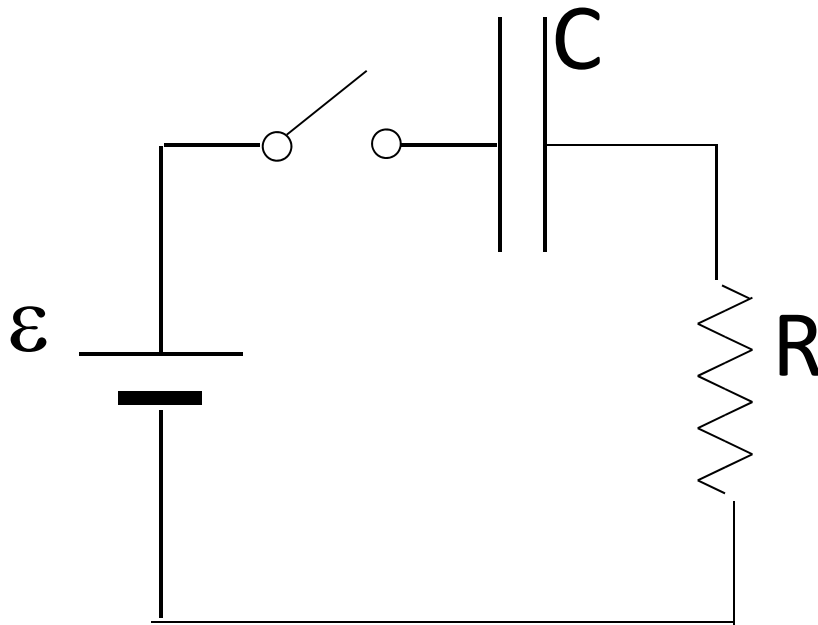


Class 22 More RC Circuits

RC Circuits – Charging



$$\varepsilon = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R \frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR dq = (C\varepsilon - q) dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR} dt \quad \text{Integration constant}$$

$$\Rightarrow \ln(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \quad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + Ke^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = 0 \Rightarrow 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore q = \underline{\underline{C\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR} e^{-\frac{t}{CR}} = \underline{\underline{\frac{\varepsilon}{R} e^{-\frac{t}{CR}}}}$$

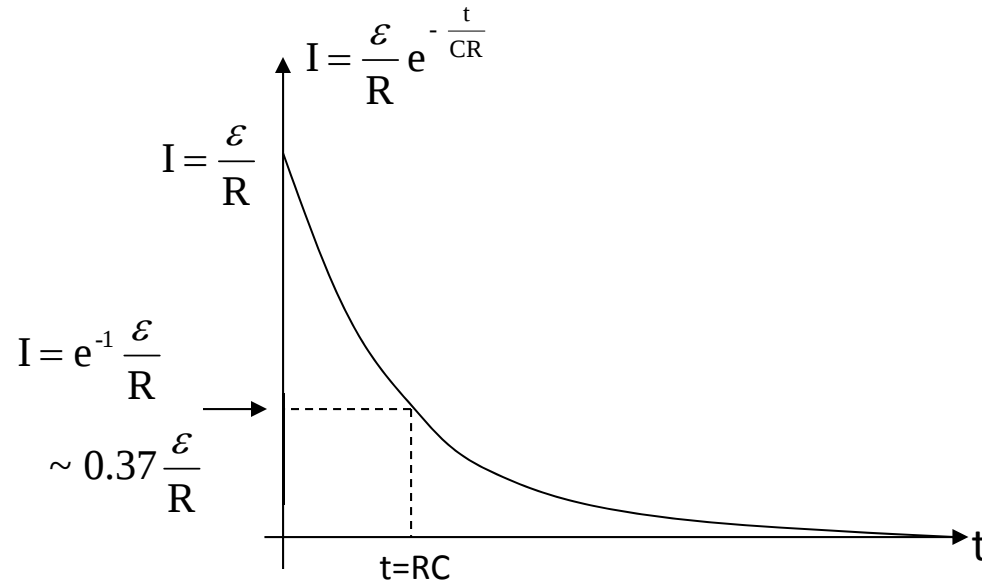
$$\Delta V_R = IR = \underline{\underline{\varepsilon e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$\left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \Delta V_R + \Delta V_C = \varepsilon$$

RC time constant

$\tau=RC$ is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.

