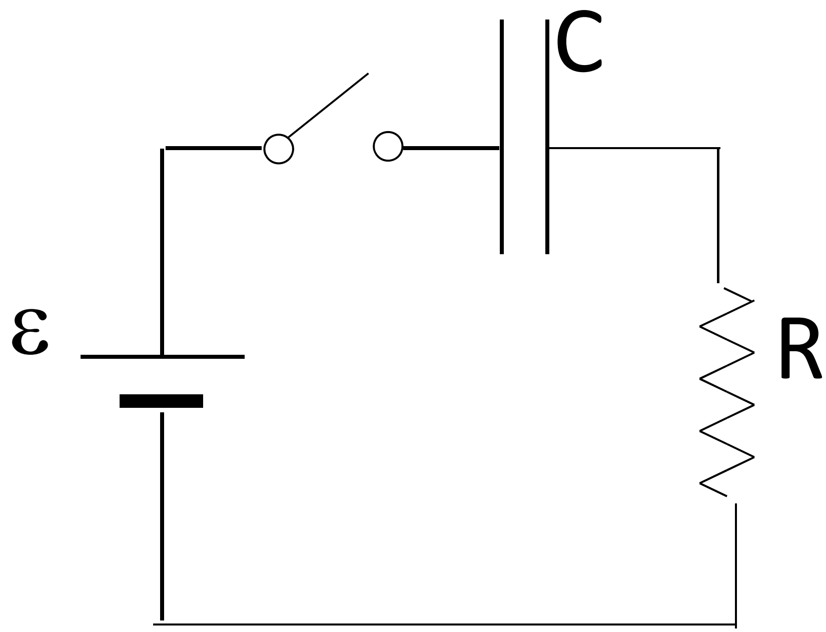
Class 21 RC Circuits

RC Circuits – Charging

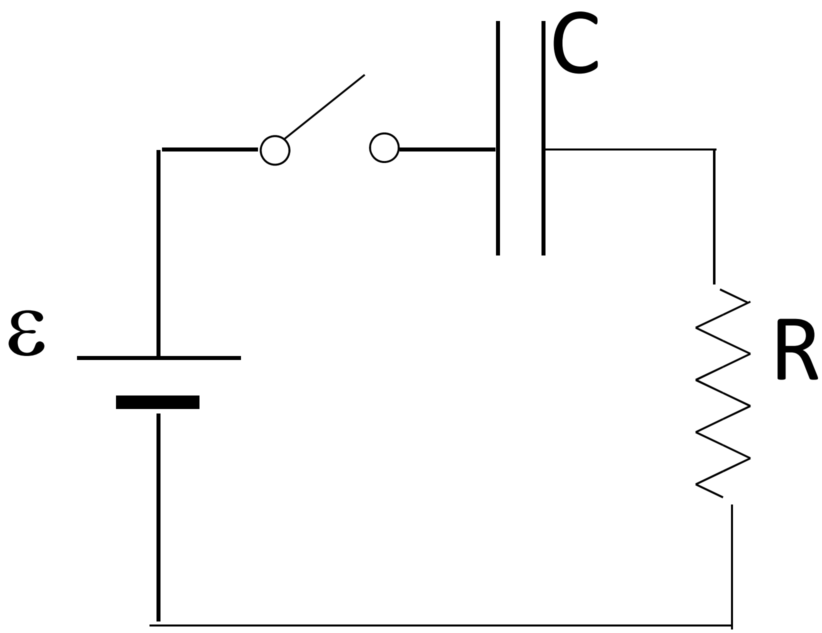


At t=0, capacitance is uncharged and Q=0 (initial condition).

At t=0, switched is closed, it the capacitor has no charge, it behaves like a conductor and $I=\varepsilon/R$.

After the capacitor is completely charged, $Q=C\,\varepsilon$, $\Delta V_C= \varepsilon$ and $\Delta V_R=0$. $I=0$ and the capacitors behave like an insulator.

