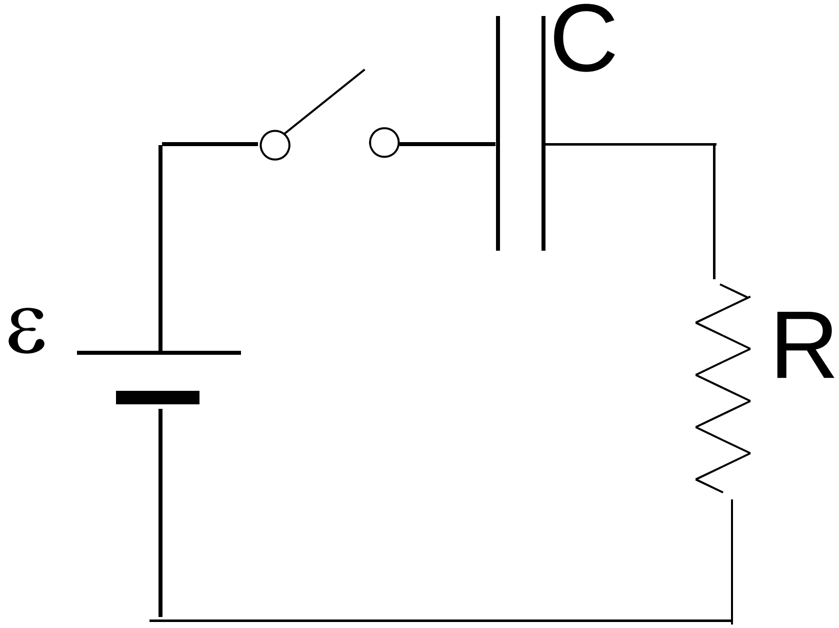
Class 35 RL Circuits

From Class 21

Slide #3

RC Circuits – Charging

← Charge



$$\varepsilon = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR\,dq = (C\varepsilon - q)\,dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + K e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = 0 \Rightarrow 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore q = \underline{\underline{C\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

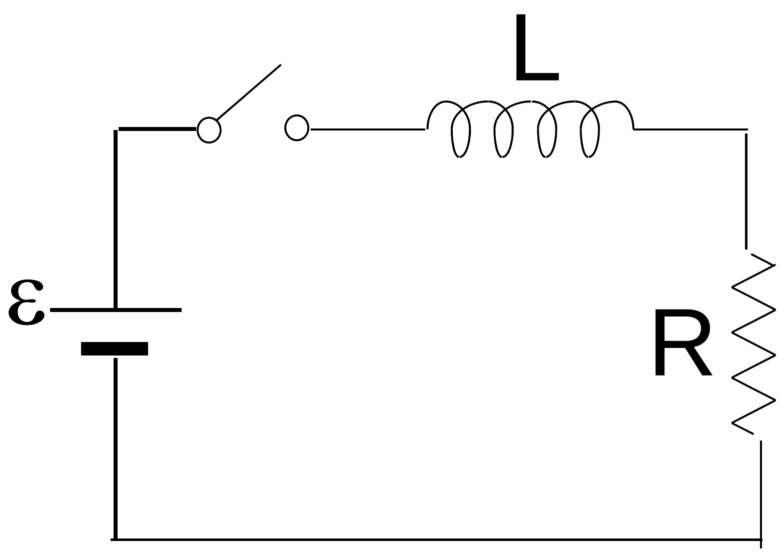
$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR}e^{-\frac{t}{CR}} = \underline{\underline{\frac{\varepsilon}{R}e^{-\frac{t}{CR}}}}$$

$$\Delta V_R = IR = \underline{\underline{\varepsilon e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$\Delta V_R + \Delta V_C = \varepsilon$

RL Circuits – Charging ← Current



$$\varepsilon = L\frac{dI}{dt} + IR \Rightarrow L\,dI + I\,R\,dt = \varepsilon\,dt$$

$$\Rightarrow L\,dI = (\varepsilon - IR)dt$$

$$\Rightarrow \frac{L\,dI}{\varepsilon - IR} = dt$$

Integration constant

$$\Rightarrow \frac{L}{R}\ell n(\varepsilon - IR) = t + K'$$

$$\Rightarrow \ell n(\varepsilon - IR) = \frac{R}{L}t + \frac{R}{L}K'$$

$$\Rightarrow \varepsilon - IR = Ke^{-\frac{R}{L}t} \qquad (K = e^{\frac{R}{L}K'})$$

