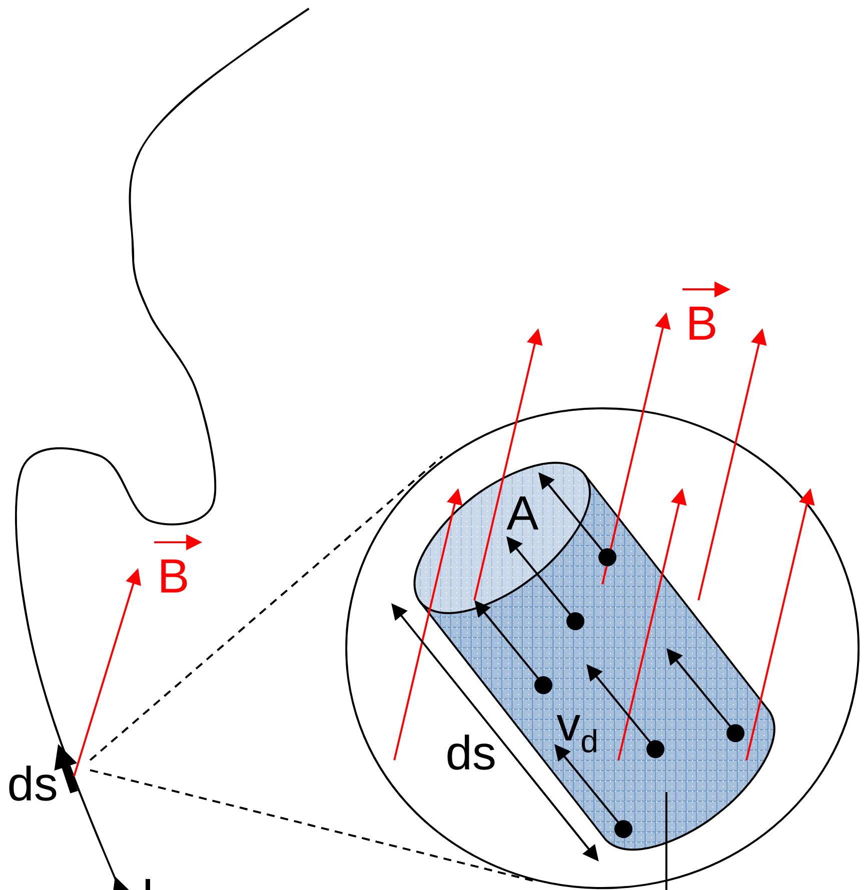
Class 27 Magnetic Torque and Hall Effect

Magnetic Force Acting on a Current



Force acting on one electron (note that in a current electron is considered as positive in charge):

$$\vec{F}_B = q\vec{v} \times \vec{B} = e\,\vec{v}_d \times \vec{B}$$

Force acting on the infinitesimal element:

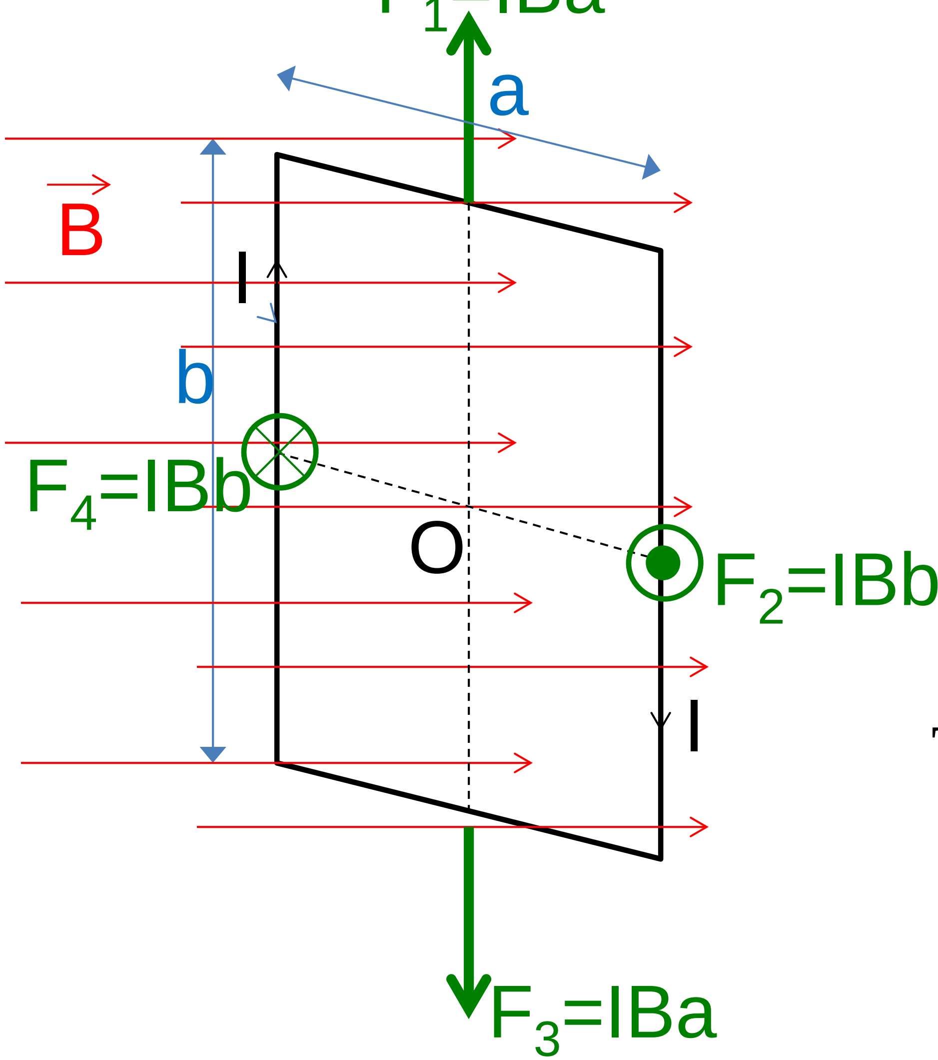
$$d\vec{F}_B = (e\,\vec{v}_d \times \vec{B})(nAds)$$

$$\Rightarrow \boxed{d\vec{F}_B = I\,d\vec{s} \times \vec{B}}$$

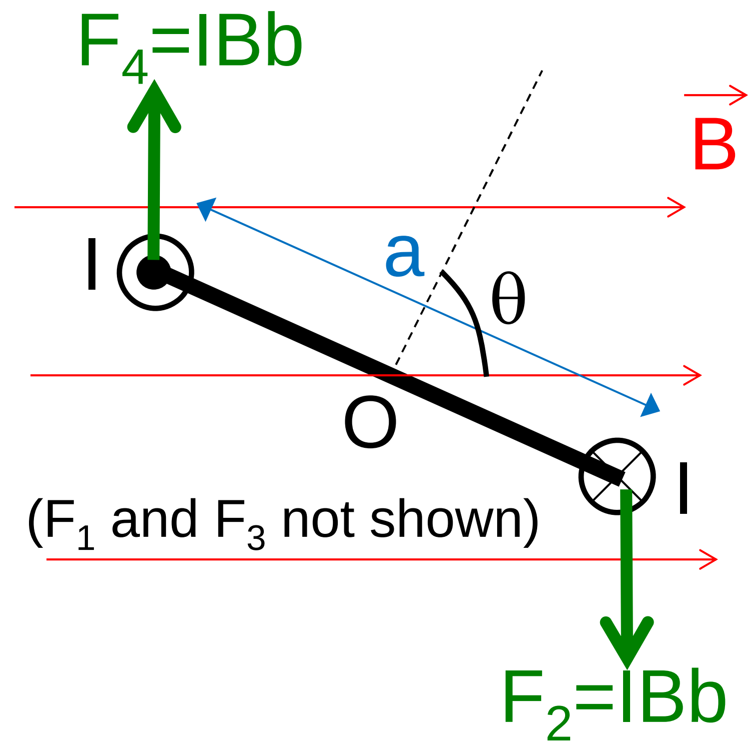
Force on the whole wire:

$$\vec{F}_B = \int_{\text{wire}} d\vec{F}_B = I \int_{\text{wire}} d\vec{s} \times \vec{B}$$