RC Circuits – Charging



$$\varepsilon = \frac{q}{C} + IR \;\Rightarrow\; \frac{q}{C} + R\frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR\,dq = (C\varepsilon - q)\,dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR}\,dt$$

Integration constant

$$\Rightarrow \ell n(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + K\,e^{-\frac{t}{CR}}$$

$$\text{At } t = 0,\; q = 0 \;\Rightarrow\; 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore\; q = \underline{\underline{C\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR}e^{-\frac{t}{CR}} = \underline{\underline{\frac{\varepsilon}{R}e^{-\frac{t}{CR}}}}$$

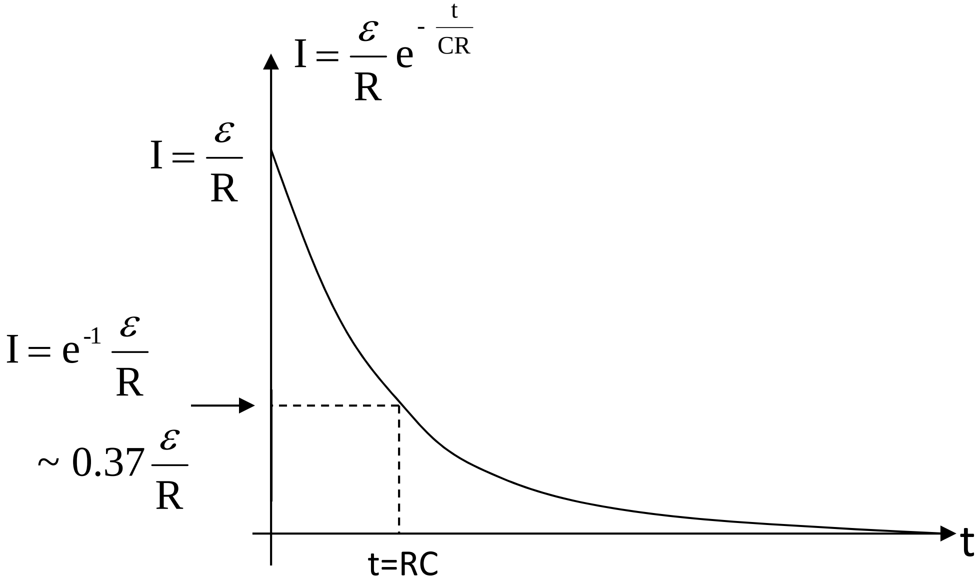
$$\Delta V_R = IR = \underline{\underline{\varepsilon\, e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\varepsilon\,(1 - e^{-\frac{t}{CR}})}}$$

