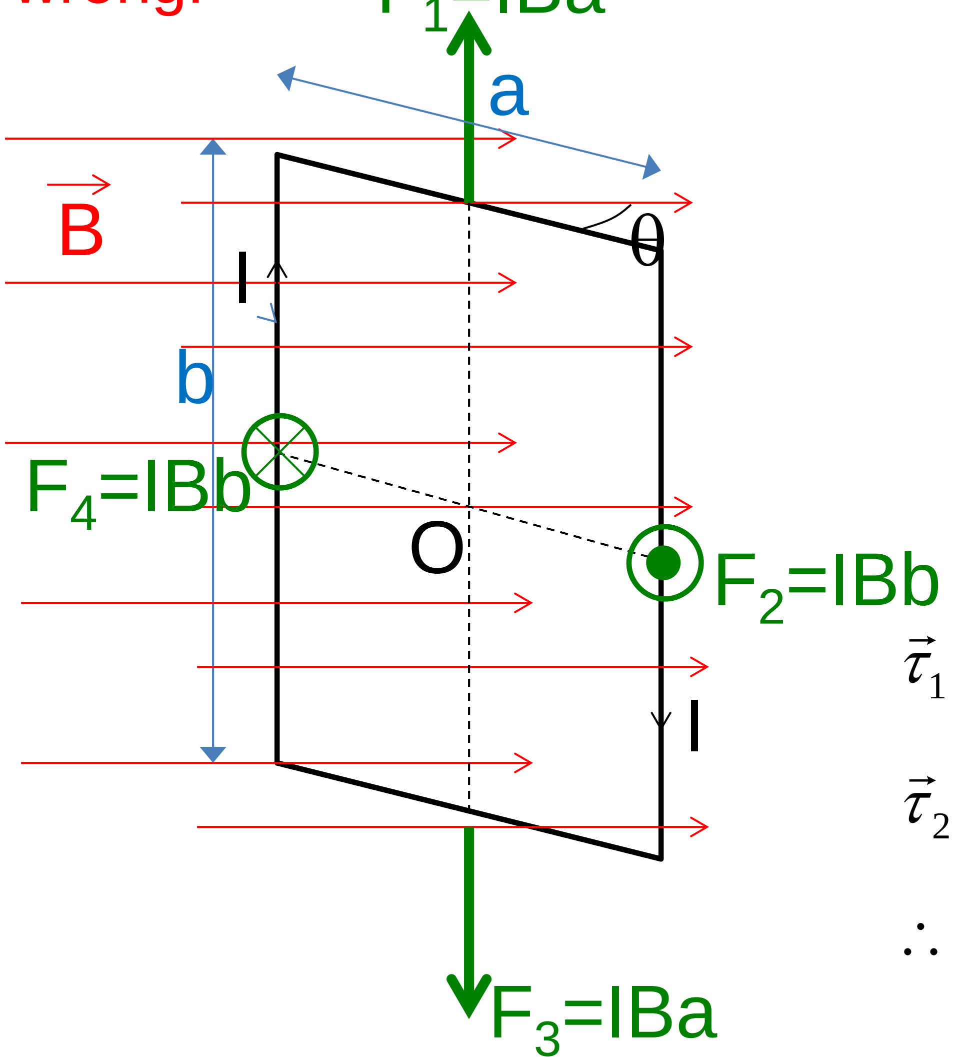
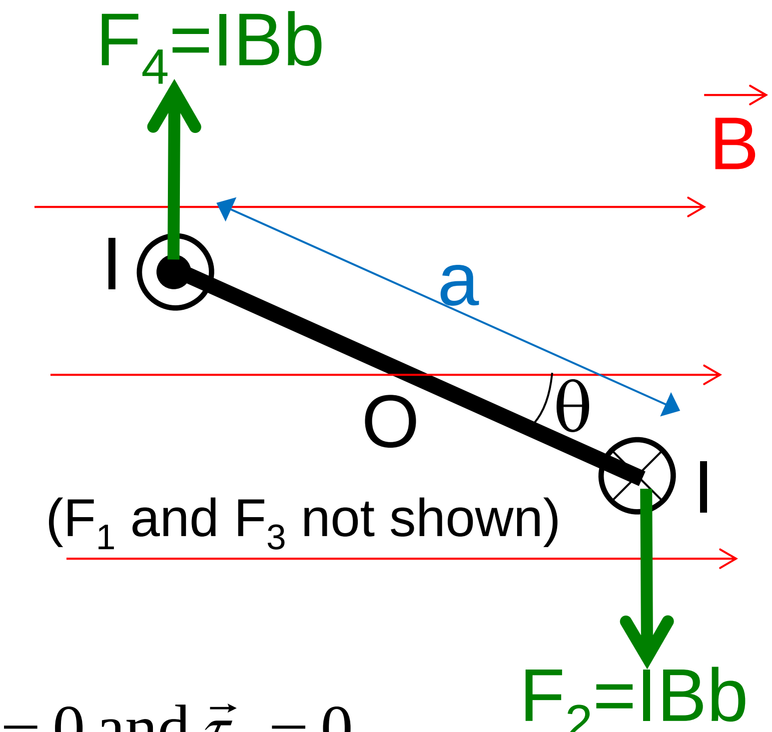
Class 28 Biot – Savart Law and Ampere's Law