Magnetic Force on a Rectangular Loop



Top view

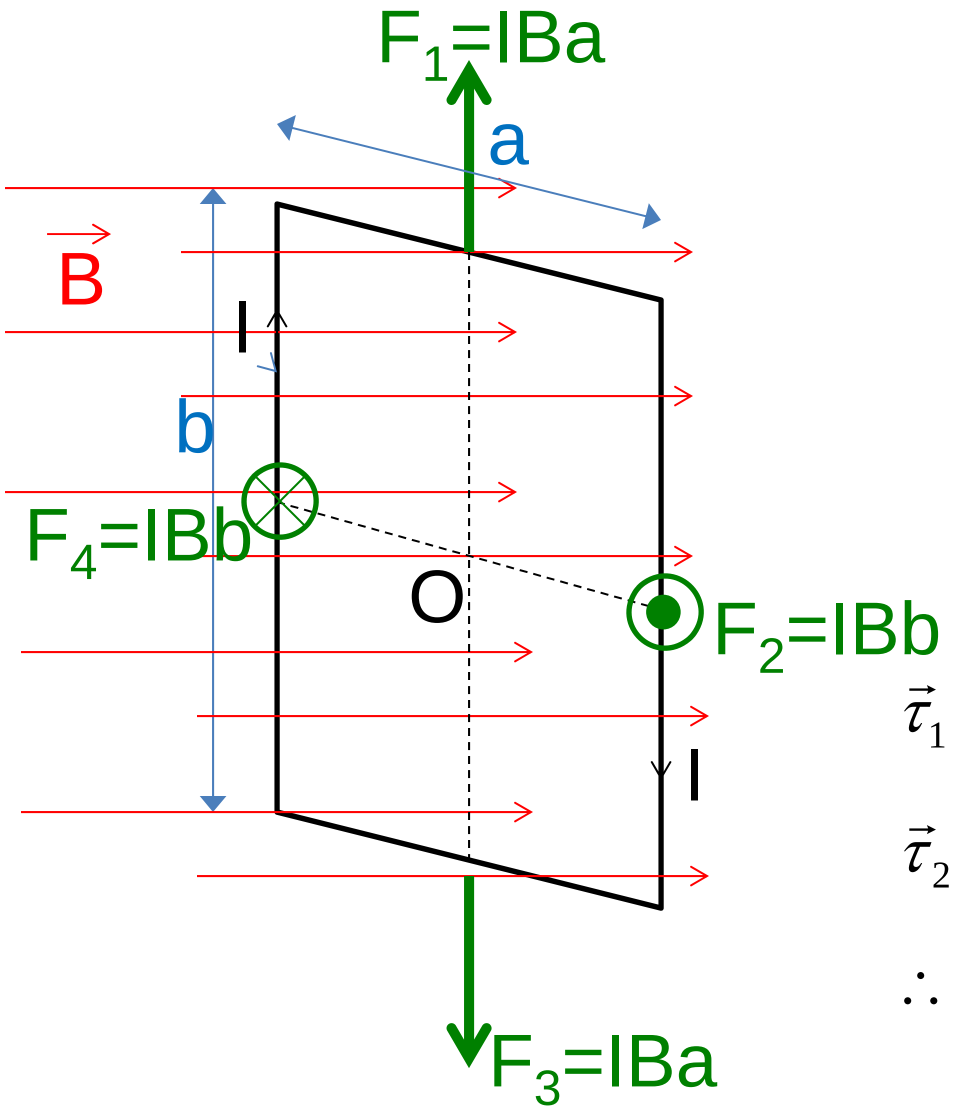


Total force acting on the loop

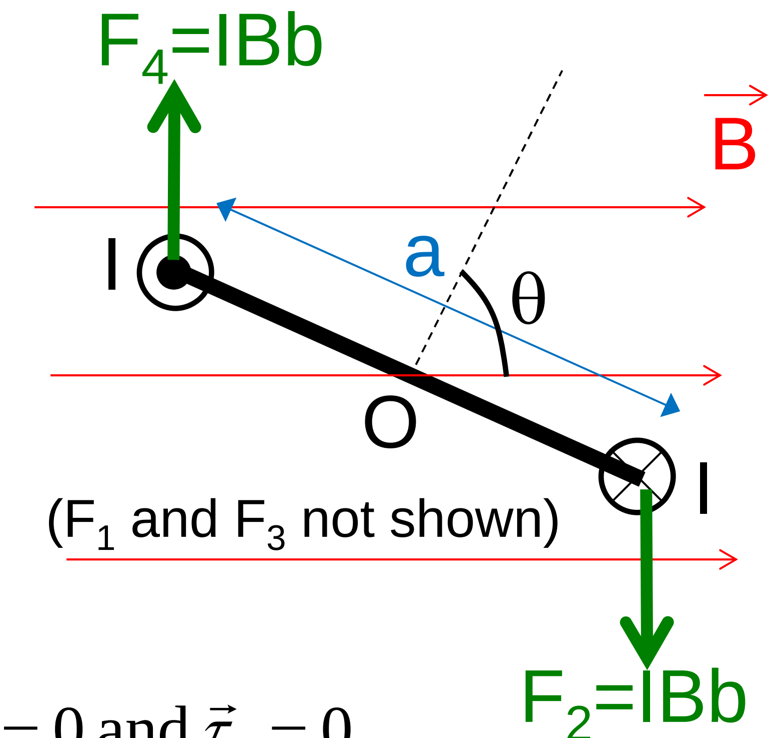
$$= \underbrace{\vec{F}_1 + \vec{F}_3}_{=0} + \underbrace{\vec{F}_2 + \vec{F}_4}_{=0}$$

$$= 0$$

Magnetic torque on a Rectangular Loop



Top view



$$\vec{\tau}_1 = 0 \text{ and } \vec{\tau}_3 = 0$$