$\Delta V_R + \Delta V_C = \varepsilon$

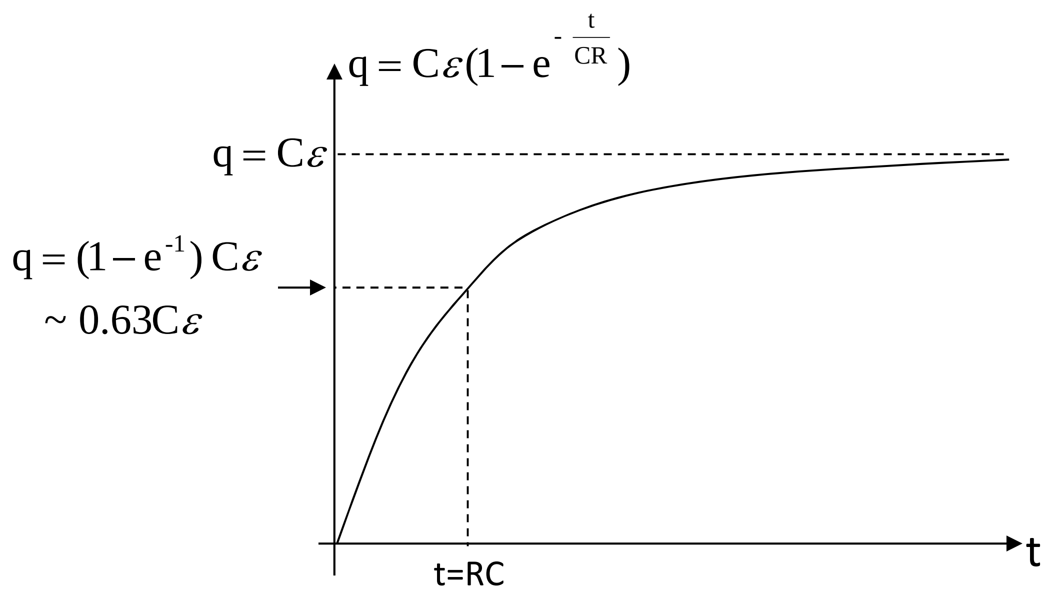
RC time constant

τ=RC is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.



$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

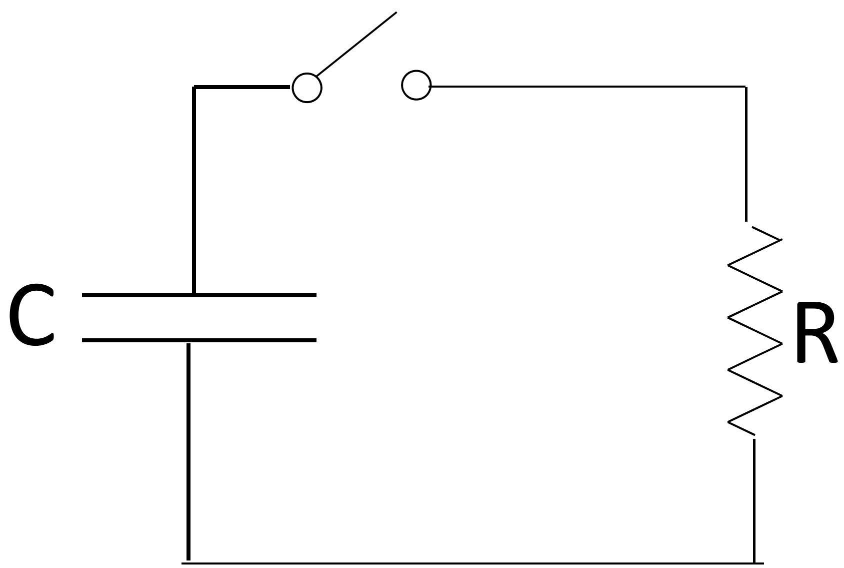


$$\sqrt{2} \approx 1.414$$

$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits

RC Circuits – Discharging

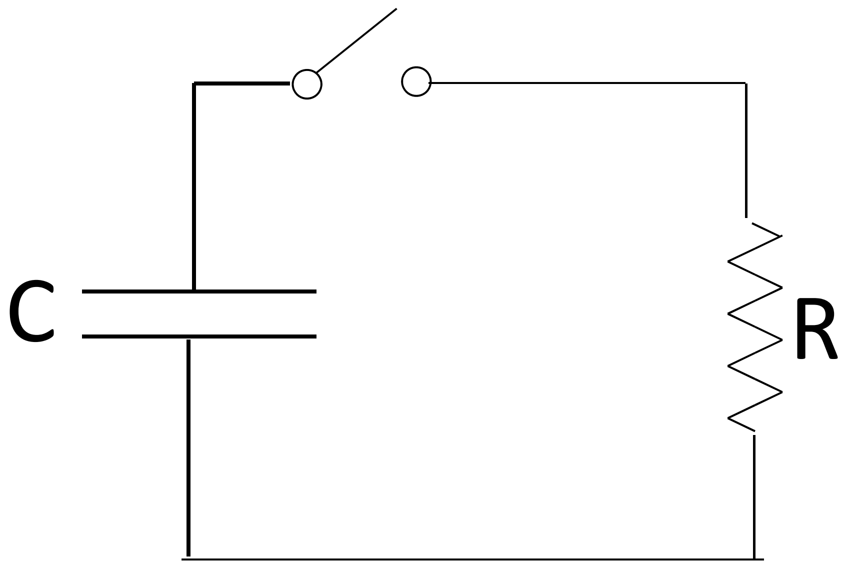


At t=0, capacitance is charged with a charge Q (initial condition).

