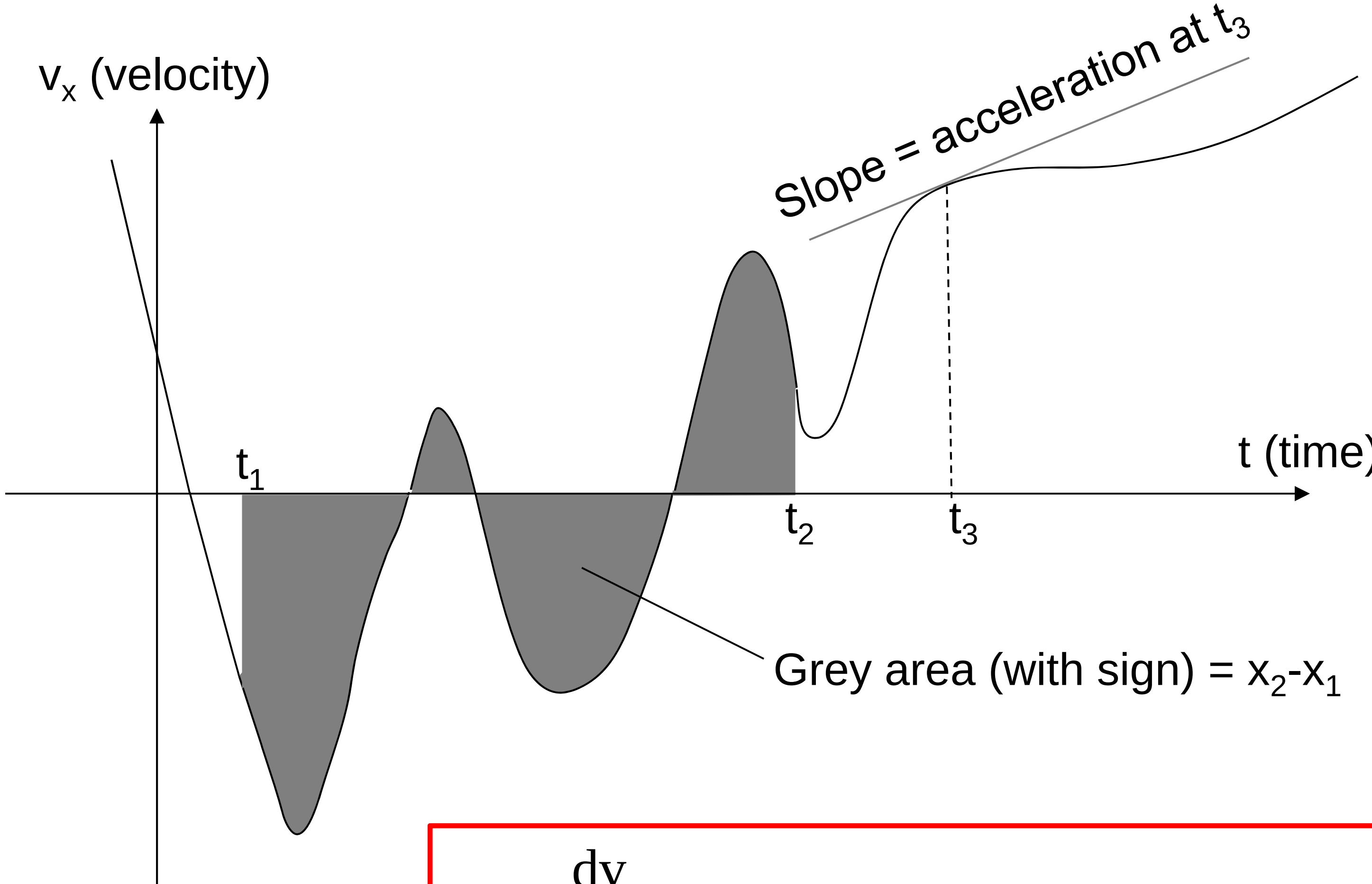
Class 4: Constant acceleration motion

Review

$$x\ (\text{m}) \quad \underset{\int dt}{\overset{\frac{d}{dt}}{\rightleftarrows}} \quad v\ (\text{m/s}) \quad \underset{\int dt}{\overset{\frac{d}{dt}}{\rightleftarrows}} \quad a\ (\text{m/s}^2)$$

v-t graph



$$a_x = \frac{dv_x}{dt} = \text{ slope of curve}$$

$$\Delta x = x_2 - x_1 = \int_{t_1}^{t_2} v_x dt = \text{ Area under curve}$$

Instantaneous acceleration (1D)