Magnetic torque on a Rectangular Loop

The way we define θ here does not follow convention, but nothing wrong!



Top view



$$\vec{\tau}_1 = 0 \text{ and } \vec{\tau}_3 = 0$$

$$\vec{\tau}_2 = \vec{\tau}_4 = \mathrm{IBb} \cdot \frac{a}{2} \cos\theta \otimes$$

$$\therefore \ \vec{\tau} = \mathrm{IBa} \cdot \mathrm{b} \cos\theta \otimes = \mathrm{IBA} \cos\theta \otimes$$

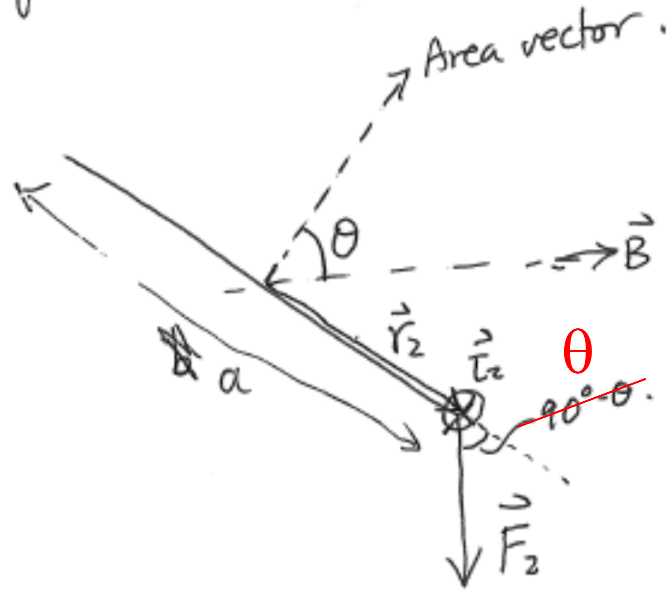
$$= \mathrm{I}\vec{\mathrm{A}} \times \vec{\mathrm{B}}$$

Correction

Class 25

Written notes pg.2

Torque $\vec{\tau} = \vec{r} \times \vec{F}$



$$|\vec{\tau}_2| = \frac{a}{2} \cdot IBb \cdot \sin(\cancel{90^\circ - \theta})\ \theta .$$

