Ex 4.9

(a). Since $\psi(x,0)$ is normalized already.

$$\langle E\rangle = \langle\psi|H|\psi\rangle$$

$$= \left(\frac{5}{\sqrt{50}}\langle\phi_0| + \frac{4}{\sqrt{50}}\langle\phi_1| + \frac{3}{\sqrt{50}}\langle\phi_2|\right)\left(\hbar\omega(n+\tfrac{1}{2})\right)\left(\frac{5}{\sqrt{50}}|\phi_0\rangle + \frac{4}{\sqrt{50}}|\phi_1\rangle + \frac{3}{\sqrt{50}}|\phi_2\rangle\right).$$

$$= \frac{\hbar\omega}{2}\cdot\frac{25}{50} + \frac{3}{2}\hbar\omega\cdot\frac{16}{50} + \frac{5}{2}\hbar\omega\cdot\frac{9}{50}.$$

$$= \frac{\hbar\omega}{100}(25+48+45).$$

$$= \frac{118}{100}\hbar\omega$$

$$= \frac{59}{50}\hbar\omega \,//$$

(b) $$\psi(x,t) = \frac{5}{\sqrt{50}}\phi_0 e^{-i\frac{E_0}{\hbar}t} + \frac{4}{\sqrt{50}}\phi_1 e^{-i\frac{E_1}{\hbar}t} + \frac{3}{\sqrt{50}} e^{-i\frac{E_2}{\hbar}t}\phi_2$$

$$= \frac{5}{\sqrt{50}} e^{-\frac{1}{2}i\omega t}\phi_0 + \frac{4}{\sqrt{50}} e^{-\frac{3}{2}i\omega t}\phi_1 + \frac{3}{\sqrt{50}} e^{-\frac{5}{2}i\omega t}\phi_2.$$

$$\langle E\rangle = \langle\psi(x,t)|E|\psi(x,t)\rangle$$

$$= \left(\frac{5}{\sqrt{50}}\langle\phi_0| e^{\frac{1}{2}i\omega t} + \frac{4}{\sqrt{50}}\langle\phi_1| e^{\frac{3}{2}i\omega t} + \frac{3}{\sqrt{50}} e^{\frac{5}{2}i\omega t}\right|\left(\hbar\omega(n+\tfrac{1}{2})\right)\left(\frac{5}{\sqrt{50}} e^{-\frac{1}{2}i\omega t}|\phi_0\rangle + \frac{4}{\sqrt{50}} e^{-\frac{3}{2}i\omega t}|\phi_1\rangle + \frac{3}{\sqrt{50}} e^{-\frac{5}{2}i\omega t}|\phi_2\rangle\right)$$

$$= \frac{59}{50}\hbar\omega, \text{ same as (a)} \,//$$

Answer is obvious ~~a~~ as energy is a constant of motion.

(c).

$$\langle \psi(x,t) | \hat{x} | \psi(x,t) \rangle$$

$$= \frac{25}{50} \int_{-\infty}^{\infty} x \phi_0^2(x)\, dx + \frac{16}{50} \int_{-\infty}^{\infty} x \phi_1^2(x)\, dx + \frac{9}{50} \int x \phi_2^2(x)\, dx$$

$$= 0 + 0 + 0$$

$$= 0 //$$