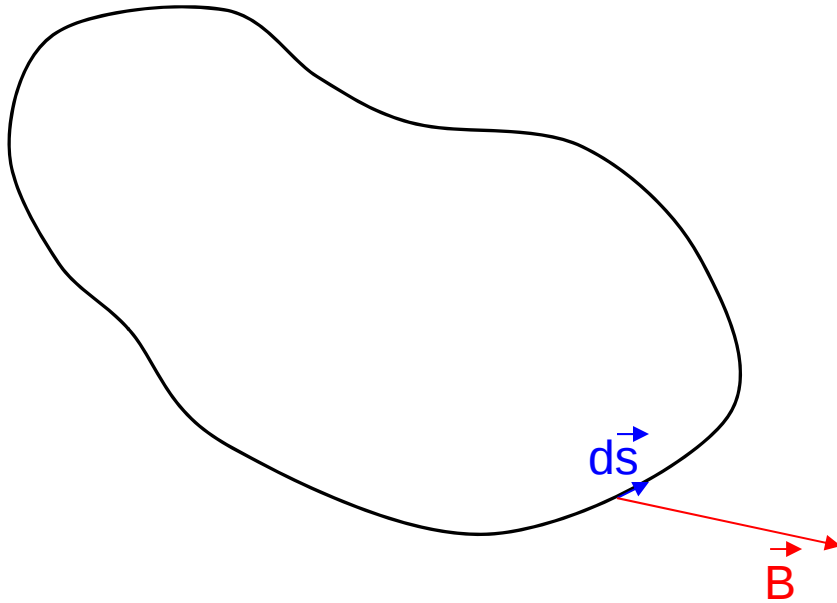


## Class 29 Long straight wire and solenoid

# Path Integral of Magnetic Field



Path Integral:

$$\oint \vec{B} \cdot d\vec{s}$$

Do you remember:

$$\oint \vec{E} \cdot d\vec{s} = ? \quad (\text{for stationary case})$$

# Gauss's Law (Maxwell's first equation)

For *any* closed surface,

From Class 5

$$\epsilon_0 \Phi_E = q_{\text{in}} \quad \text{or} \quad \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{in}}$$

Two types of problems that involve Gauss's Law:

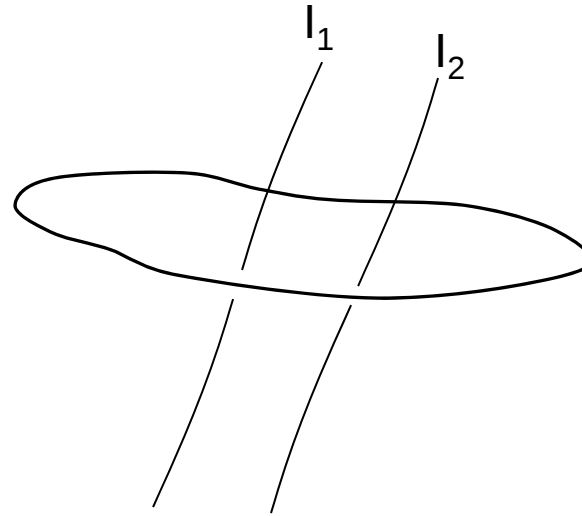
1. Give you left hand side (i.e. flux through a given surface), calculate the right hand side (i.e. charge enclosed by that surface).
2. Give you right hand side (i.e. a charge distribution) , calculate the left hand side (i.e. flux and the electric field).

# Ampere's Law (Maxwell's third equation - partial)

For *any* closed loop,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$$



Two types of problems that involve Ampere's Law:

1. Give you left hand side (i.e. line integral of a given loop), calculate the right hand side (i.e. current enclosed by that loop).
2. Give you right hand side (i.e. current) , calculate the left hand side (i.e. the line integral and the magnetic field).

## Calculating Magnetic Field Using Ampere's Law

1. By symmetry argument, construct a loop so that the path integral  $\oint \vec{B} \cdot d\vec{s}$  can be easily calculated. In most cases when Ampere's Law is applicable,

$$\oint \vec{B} \cdot d\vec{s} = BL$$

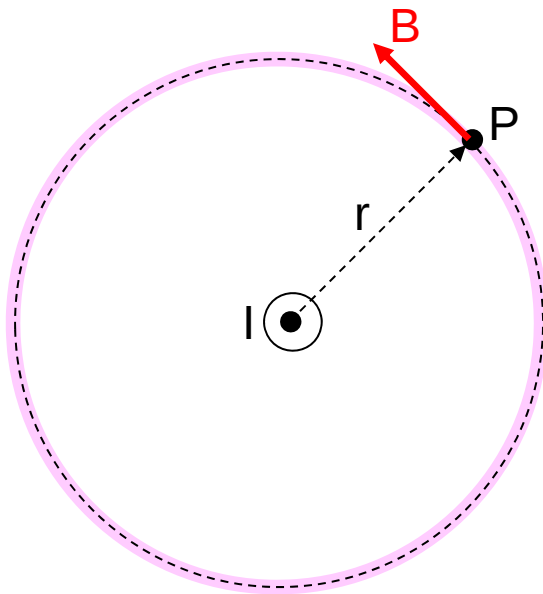
where  $L$  is the length of the loop.

2. You can then apply Ampere's Law and solve for  $B$ :

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}} \Rightarrow BL = \mu_0 I_{\text{in}} \Rightarrow B = \frac{\mu_0 I_{\text{in}}}{L}$$

3. Two common cases when Ampere's Law can be used to calculate magnetic field: infinite long wire and infinite long solenoid / toroid.

# Magnetic field due to a long wire



Want to calculate the magnetic field  $B$  at point  $P$ .

