Class 14: Capacitance

Capacitance

Capacitance is defined between *two conductors*, with equal magnitude but opposite charges:

$$Q \propto \Delta V$$

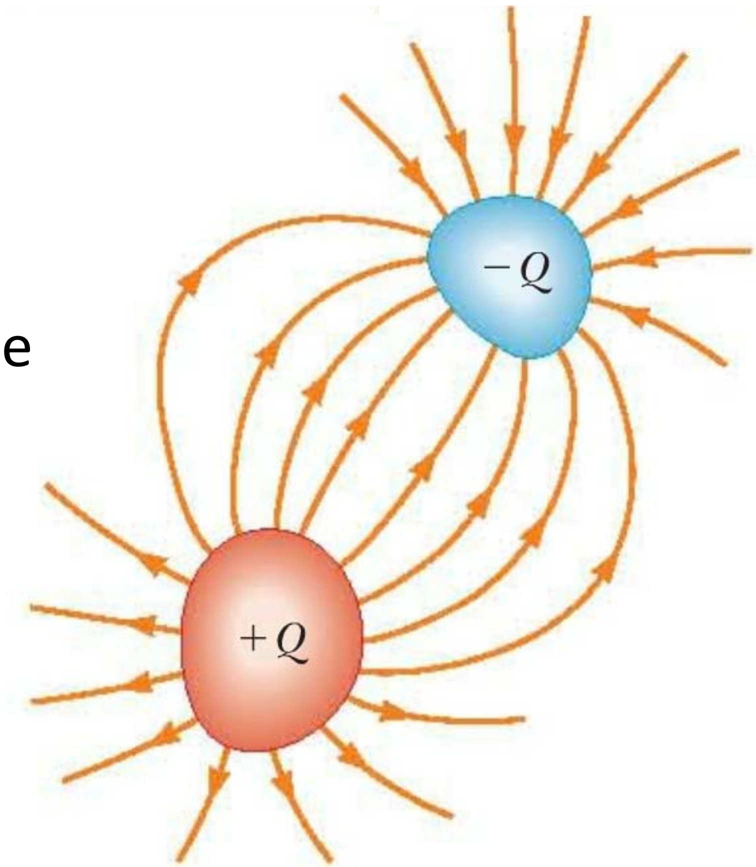
Capacitace is the charge needed to raise the potential difference by 1V:

$$C = \frac{Q}{\Delta V}$$

Very often the conductor at the lower potential is defined as the zero potential, then

$$C = \frac{Q}{V}$$

V is the potential of the other conductor.

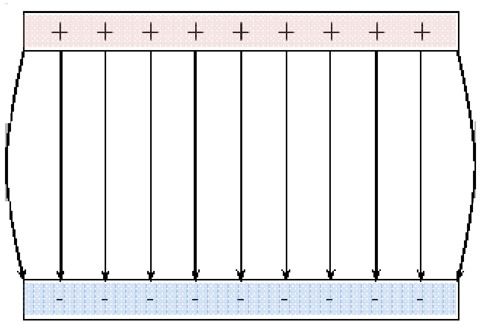


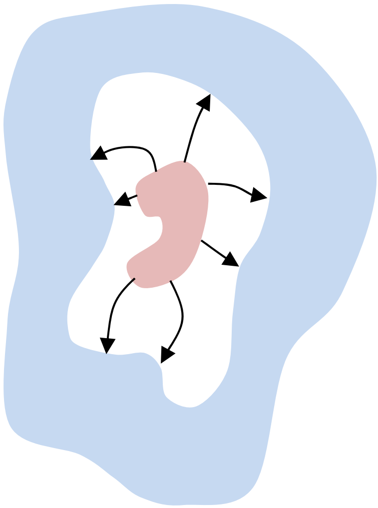
Units for capacitance:
Farad (F) $\equiv$ C/V

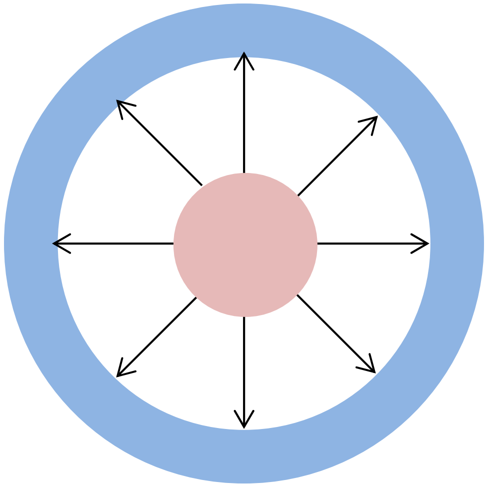
Special case

If all the field lines from one conductor ends at the other, then we just need to vary the charge in one conductor and the charge of the other conductor will follow (by induction).

