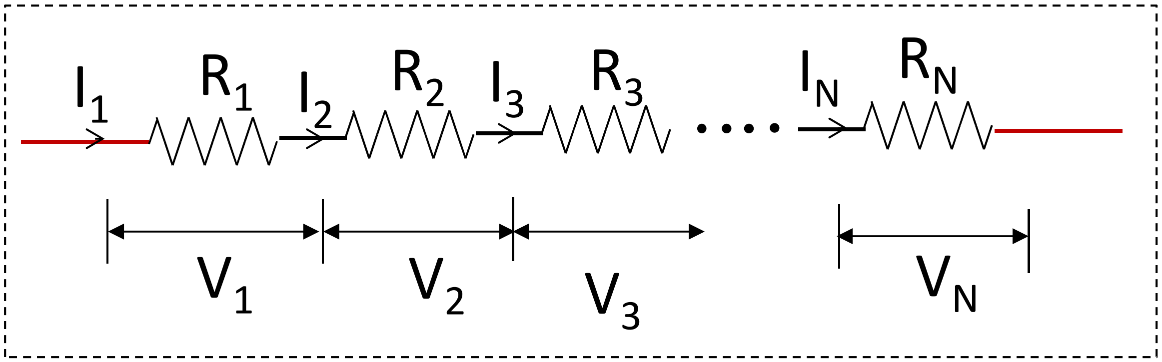
Class 19 Kirchhoff’s Rule

Connecting Resistors in Series



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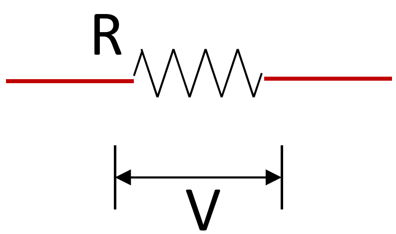


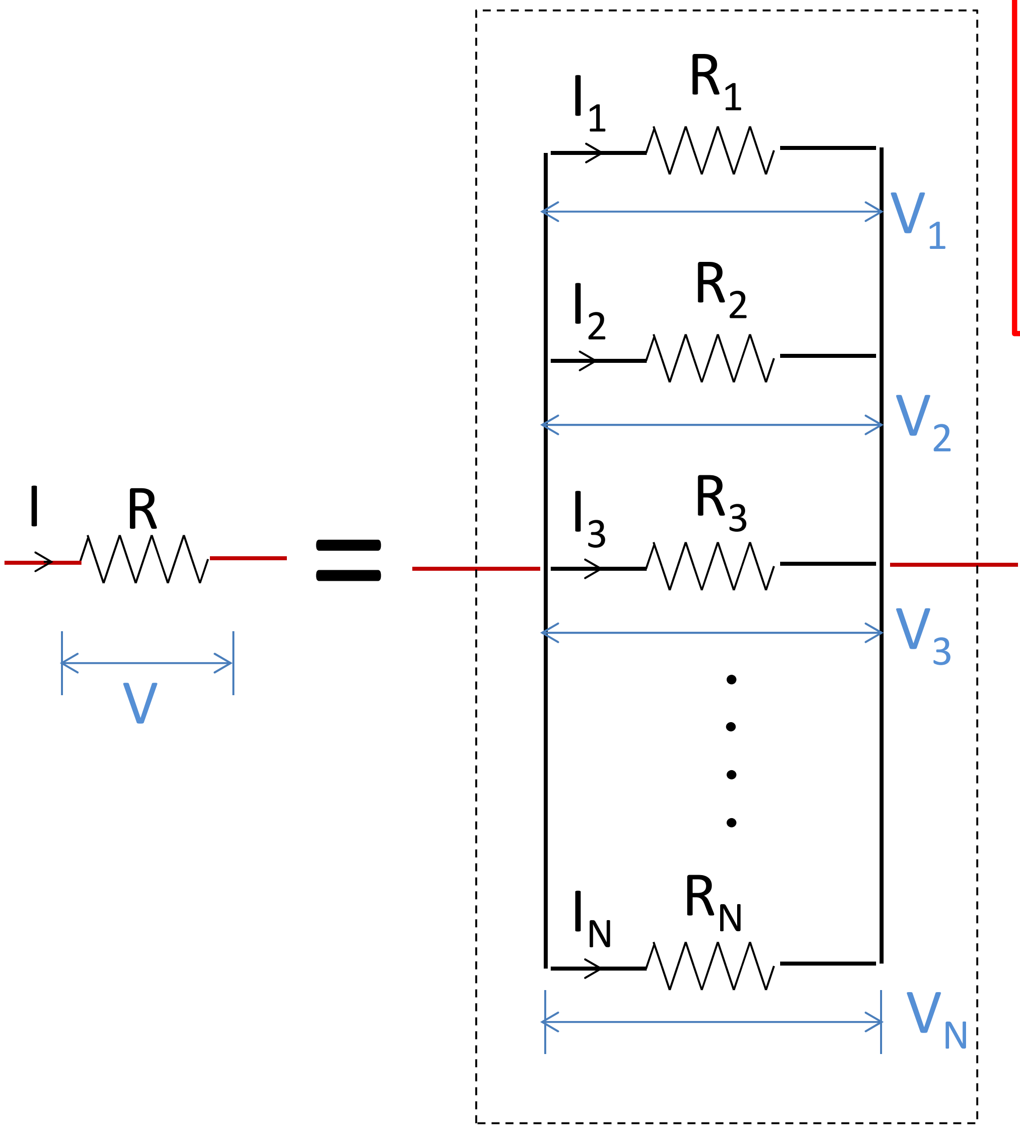
$$R_{eff} = R_1 + R_2 + R_3 + \cdots + R_N$$
$$I = I_1 = I_2 = I_3 = \cdots = I_N$$
$$V = V_1 + V_2 + V_3 + \cdots + V_N$$