At t=0, switched is closed, the capacitor starts to discharge.

After the capacitor is completely discharged, Q=0, ΔV_C= 0, ΔV_R=0 and I=0.

RC Circuits – Discharging



$$0 = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = 0$$

$$\Rightarrow CR\,dq = -q\,dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n\, q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = K\,e^{-\frac{t}{CR}}$$

$$\text{At } t = 0,\ q = Q \Rightarrow Q = K$$

$$\therefore\ q = \underline{\underline{Qe^{-\frac{t}{CR}}}}$$

$$I = \frac{dq}{dt} = \underline{\underline{-\frac{Q}{RC}e^{-\frac{t}{CR}}}}$$

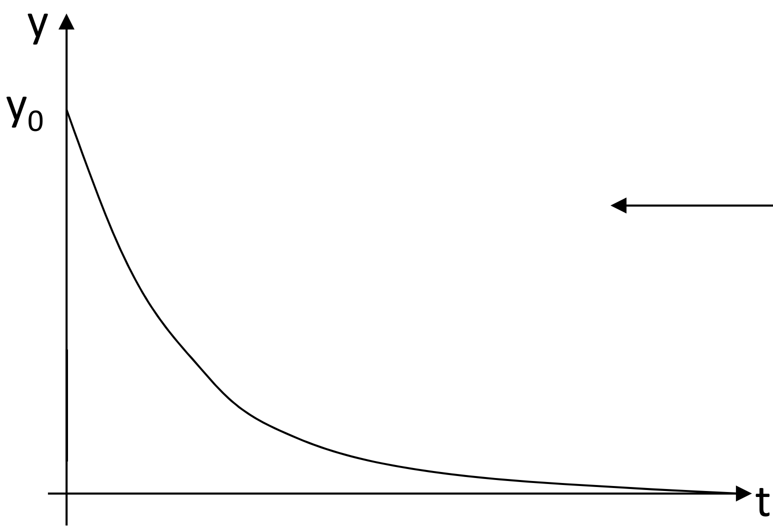
$$\Delta V_R = IR = \underline{\underline{-\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$\Delta V_R + \Delta V_C = 0$

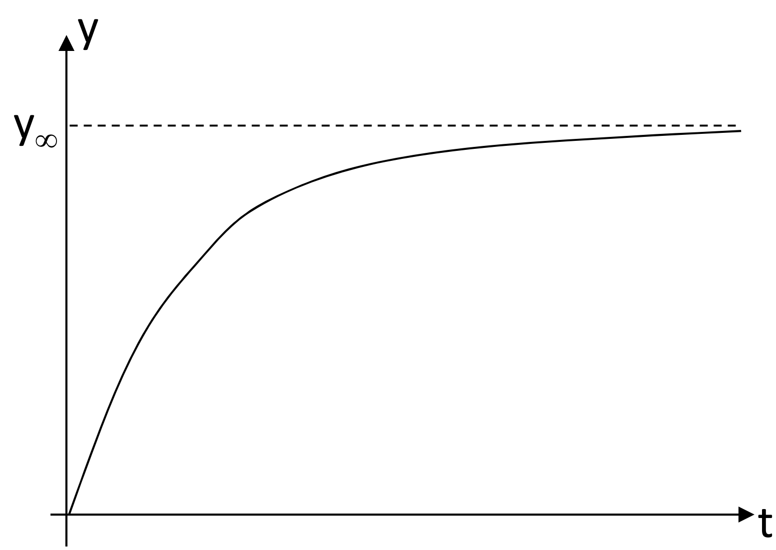
In Summary

For both charge and discharge, Q, I, ΔV_C, and ΔV_R must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

$$y = y_\infty (1 - e^{-\frac{t}{RC}})$$



y can be Q, I, ΔV_C, or ΔV_R