$$\vec{\tau}_2 = \vec{\tau}_4 = \text{IBb} \cdot \frac{a}{2} \cos\theta \otimes$$

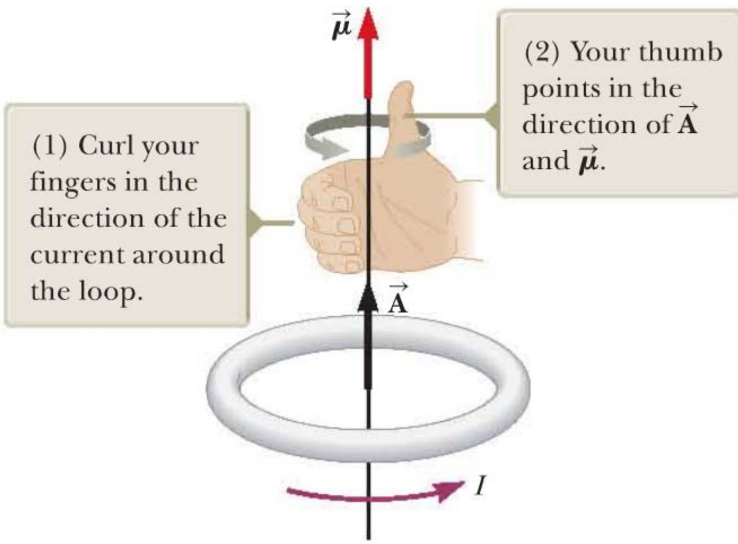
$$\therefore\ \vec{\tau} = \text{IBa} \cdot \text{b} \cos\theta \otimes = \text{IBA}\cos\theta \otimes$$

$$= \text{I}\vec{\text{A}} \times \vec{\text{B}}$$

Magnetic moment and Magnetic torque

For any current loop of arbitrary shape in a plane:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