Examples of this type of configuration:

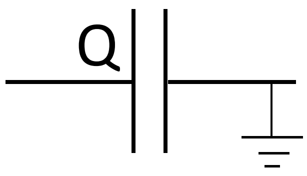






Calculate the Capacitance of Two Conductors

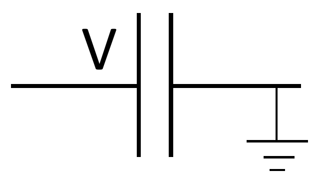
Step 1. Fix voltage of one conductor at 0 and put some charges Q to the other conductor.



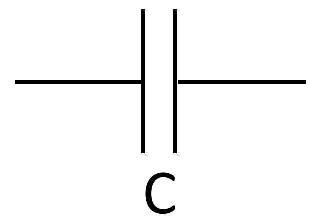
Step 2. Calculate the electric field due to such a charge configuration.

Step 3. From the electric field calculate the electric potential at the other conductor.

$$V = -\int_{0}^{\text{Conductor}} \vec{E}(\vec{r}) \cdot d\vec{r}$$



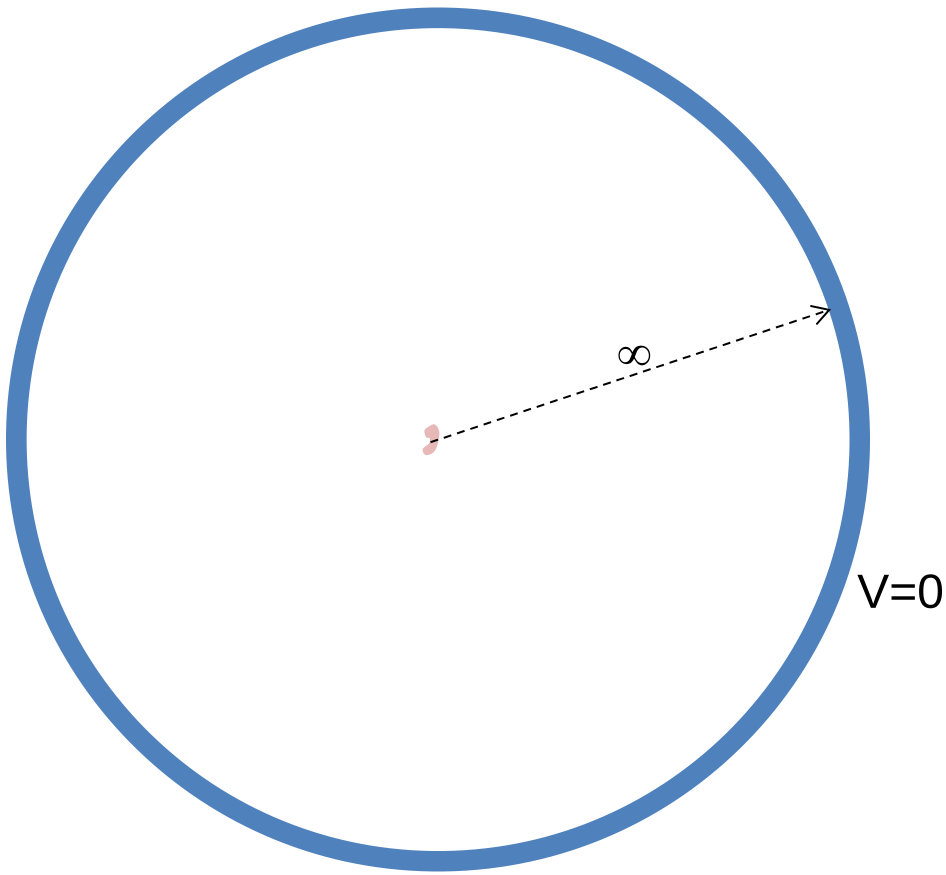
Symbol of a capacitor:



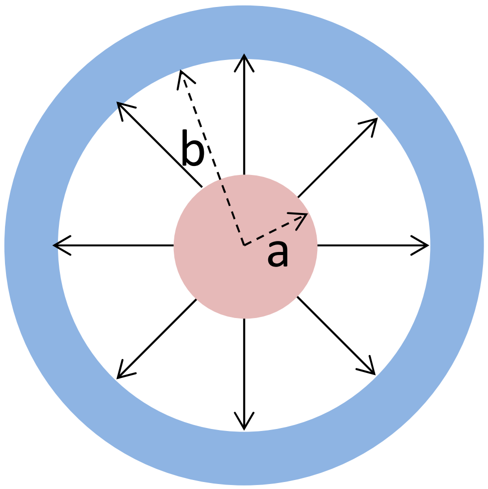
Step 4. C = Q/V

Confusion

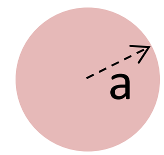
Sometimes we talk about the capacitance of a single conductor (e.g. the Earth). In this case we can imagine the conductor is inside a big cavity of an infinite large conductor at zero potential.



Spherical capacitor



$$b \rightarrow \infty$$



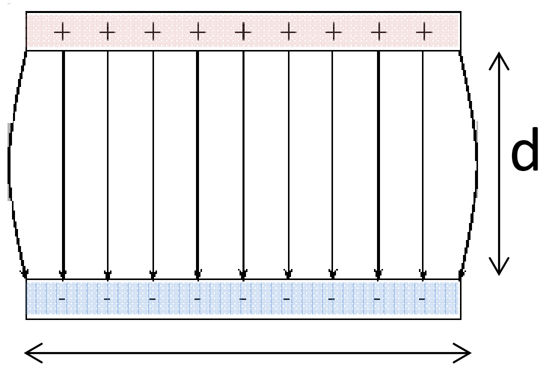
$$C = \frac{4\pi\varepsilon_0 ab}{b - a}$$

or $$\frac{1}{C} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Capacitance of a single conductance sphere:

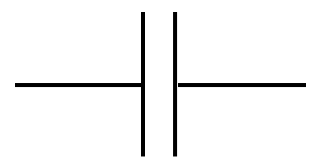
$$C = 4\pi\varepsilon_0 a \quad (C \propto a)$$

Parallel plate capacitor



$$C = \frac{\varepsilon_0 A}{d}$$

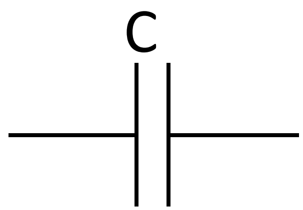
Symbol of a capacitor:



$$V = E\,d$$

Energy Stored in a Capacitor

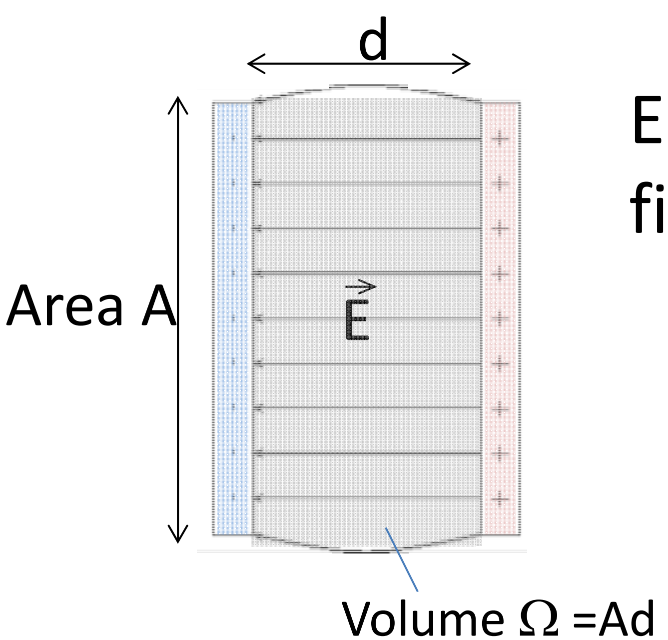
This is actually the energy you need to install the charges into the capacitor.



