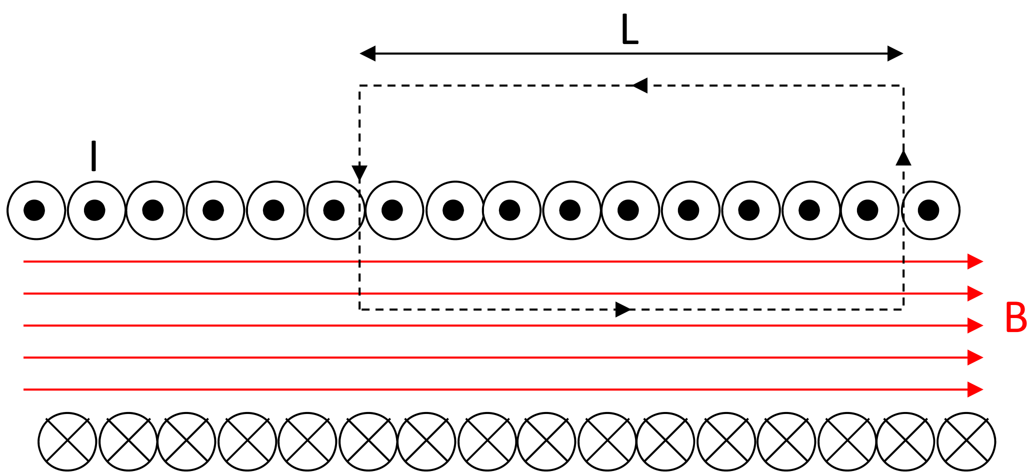
Class 30 Magnetism

Solenoid



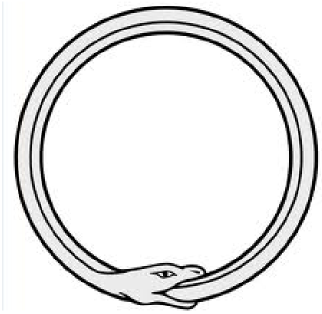
If n = number of turns per unit length

$$\therefore \oint \vec{B} \cdot d\vec{s} = B \cdot L$$

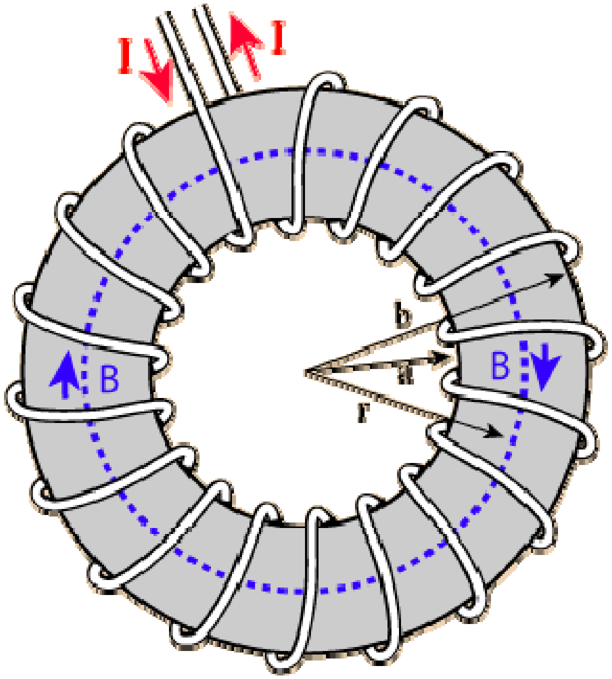
Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 \, I \Rightarrow \; B \cdot L = \mu_0 \, (nL) \, I$

$$\Rightarrow \boxed{B = \mu_0 \, n \, I}$$

Note that B is proportional to the number of turns per unit length, but not the total number of turns.