If the coil has N turns,

$$\vec{\tau} = N\vec{\mu} \times \vec{B}$$

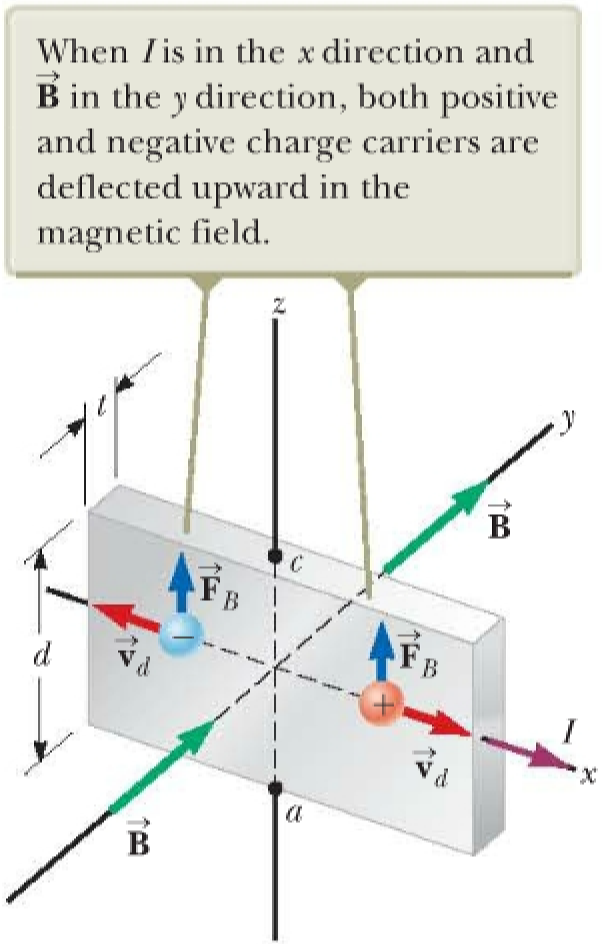
Torque tends to align μ and $\vec{A}$ with $\vec{B}$
(i.e. lowest potential energy at θ=0)

Magnetic moment:

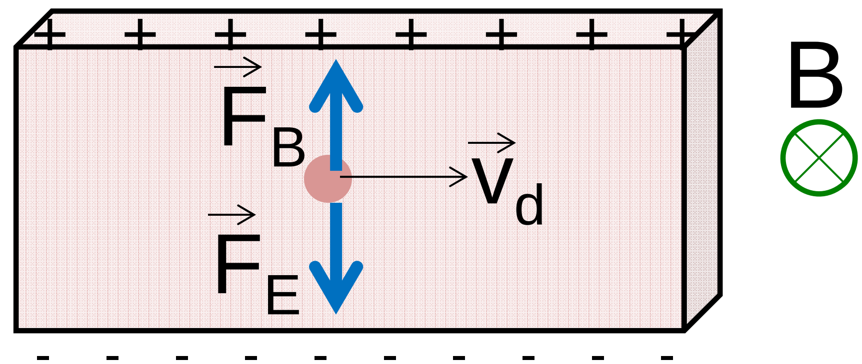
$$\mu = IA$$ (magnitude)