$$= \frac{a}{2} IBb \cancel{\cos\theta}\ \sin\theta .$$

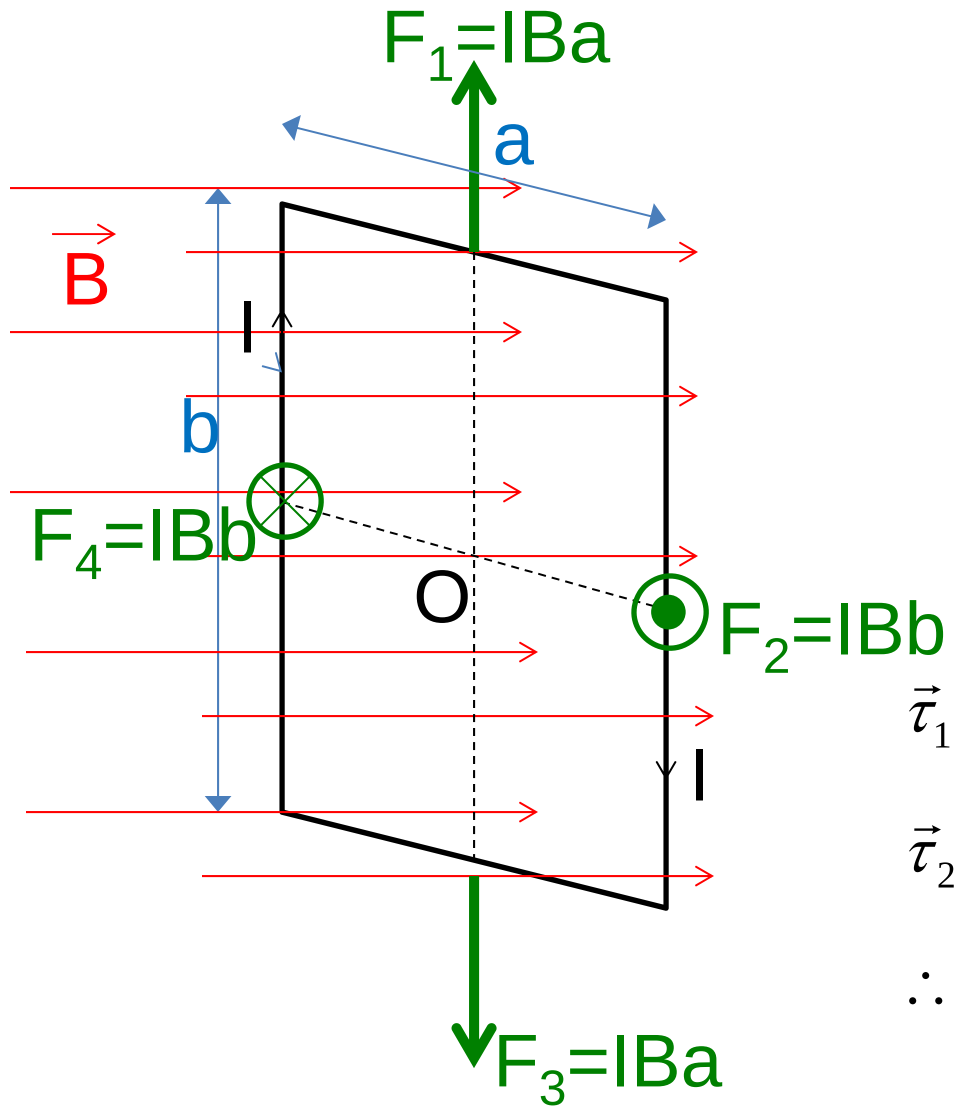
$$= \frac{A}{2} \cdot IB \cancel{\cos\theta}\ \sin\theta .$$