Toroid



$$\therefore \oint \vec{B}\cdot d\vec{s} = B\cdot 2\pi\, r$$

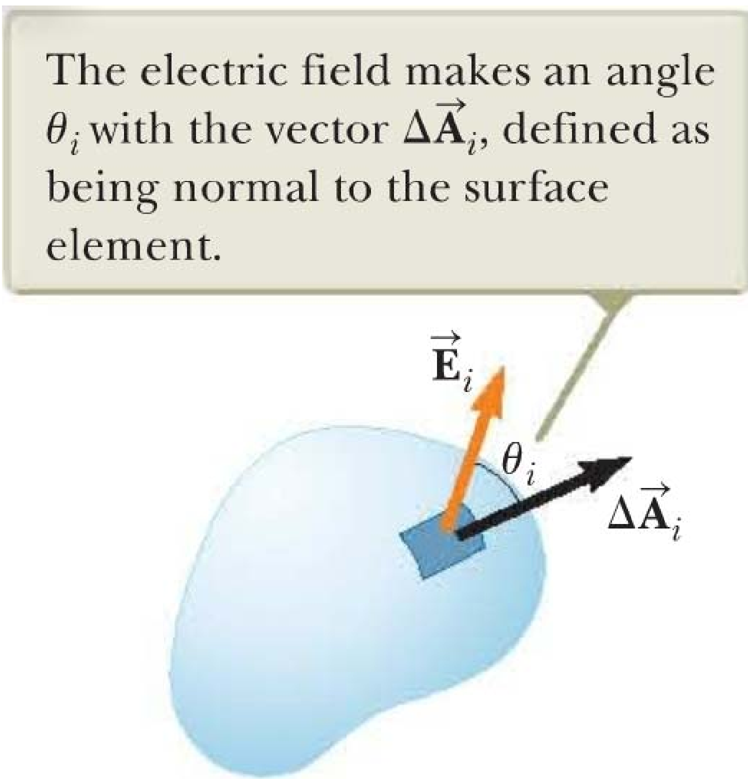
Ampere's Law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\, I \Rightarrow \quad B\cdot 2\pi\, r = \mu_0\, (n\cdot 2\pi\, r)\, I$$

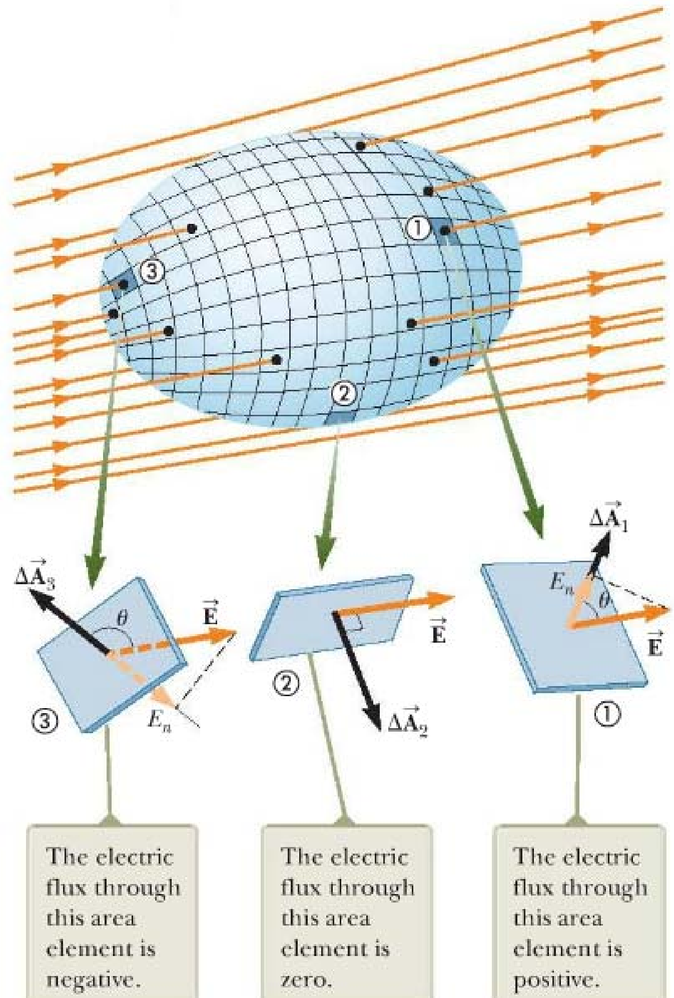
$$\Rightarrow\ B = \mu_0 \left(\frac{N}{2\pi\, r}\right) I$$

N = Total number of turns

Flux for curve surface



$$d\Phi_E = \vec{E}\cdot d\vec{A} \quad \Rightarrow \quad \Phi_E = \int \vec{E}\cdot d\vec{A}$$



For closed surface, surface vector is positive when it is pointing outward.