$$U = -\vec{\mu} \cdot \vec{B}$$

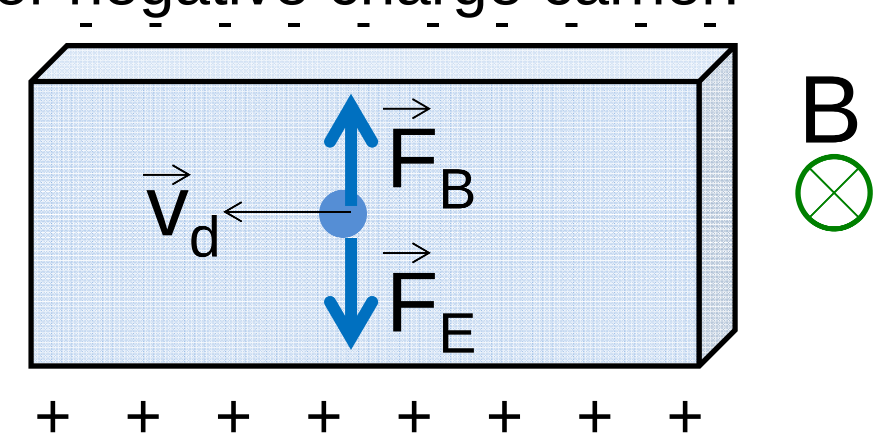
Balance of Magnetic Force and Electric Force



For positive charge carrier:



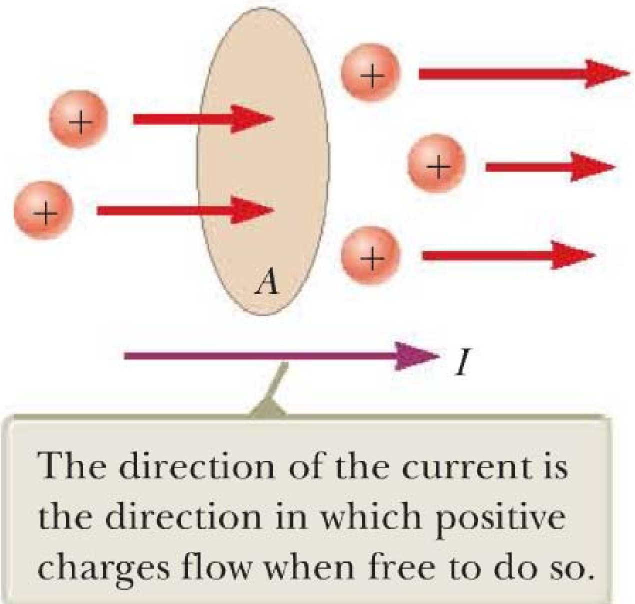
For negative charge carrier:



$$\vec{F}_B = \vec{F}_E$$

Slide from Class 13,
June 30, 2014

Current



If dQ is the amount of charge passes through A in a short time interval dt, current is defined as:

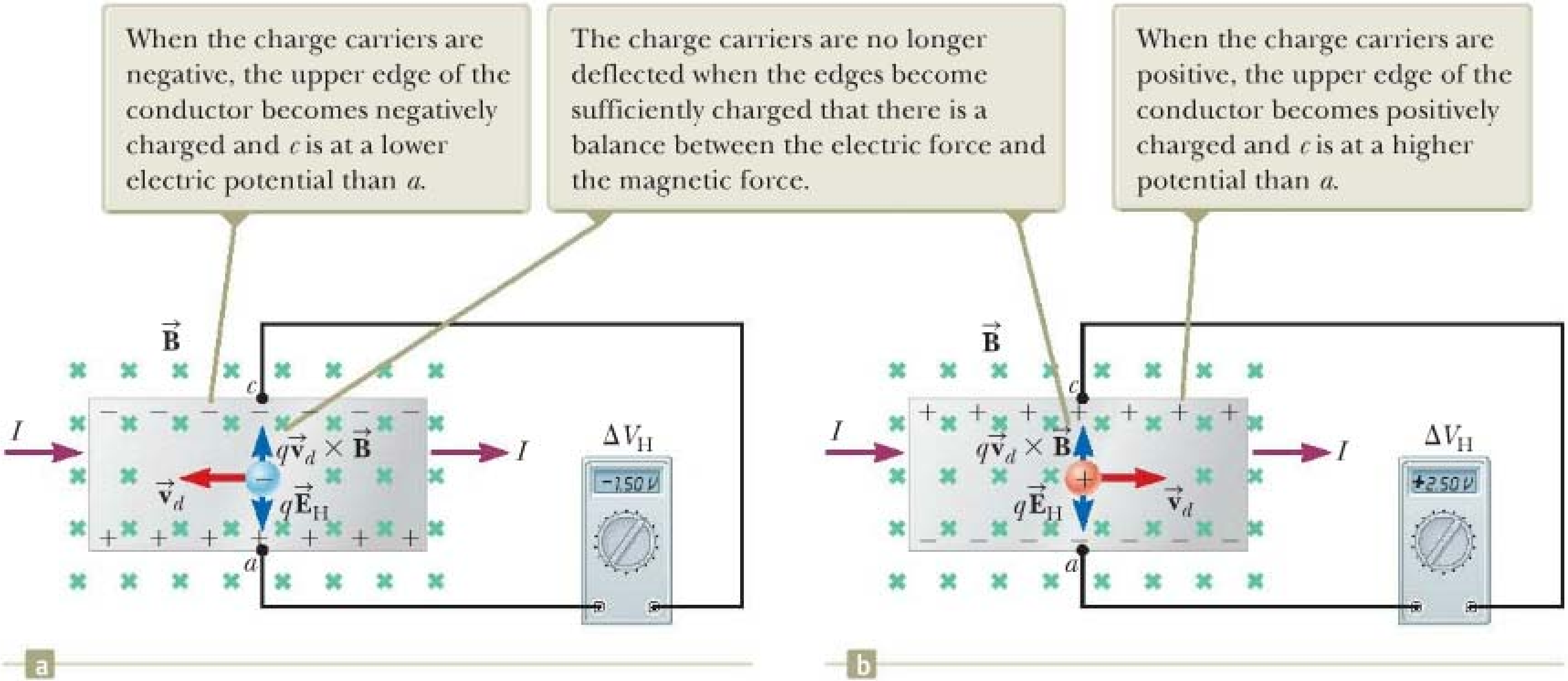
$$I = \frac{dQ}{dt}$$

Units of current:
Ampere (A) $\equiv$ C/s

These two kinds of currents are actually different!

Electrically these two cases produce the same current,
but they can be distinguished with a magnetic field.

Equal Current with Opposite Carriers



These two cases produce the same current, but can be distinguished with a magnetic field by Hall effect.

Application of Hall Effect

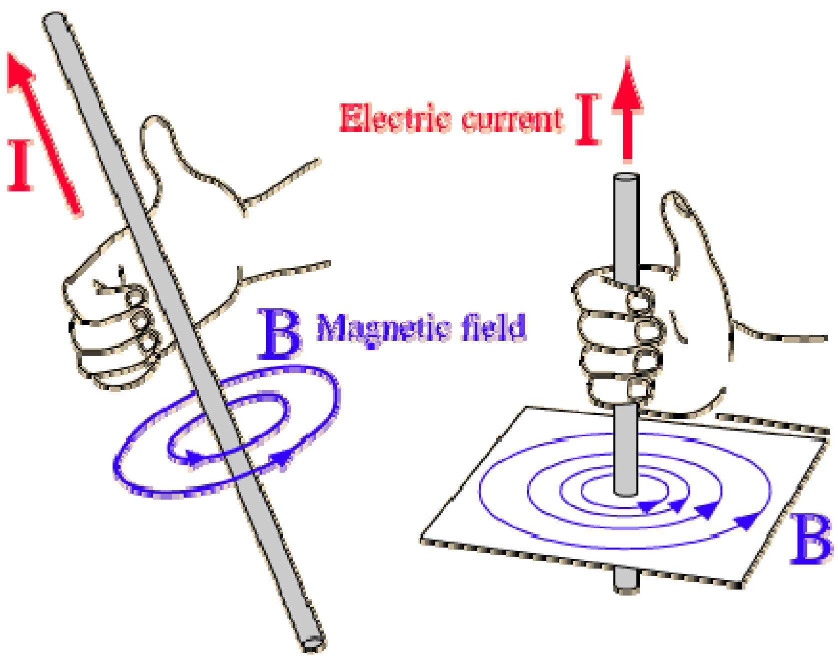
$$\Delta V_H = \frac{IB}{n\,e\,t}$$

1. Hall effect can be used to measure magnetic field.
2. Hall effect can be used to measure the carrier density n.
3. Hall effect can determine the sign of the carriers.
4. (Quantum) Hall effect provides an international standard of resistance.