Correction

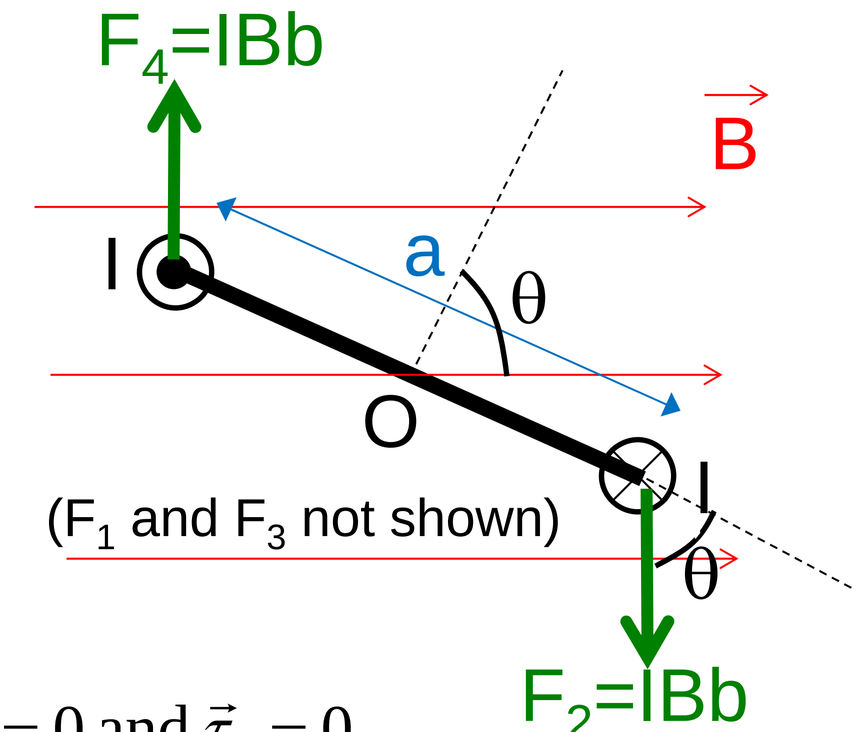
Class 25 slide 4

Class 27 slide 3

Magnetic torque on a Rectangular Loop



Top view



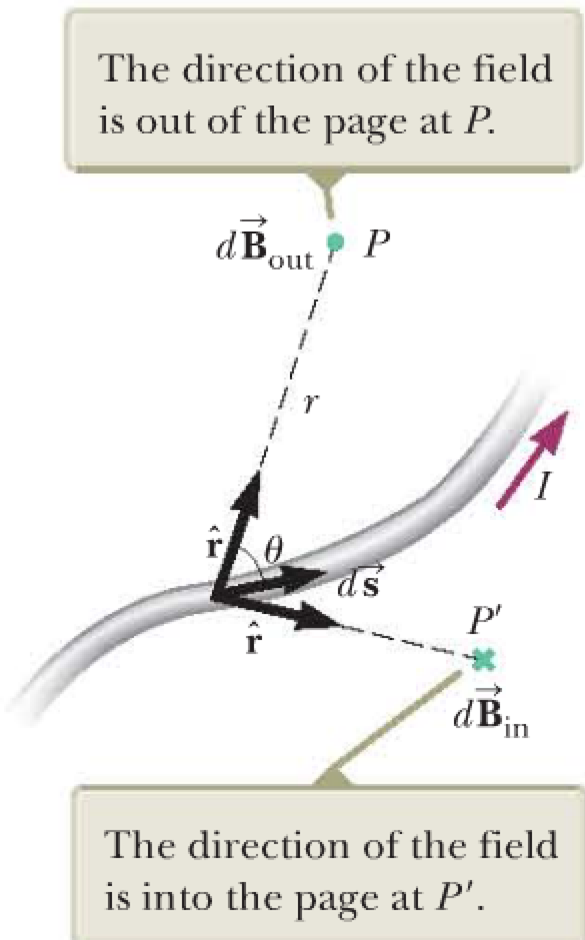
$$\vec{\tau}_1 = 0 \text{ and } \vec{\tau}_3 = 0$$

$$\vec{\tau}_2 = \vec{\tau}_4 = IBb \cdot \frac{a}{2} \cancel{\cos\theta}\ \sin\theta \otimes$$

$$\therefore\ \vec{\tau} = IBa \cdot b \cancel{\cos\theta}\ \sin\theta \otimes = IBA \cancel{\cos\theta}\ \sin\theta \otimes$$

$$= I\vec{A} \times \vec{B}$$

Biot-Savart Law



Magnetic field at point P due to the infinitesimal element ds:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} \quad \text{or} \quad \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$$

Magnetic field due to the whole wire:

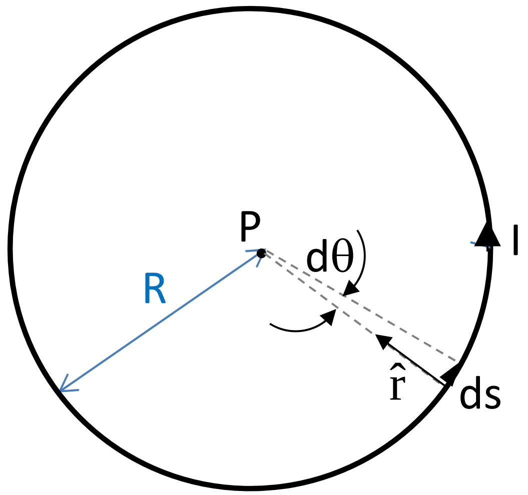
$$\vec{B} = \int_{\text{wire}} \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

μ_0 is a constant called permeability of free space:

$\mu_0 = 4\pi\times10^{-7}\ \mathrm{TmA^{-1}}$

In the calculation of magnetic field, Biot-Savart Law play the same role as the Coulomb's Law in electric field.

Magnetic Field at the Center of a Circular Current Loop



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{ds}{R^2}\odot$$

$$ds = Rd\theta$$

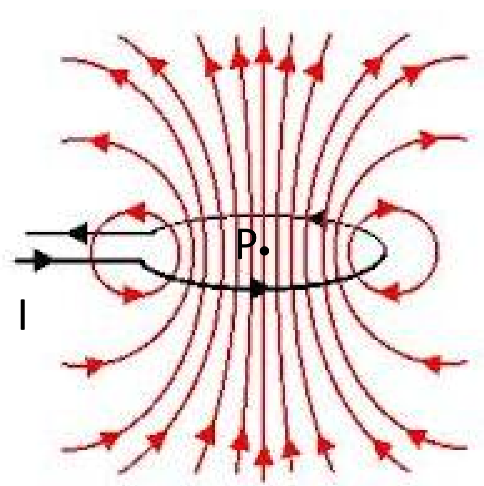
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{Rd\theta}{R^2}\odot$$

$$= \frac{\mu_0 I}{4\pi R}d\theta\odot$$

$$\therefore\ \vec{B} = \int_0^{2\pi}\frac{\mu_0 I}{4\pi R}d\theta\odot = \odot\frac{\mu_0 I}{4\pi R}\int_0^{2\pi}d\theta = \frac{\mu_0 I}{4\pi R}[\theta]_0^{2\pi}\odot$$

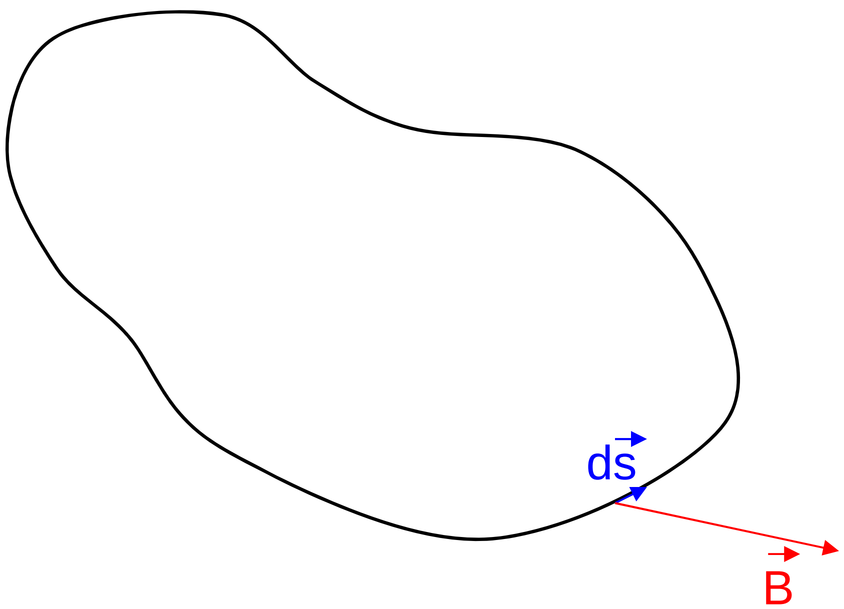
$$= \frac{\mu_0 I}{4\pi R}\cdot 2\pi\odot$$