By symmetry argument,  $B$  is in the plane of the paper (infinite long wire), has the same magnitude for all points on the dotted circular loop (azimuthal symmetry), and tangent to the circular loop (so  $\cos \theta = 1$ ).

$$\therefore \oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi r$$

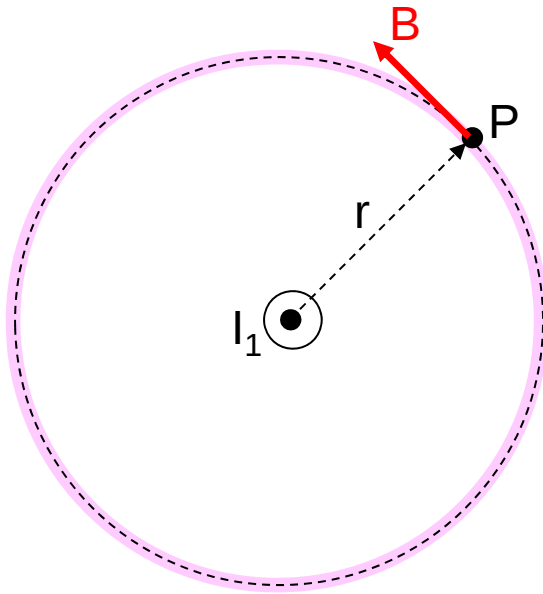
$$\text{Ampere's Law: } \oint \vec{B} \cdot d\vec{s} = \mu_0 I \Rightarrow B \cdot 2\pi r = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

# Magnetic Force Between Two Parallel Long Wires

Magnetic field at point P due to  $I_1$ :

$$B = \frac{\mu_0 I_1}{2\pi r}$$

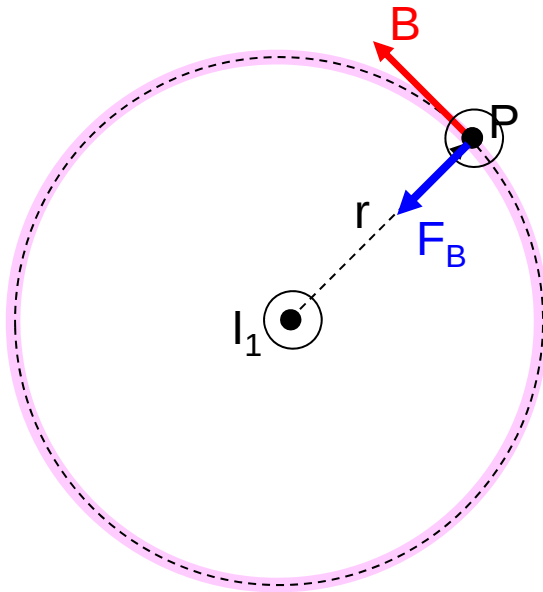


If another current  $I_2$  parallel to  $I_1$  is passing through point P, it will experience a force because of the field there.

# Magnetic Force Between Two Parallel Long Wires

Magnetic field at point P due to  $I_1$ :

$$B = \frac{\mu_0 I_1}{2\pi r}$$



If another current  $I_2$  parallel to  $I_1$  is passing through point P, it will experience a force because of the field there.

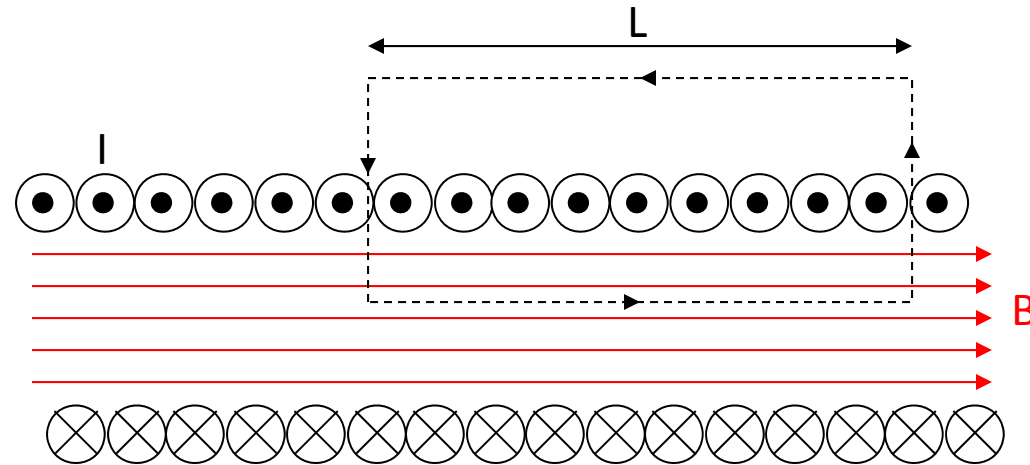
$$\vec{F}_B = I_2 \vec{L} \times \vec{B} \Rightarrow F_B = I_2 B L \sin 90^\circ = I_2 B L$$

$$\Rightarrow F_B = I_2 L \cdot \frac{\mu_0 I_1}{2\pi r}$$

$$\Rightarrow \frac{F_B}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Force is attractive if the two currents are in the same direction, repulsive if the two currents are in opposite direction.

# Solenoid



If  $n$  = number of turns per unit length

$$\therefore \oint \vec{B} \cdot d\vec{s} = B \cdot L$$

$$\text{Ampere's Law: } \oint \vec{B} \cdot d\vec{s} = \mu_0 I \Rightarrow B \cdot L = \mu_0 (nL) I$$

$$\Rightarrow B = \mu_0 n I$$

Note that  $B$  is proportional to the number of turns per unit length, but not the total number of turns.