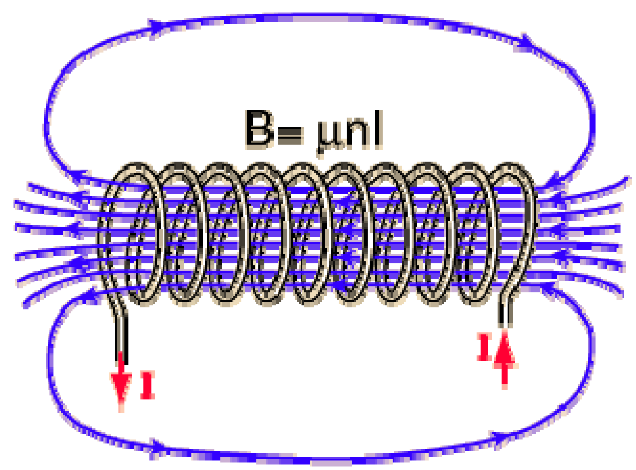
Biot – Savart Law

Origin of Magnetic field

A current (or moving charge) experience a magnetic force when it is in a magnetic field. The magnetic field is the result of another current (or moving charge).
If electric field describes the interaction between two charges, then magnetic field describes the interaction between two currents (or moving charges).

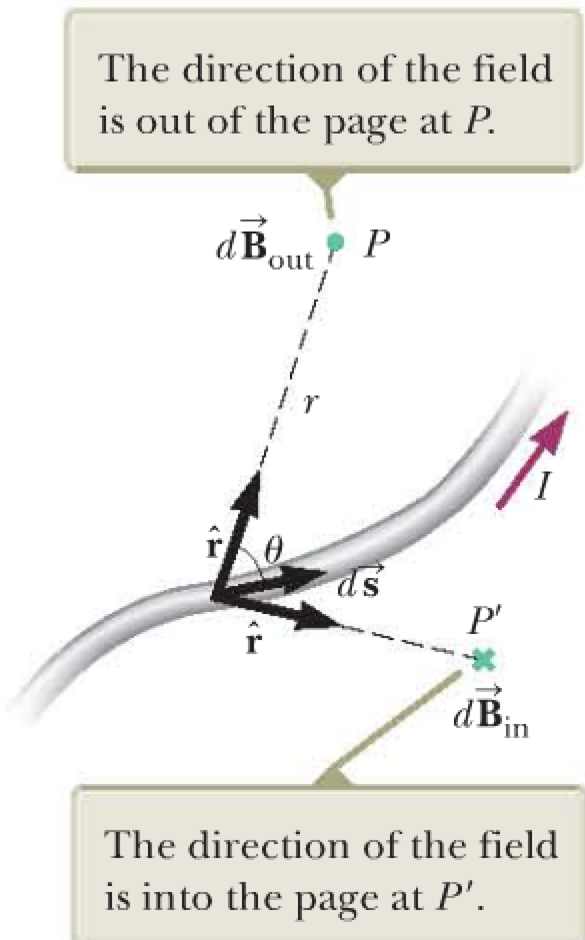


Magnetic field due to a long current



Magnetic field of a solenoid

Biot-Savart Law



Magnetic field at point P due to the infinitesimal element ds:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} \quad \text{or} \quad \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$$

Magnetic field due to the whole wire:

$$\vec{B} = \int_{wire}\frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

μ_0 is a constant called permeability of free space:

$\mu_0 = 4\pi\times10^{-7}\ TmA^{-1}$

In the calculation of magnetic field, Biot-Savart Law play the same role as the Coulomb's Law in electric field.