$$\therefore\ \boxed{\vec{B} = \frac{\mu_0 I}{2R}\odot}$$



Ampere's Law

Path Integral of Magnetic Field



Path Integral:

$$\oint \vec{B} \cdot d\vec{s}$$

Do you remember:

$$\oint \vec{E} \cdot d\vec{s} = ?$$ (for stationary case)

Gauss's Law (Maxwell's first equation)

From Class 5

For *any* closed surface,

$$\varepsilon_0 \Phi_E = q_{in} \quad \text{or} \quad \varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{in}$$

Two types of problems that involve Gauss's Law:

1. Give you left hand side (i.e. flux through a given surface), calculate the right hand side (i.e. charge enclosed by that surface).

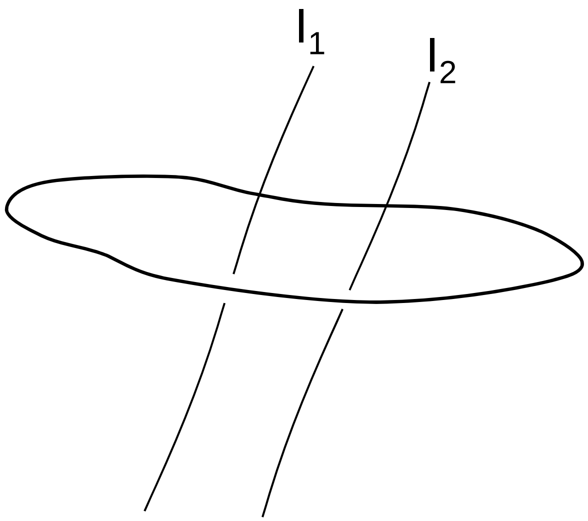
2. Give you right hand side (i.e. a charge distribution) , calculate the left hand side (i.e. flux and the electric field).

Ampere's Law (Maxwell's third equation - partial)

For *any* closed loop,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \ I_{in}$$

$$\mu_0 = 4\pi \times 10^{-7} \ \mathrm{TmA}^{-1}$$



Two types of problems that involve Ampere's Law:

1. Give you left hand side (i.e. line integral of a given loop), calculate the right hand side (i.e. current enclosed by that loop).

2. Give you right hand side (i.e. current) , calculate the left hand side (i.e. the line integral and the magnetic field).

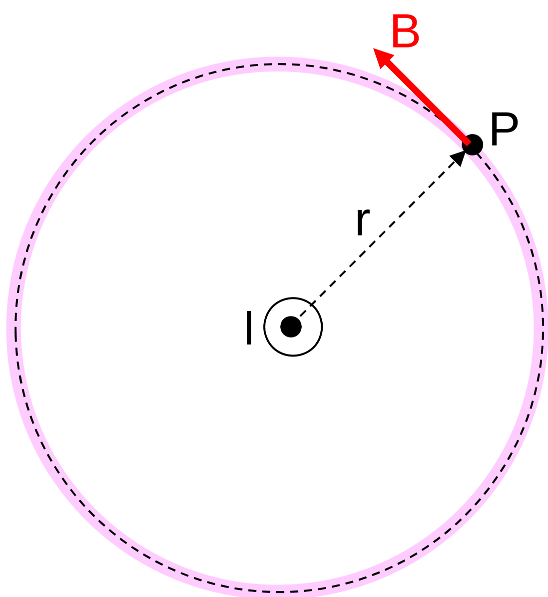
Calculating Magnetic Field Using Ampere's Law

1. By symmetry argument, construct a loop so that the path integral $\oint \vec{B} \cdot d\vec{s}$ can be easily calculated. In most cases when Ampere's Law is applicable,
$$\oint \vec{B} \cdot d\vec{s} = BL$$
where L is the length of the loop.