1. Current through each individual resistor is the same: (why?)

$$I = I_1 = I_2 = I_3 = \cdots = I_N$$
$$\Rightarrow \frac{V}{R} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3} = \cdots = \frac{V_N}{R_N}$$

2. Potential difference across each individual resistor should be different (unless they have the same resistance).

Connecting Resistors in Parallel



$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N}$$

$$I = I_1 + I_2 + I_3 + \cdots I_N$$

$$V = V_1 = V_2 = V_3 = \cdots V_N$$

1. Potential difference across each individual resistor is the same: (why?)

$$V = V_1 = V_2 = V_3 = \cdots V_N$$

$$\Rightarrow I_1R_1 = I_2R_2 = I_3R_3 = \cdots I_NR_N$$

2. Current through each individual resistor should be different (unless they have the same resistance).

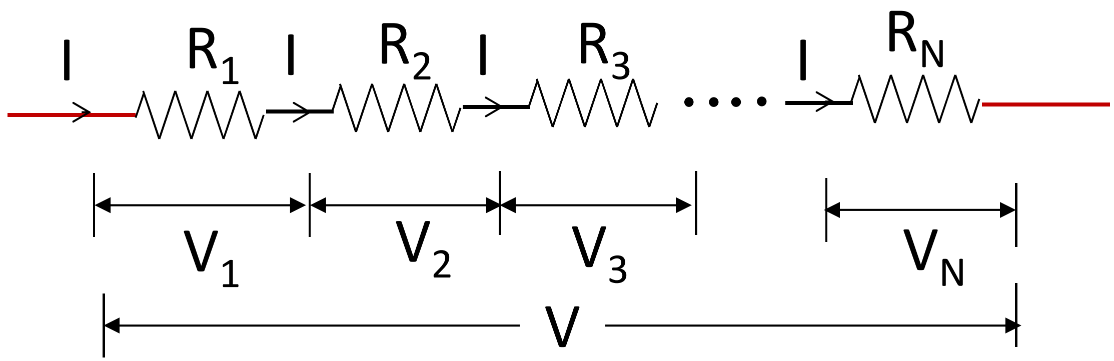
Why so similar, but yet so opposite?