Energy stored in a charged capacitor:

$$U = \frac{1}{2} C V^2$$

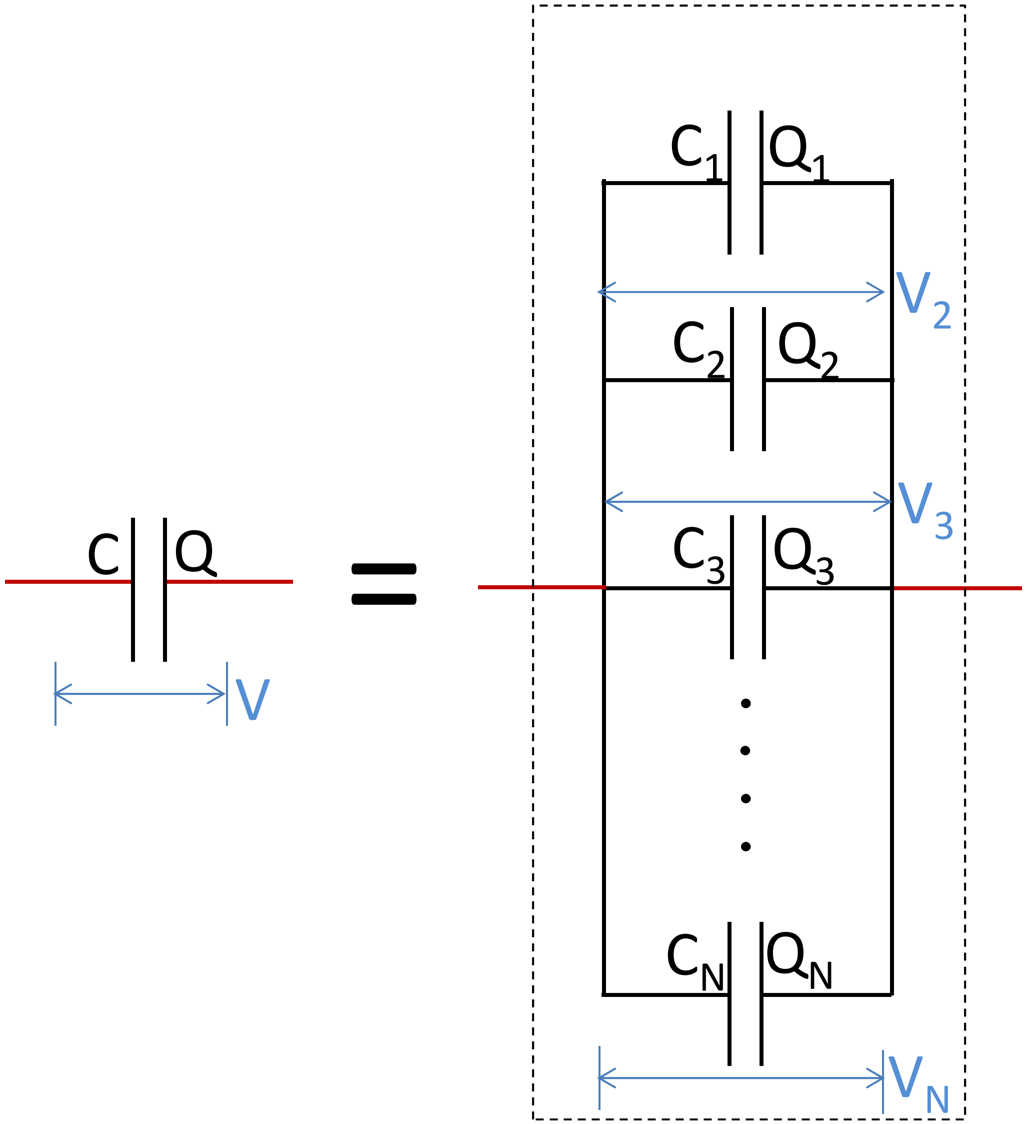
(Do not forget $C = \frac{Q}{V}$.)



Energy density stored in an electric field:

$$u_E = \frac{U}{\Omega} = \frac{1}{2} \varepsilon_0 E^2$$

Connecting Capacitors in Parallel



$$C = C_1 + C_2 + C_3 + \cdots C_N$$
$$Q = Q_1 + Q_2 + Q_3 + \cdots Q_N$$
$$V = V_1 = V_2 = V_3 = \cdots V_N$$

1. Potential difference across each individual capacitor is the same: (why?)

$$V = V_1 = V_2 = V_3 = \cdots V_N$$
$$\Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \cdots \frac{Q_N}{C_N}$$

2. Charge stored in each individual capacitor should be different (unless they have the same capacitance).