Instantaneous acceleration

$$= a_x$$

$$= \lim_{\Delta t \to 0} \text{Average acceleration} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$a_x = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt}$$

In mechanics, only instantaneous acceleration is important. Average acceleration is just introduced to define its instantaneous values. So from now on, by acceleration, we mean instantaneous acceleration.

Constant acceleration motion (1D)

$$a_x = \text{constant}$$

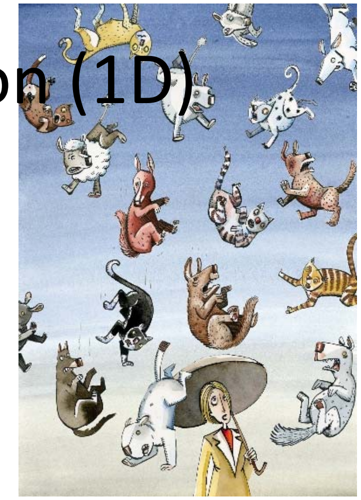
$$\Downarrow$$

$$v_{xf} = v_{xi} + a_x t$$

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

These two equations are enough to solve any problem of this type. Five variables are involved: x_f-x_i (=Δx), v_f, v_i, a, and t. So we can afford to solve for two variables as unknown with these two equations.

Free falling is a constant acceleration motion (1D)



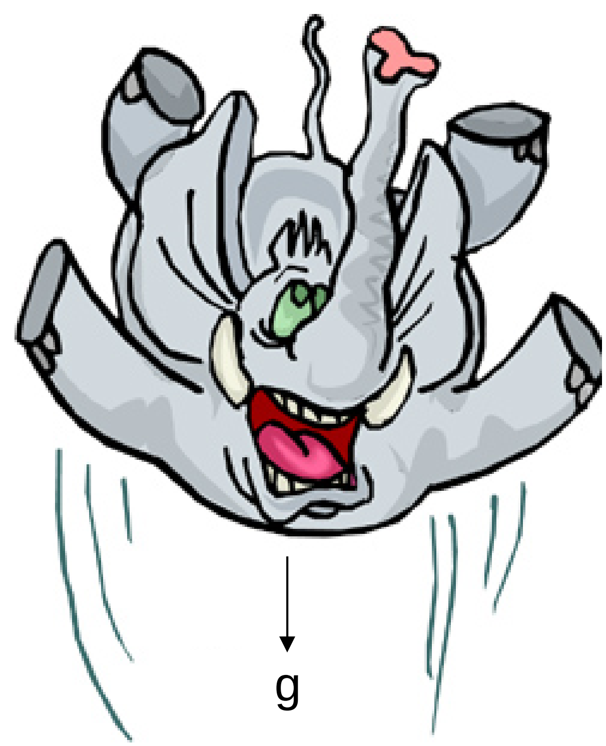
a_y = constant = 9.8 m/s^2 for any object

$$\Downarrow$$

$$v_{yf} = v_{yi} + a_y t$$

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

g = 9.8 m/s^2



Constant acceleration motion (1D)

Example problem (Serway & Jewette 9^{th} Ed. Prob. 2-67)

An elevator moves downward in a tall building at a constant speed of 5.00 m/s. Exactly 5.00s after the top of the elevator car passes a bolt loosely attached to the wall of the elevator shaft, the bolt falls from rest. (a) At what time des the bolt hit the top of the still descending elevator? (b) Estimate the highest floor from which the bolt can fall if the elevator reaches the ground floor before the bolt hits the top of the elevator.