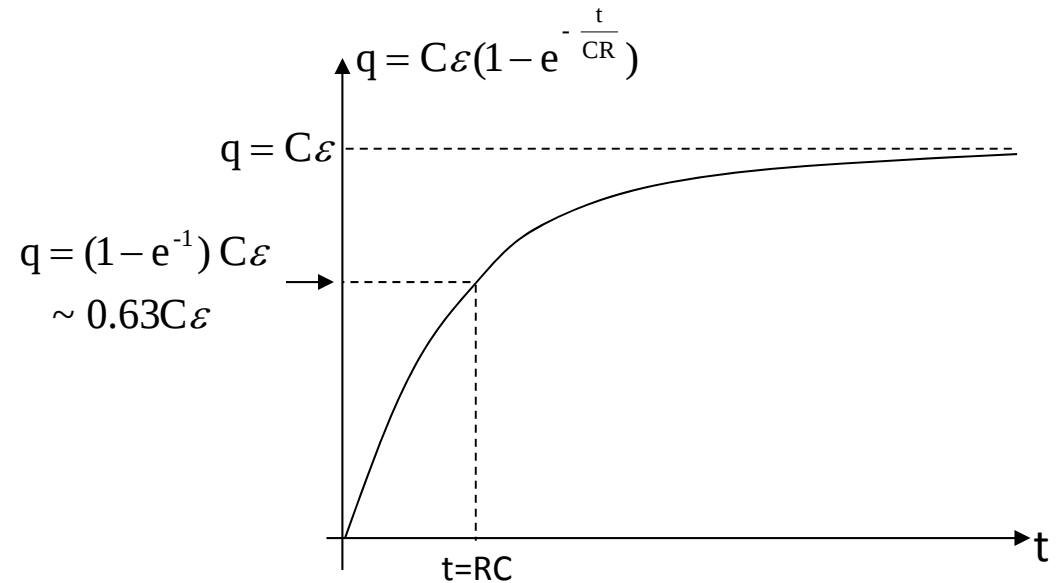


$$e \approx 2.72$$

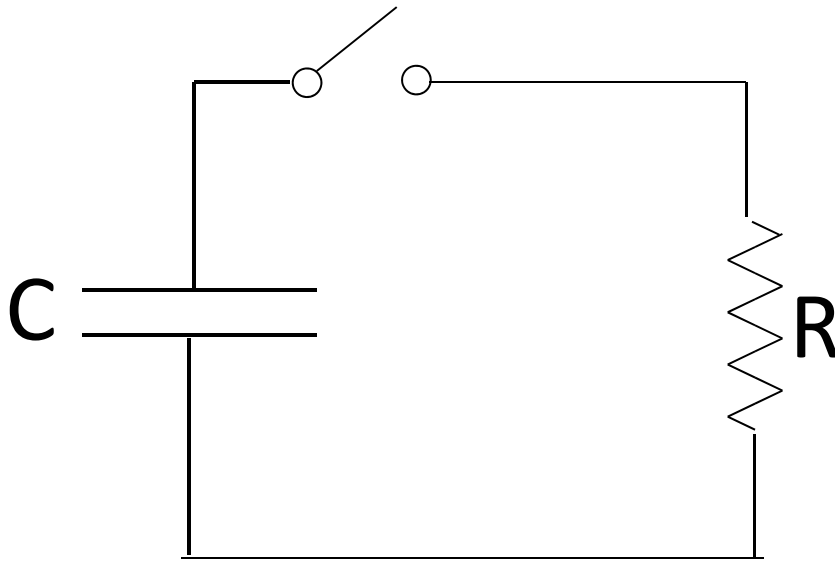
$$e^{-1} \approx 0.37$$

$$\sqrt{2} \approx 1.414$$
$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits



RC Circuits – Discharging



$$0 = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R \frac{dq}{dt} = 0$$

$$\Rightarrow CR \, dq = -q \, dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR} dt$$

Integration constant

$$\Rightarrow \ln q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = K e^{-\frac{t}{CR}} \quad (K = e^{K'})$$

$$\Rightarrow q = K e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = Q \Rightarrow Q = K$$

$$\therefore q = \underline{\underline{Q e^{-\frac{t}{CR}}}}$$

$$I = \frac{dq}{dt} = -\frac{Q}{RC} e^{-\frac{t}{CR}}$$

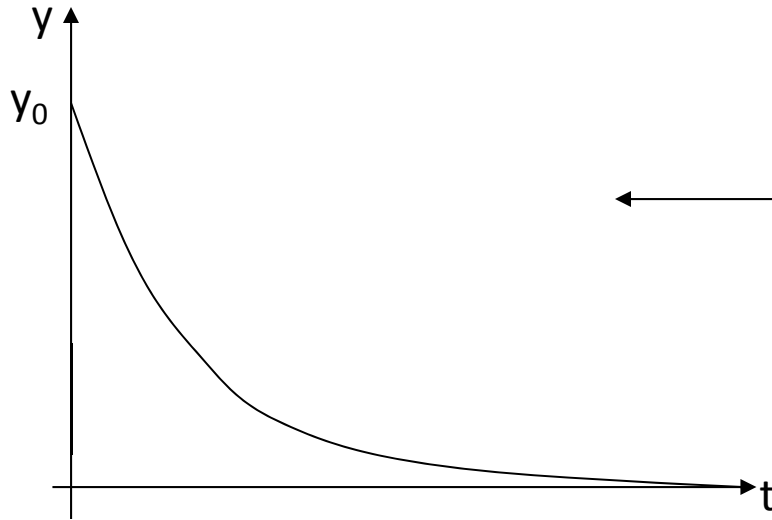
$$\Delta V_R = IR = -\frac{Q}{C} e^{-\frac{t}{CR}}$$

$$\Delta V_C = \frac{q}{C} = \frac{Q}{C} e^{-\frac{t}{CR}}$$

$$\left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \Delta V_R + \Delta V_C = 0$$

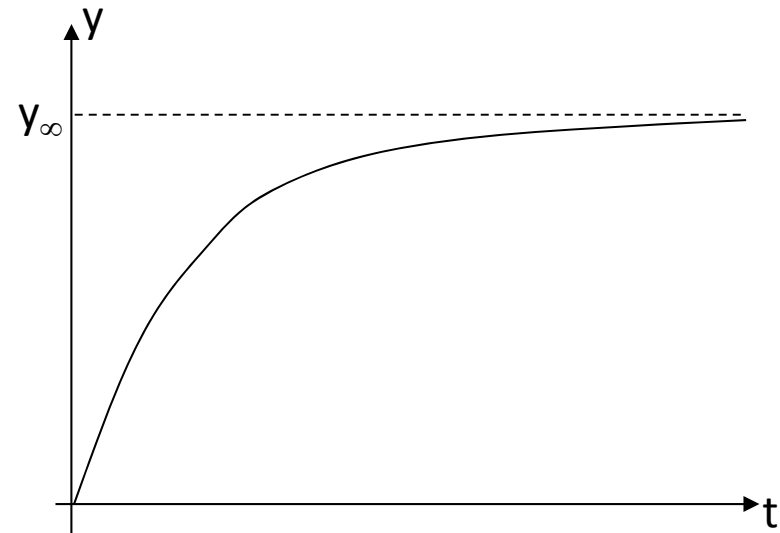
In Summary

For both charge and discharge, Q , I , ΔV_C , and ΔV_R must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

$$y = y_\infty (1 - e^{-\frac{t}{RC}})$$



y can be Q , I , ΔV_C , or ΔV_R