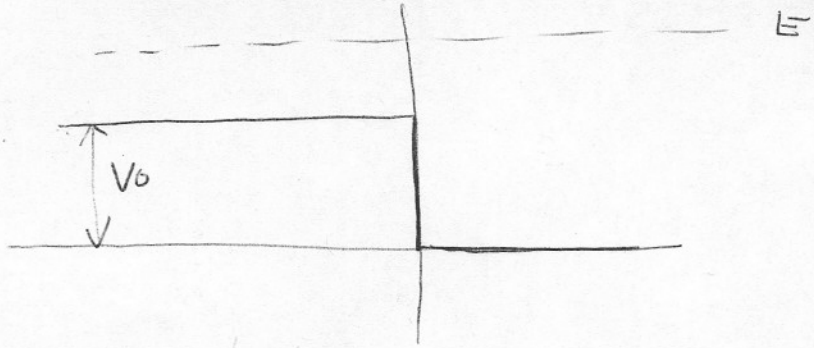
(a). From Textbook Eq. 4.27



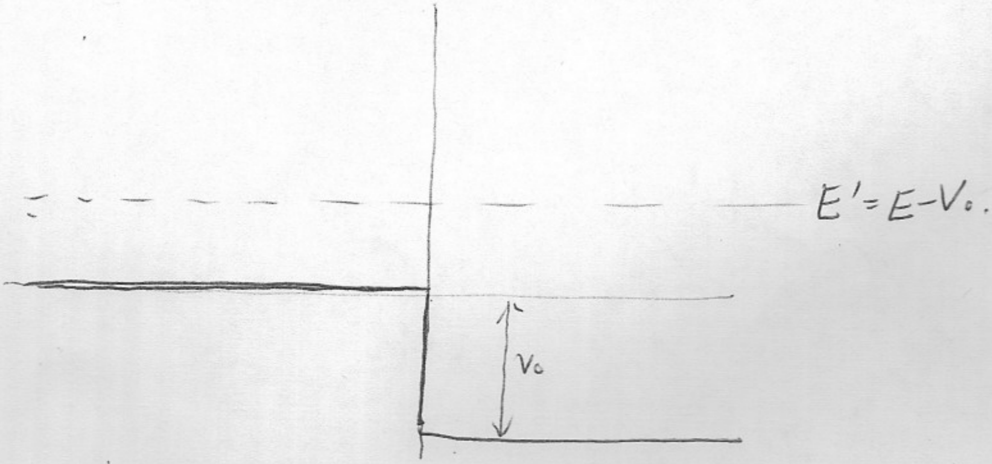
$k_1^2 = \frac{2mE}{\hbar^2}$ $\qquad$ $k_2^2 = \frac{2m(E-V_0)}{\hbar^2}$.

$$R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} \qquad T = \frac{4k_1k_2}{(k_1+k_2)^2}.$$

Now apply this result to the present problem:



which is the same as



$\cancel{k_2}$

$$k_1^2 = \frac{2mE'}{\hbar^2} = \frac{2m(E-V_0)}{\hbar^2} \qquad k_2^2 = \frac{2m(E'+V_0)}{\hbar^2} = \frac{2mE}{\hbar^2}.$$

With $V_0 = 6\,eV$ and $E = 8\,eV$.

$$R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \left(\frac{1 - \frac{k_2}{k_1}}{1 + \frac{k_2}{k_1}} \right)^2.$$

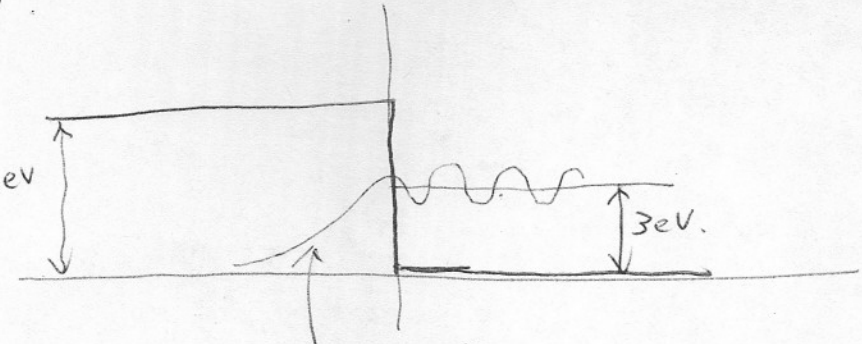
$$\frac{k_2}{k_1} = \sqrt{\frac{E}{E - V_0}} = \sqrt{\frac{8}{8-6}} = \sqrt{4} = 2.$$

$$R = \left(\frac{1-2}{1+2} \right)^2 = \frac{1}{9} \; //$$

$$T = \frac{4 k_1 k_2}{(k_1 + k_2)^2} = \frac{4 \cdot \frac{k_2}{k_1}}{\left(1 + \frac{k_2}{k_1}\right)^2} = \frac{4 \cdot 2}{(1+2)^2} = \frac{8}{9} \; //.$$

$R + T = 1$ as expected.

(b)



$\psi(x) \sim e^{+\kappa x}$.

penetration depth ~~te~~ $\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{\cancel{2k}\, 2mc^2 (V_0 - E)}{(\hbar c)^2}}.$

$$= \sqrt{\frac{2 \times 0.511 \times (6-3) \times 10^{-6}}{197.3^2}}$$

$$= \cancel{1.270 \times 10^{10}} \; 8.875 \times 10^{-6} \text{ fm}^{-1}$$

$\therefore$ penetration depth $= \frac{1}{\kappa} = 1.13 \times 10^5 \text{ fm} = \underline{\underline{1.13\ \text{Å}}}$

($\cancel{e}$ ii).

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2m(4E - E)}{\hbar^2}}$$

$$= \sqrt{\frac{6mE}{\hbar^2}}.$$

$$= \sqrt{\frac{6m \cdot \frac{1}{2}mv^2}{\hbar^2}}$$

$$= \sqrt{3}\,\frac{p}{\hbar}.$$

$$= \sqrt{3} \cdot \frac{(70)(4)}{1.055 \times 10^{-34}}$$

$$= 4.597 \times 10^{36} \ m^{-1}.$$

Penetration depth $= \frac{1}{\kappa} = 2.18 \times 10^{-37}\ m$ //