2. You can then apply Ampere's Law and solve for B:
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in} \Rightarrow BL = \mu_0 I_{in} \Rightarrow B = \frac{\mu_0 I_{in}}{L}$$

3. Two common cases when Ampere's Law can be used to calculate magnetic field: infinite long wire and infinite long solenoid / toroid.

Magnetic field due to a long wire



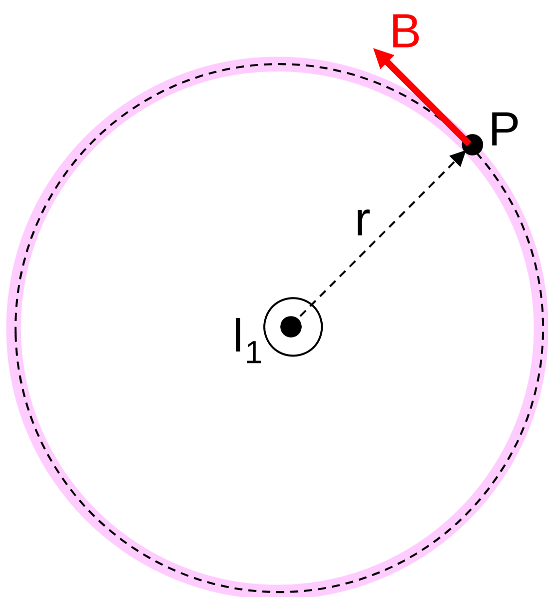
Want to calculate the magnetic field B at point P.
By symmetry argument, B is in the plane of the paper (infinite long wire), has the same magnitude for all points on the dotted circular loop (azimuthal symmetry), and tangent to the circular loop (so cos θ=1).

$$\therefore \oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi\, r$$

Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0\, I \Rightarrow \; B \cdot 2\pi\, r = \mu_0\, I$

$$\Rightarrow \; B = \frac{\mu_0\, I}{2\pi\, r}$$

Magnetic Force Between Two Parallel Long Wires

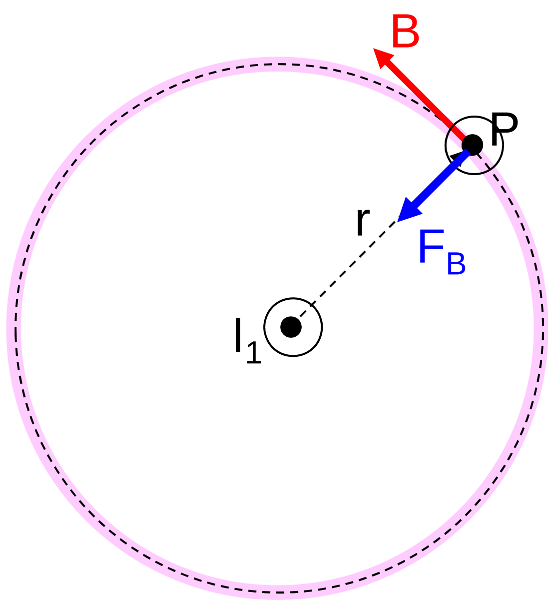


Magnetic field at point P due to I_1:

$$B = \frac{\mu_0 \, I_1}{2\pi \, r}$$

If another current I_2 parallel to I_1 is passing through point P, it will experience a force because of the field there.

Magnetic Force Between Two Parallel Long Wires



Magnetic field at point P due to I_1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

If another current I_2 parallel to I_1 is passing through point P, it will experience a force because of the field there.

$$\vec{F}_B = I_2 \vec{L} \times \vec{B} \;\Rightarrow\; F_B = I_2 BL \sin 90^o = I_2 BL$$

$$\Rightarrow\; F_B = I_2 L \cdot \frac{\mu_0 I_1}{2\pi r}$$

$$\Rightarrow\; \frac{F_B}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Force is attractive if the two currents are in the same direction, repulsive if the two currents are in opposite direction.