The formula for capacitors and resistors in parallel and series are oppositely switched:

	Resistors	Capacitors
In series	$R_{eff} = R_1 + R_2 + \cdots R_N$	$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}$
In parallel	$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N}$	$C_{eff} = C_1 + C_2 + \cdots C_N$

$$R = \frac{V}{I} \qquad C = \frac{Q}{V}$$

Voltage Divider



Current through each individual resistor is the same

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \cdots \frac{V_N}{R_N} = \frac{V}{R_{eff}}$$

or $$V_1 = V \cdot \frac{R_1}{R_1 + R_2 + \cdots R_N}$$

and similar for other resistors

Series resistors divide and share the voltage "proportionally".
(Larger R get the bigger share)

About Kirchhoff's Rules

1. There are two Kirchhoff rules – Kirchhoff's current rule (junction rule) and Kirchhoff's voltage rule (loop rule).

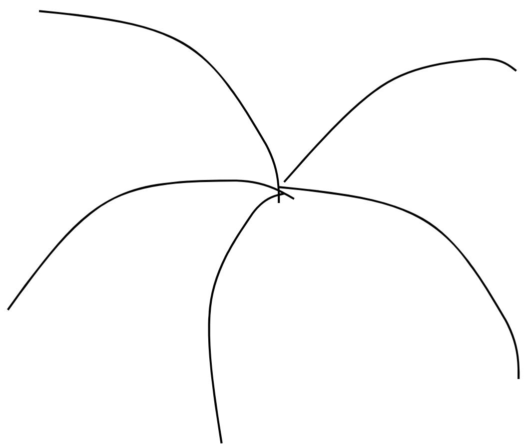
2. More complex circuits cannot be broken down into parallel and series units. The two Kirchhoff's rules allow us to solve for any circuits, no matter how complex it is (at least theoretically).

Kirchhoff’s Current Rule (Junction Rule)

Kirchhoff’s current rule is a result of conservation of charges:

At any junction, the sum of currents must equal zero.

$$\sum_{\text{junction}} I = 0$$



Junction formed by twisting many wires together.

Kirchhoff's Voltage Rule

Kirchhoff's voltage rule is a result of conservation of energy:

The sum of the potential differences across all elements around any closed circuit loop must be zero.

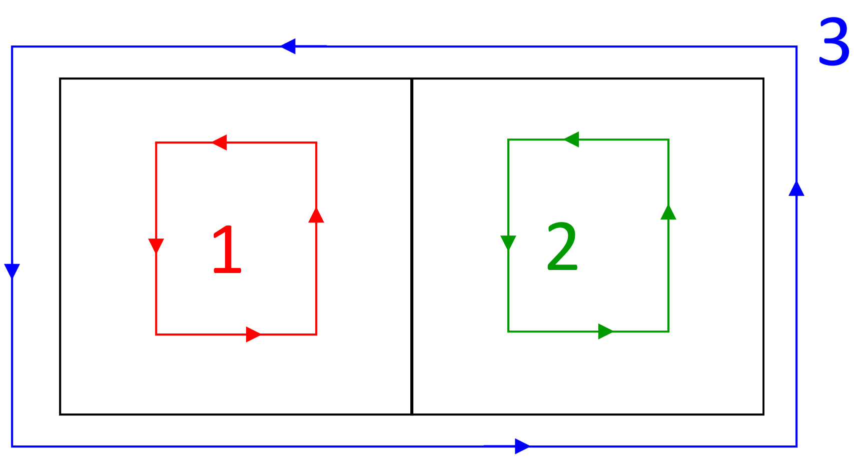
$$\sum_{\text{closed loop}} \Delta V = 0$$

Conservation of energy:
Work done by battery = Energy stored in conservative elements (e.g. capacitors) + Energy dissipated by resistive elements (e.g. resistors)

Dependence in Kirchhoff's Loop Equations

You can write down Kirchhoff's voltage equations for any closed loop you have chosen. However, the Kirchhoff's voltage equations you have written may be dependent of each other.

Example:



You can write Kirchhoff's voltage equations either for loops 1 and 2, or loops 1 and 3, or loops 2 and 3, but not 1, 2, and 3 because these three equations are dependent of each other.