$$\Rightarrow IR = \varepsilon - Ke^{-\frac{R}{L}t}$$

$$\text{At } t = 0, I = 0 \Rightarrow 0 = \varepsilon - K \Rightarrow K = \varepsilon$$

$$\therefore I\,R = \varepsilon(1 - e^{-\frac{R}{L}t}) \Rightarrow \underline{\underline{I = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})}}$$

$$\Delta V_R = IR = \underline{\underline{\varepsilon(1 - e^{-\frac{R}{L}t})}}$$

$$\Delta V_L = L\frac{dI}{dt} = \underline{\underline{\varepsilon\,e^{-\frac{R}{L}t}}}$$

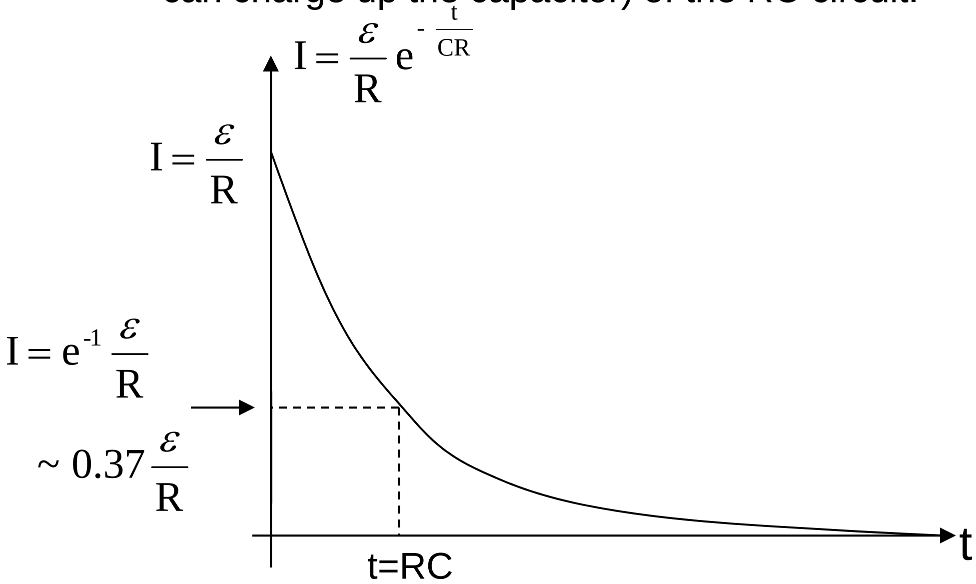
$\Delta V_R + \Delta V_C = \varepsilon$

From Class 21

Slide #4

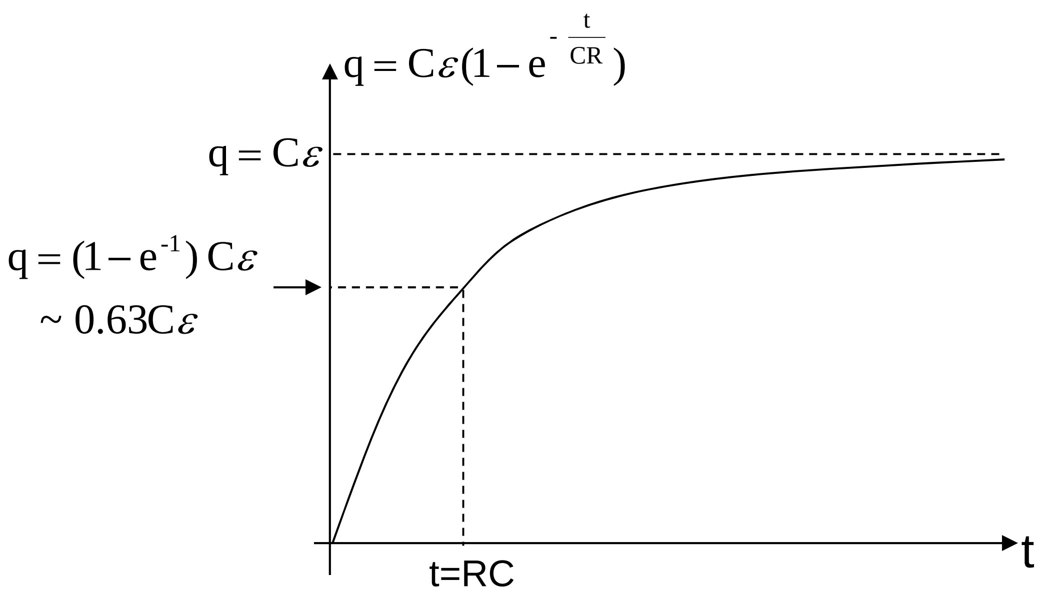
RC time constant

τ=RC is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.