Magnetic Flux for curve surface

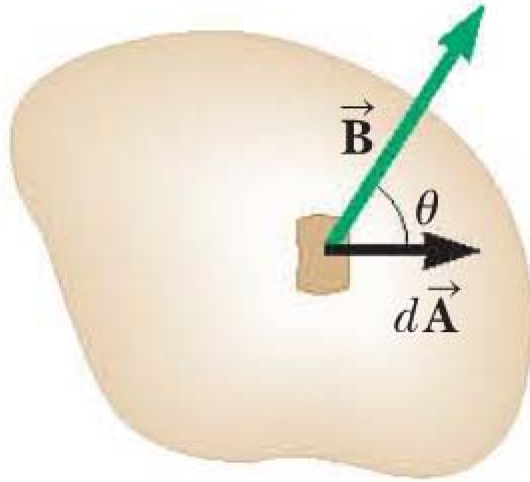


Figure 30.19 The magnetic flux through an area element dA is $\vec{B} \cdot d\vec{A} = B\,dA\cos\theta$, where $d\vec{A}$ is a vector perpendicular to the surface.

$$d\Phi_B = \vec{B} \cdot d\vec{A} \quad \Rightarrow \quad \Phi_B = \int \vec{B} \cdot d\vec{A}$$

Different Topology Between Electric and Magnetic Fields

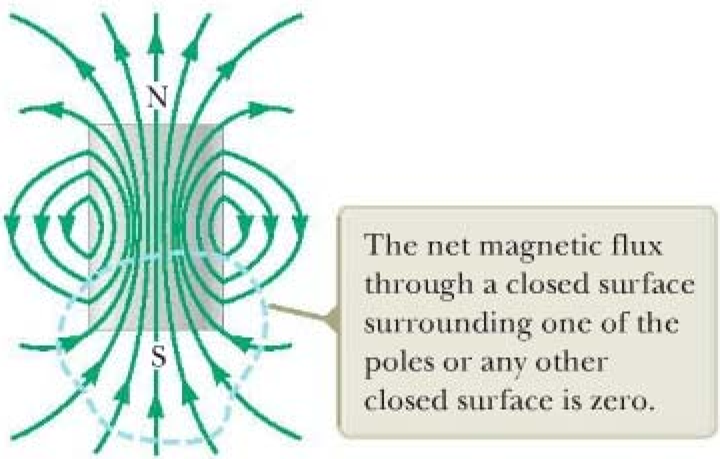


Figure 30.22 The magnetic field lines of a bar magnet form closed loops. (The dashed line represents the intersection of a closed surface with the page.)

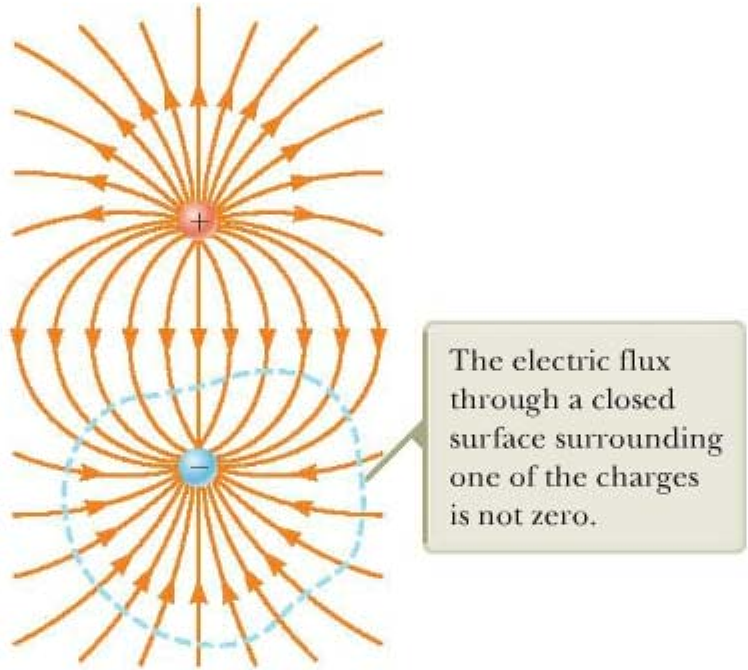


Figure 30.23 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge.

There is no magnetic point charge.