$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$



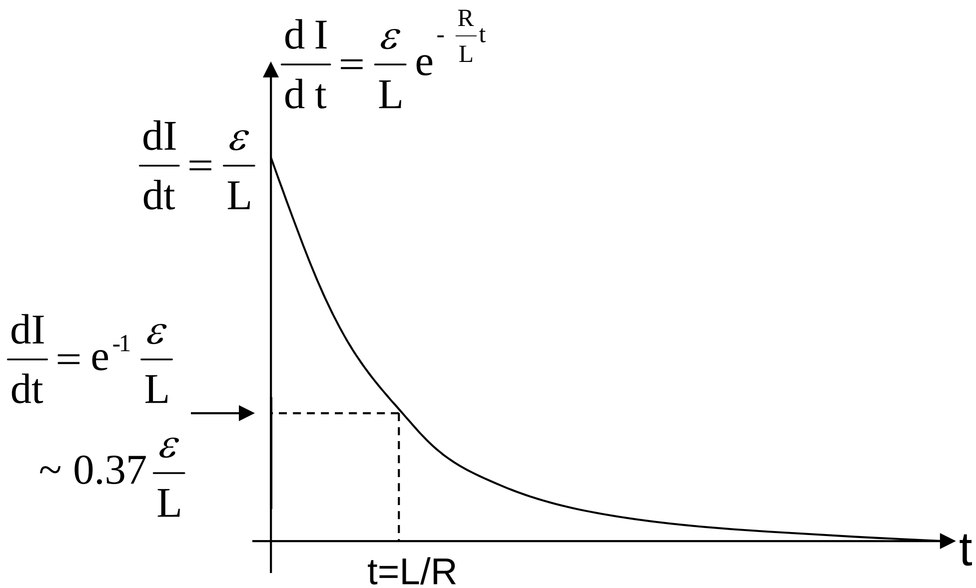
$$\sqrt{2} \approx 1.414$$

$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits

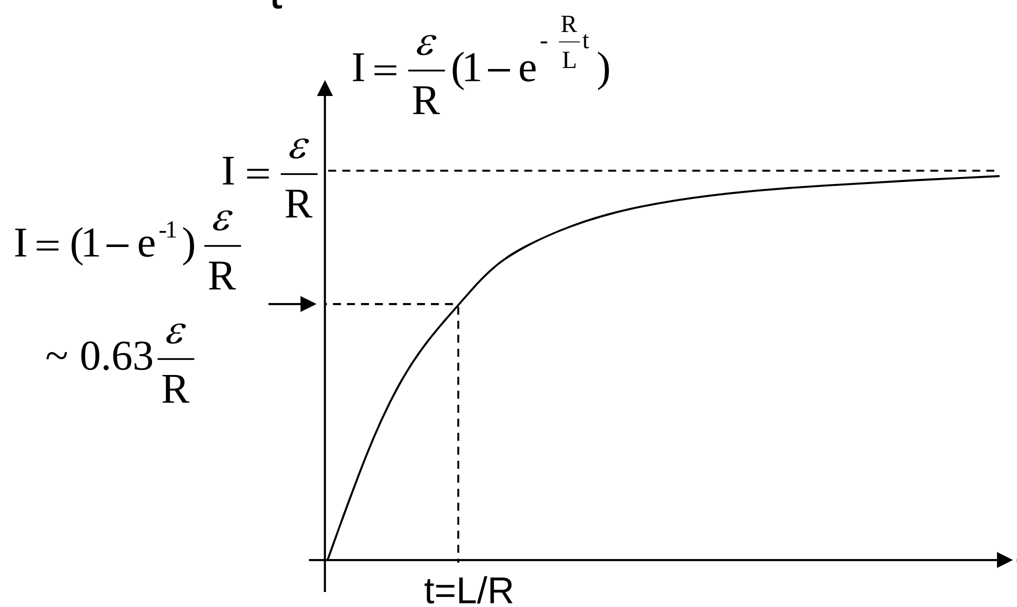
L/R time constant

τ=L/R is known as the time constant. It indicates the response time (how fast you can up a current) of the RC circuit.