Gauss's Law for magnetic field

Since there is no magnetic point charge,

$$\Phi_B = 0 \quad \text{or} \quad \oiint \vec{B} \cdot d\vec{A} = 0$$

Summary of Maxwell's Equations so far

Maxwell's 1st equation: (Gauss's Law foe electric field)

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{in}$$

Maxwell's 2nd equation: (Gauss's Law for magnetic field)

$$\oiint \vec{B} \cdot d\vec{A} = 0$$

Maxwell's 3rd equation: (Ampere's Law - incomplete)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

Maxwell's equations describe only the fields, it does not include the effect of the field on charges or currents:

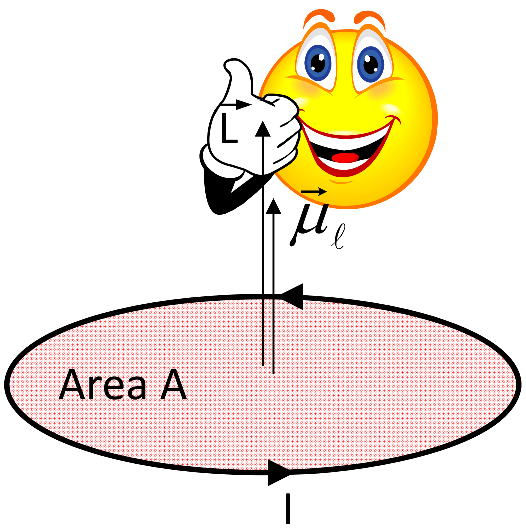
$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad (\text{or } d\vec{F}_B = I d\vec{s} \times \vec{B})$$

Not included in Maxwell's equations

Magnetic Field from Electrons in an Atom I

A charge particle possessing angular momentum will give rise to magnetic moment. Magnetic moment is a vector in the same direction (for positive charge) as the angular momentum.



$$\mu_\ell = IA$$

Magnetic moment is a result of the circular motion of the electrons, which can be described by the angular momentum.

$$L = m\,v\,r$$

$$\mu_\ell = I\,A = \frac{-e\,v}{2\pi\,r} \cdot \pi\,r^2 = \frac{-e\,v\,r}{2} \quad \Longrightarrow \quad \mu_\ell = -\frac{e}{2m} L$$

For electron (the minus sign)

Quantization of angular momentum, spin, and magnetic moment

Spin of electrons can contribute to magnetic moment too.

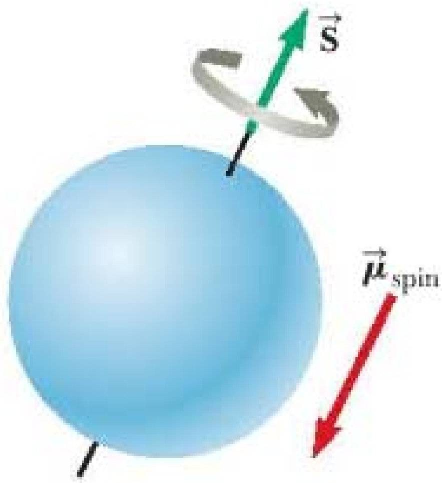


Figure 30.25 Classical model of a spinning electron. We can adopt this model to remind ourselves that electrons have an intrinsic angular momentum. The model should not be pushed too far, however; it gives an incorrect magnitude for the magnetic moment, incorrect quantum numbers, and too many degrees of freedom.

For electron

$$\mu_s = -\frac{e}{m}S$$

Quantization of spin and angular momentum:

L and S are quantized in units of ħ.
ħ = 1.05 × 10^{-34} Js.

As a result, magnetic momentum is also quantized, in units of Bohr magnetron

$$\mu_b = \frac{e\hbar}{2m} = 9.27\times10^{-24}\ JT^{-1}$$

Paramagetism and Diagmagnetism

Paramagnetism:
If an atom has a net magnetic moment, it will display parammagnetic property – the net moment will align with external field and create a slightly stronger total field. This alignment process has to compete with the thermal effect, which tends to randomize the alignment.

Diagmagnetism:
If an atom has no net magnetic moment (e.g. inert gas like He, Ar, Ne etc.), it will display diamagnetic property – an external field will induce a magnetic moment in opposite to the field direction and produce a slightly weaker field. This is a result of the Faraday's Law, which we will study in the next two chapters.

In general, the diamagnetic effect is much weaker than the paramagnetic effect.

Ferromagnetism

The individual magnetic moments in some paramagnetic materials will align together to form domains when the temperature is below a critical temperature called Curie Temperature.

When an external field is applied, the domains with moment in the field direction will grow in size. The original configuration may not be recovered after the field is removed an a permanent magnet is formed.