$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$



$$\sqrt{2} \approx 1.414$$

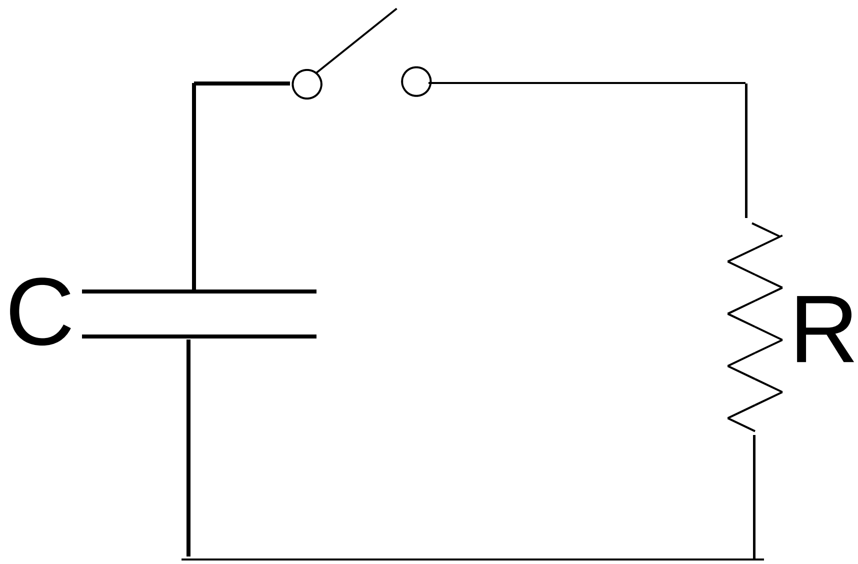
$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RL circuits

From Class 21

Slide #6

RC Circuits – Discharging ← Charge



$$0 = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = 0$$

$$\Rightarrow CR\,dq = -q\,dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n\, q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = K\,e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = Q \Rightarrow Q = K$$

$$\therefore\ q = \underline{\underline{Qe^{-\frac{t}{CR}}}}$$

$$I = \frac{dq}{dt} = -\underline{\underline{\frac{Q}{RC}e^{-\frac{t}{CR}}}}$$

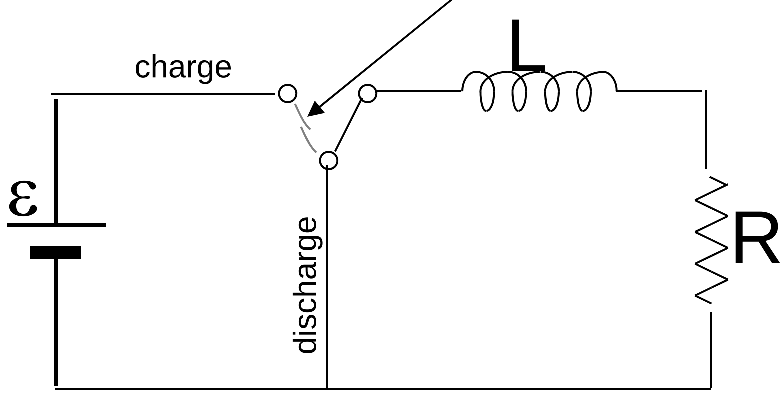
$$\Delta V_R = IR = -\underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$\Delta V_R + \Delta V_C = 0$

RL Circuits – Discharging ← Current

Note special switch



$$0 = L\frac{dI}{dt} + IR \Rightarrow LdI = -IRdt$$

$$\Rightarrow \frac{dI}{I} = -\frac{R}{L}dt$$

Integration constant

$$\Rightarrow \ell n\, I = -\frac{R}{L}t + K'$$

$$\Rightarrow I = Ke^{-\frac{R}{L}t} \qquad (K = e^{K'})$$

$$\text{At } t = 0, I = I_0 = \frac{\varepsilon}{R} \Rightarrow K = I_0 = \frac{\varepsilon}{R}$$

$$\therefore I = I_0 e^{-\frac{R}{L}t} \quad \text{or} \quad \frac{\varepsilon}{R}e^{-\frac{R}{L}t}$$

$$I = I_0 e^{-\frac{t}{CR}}$$

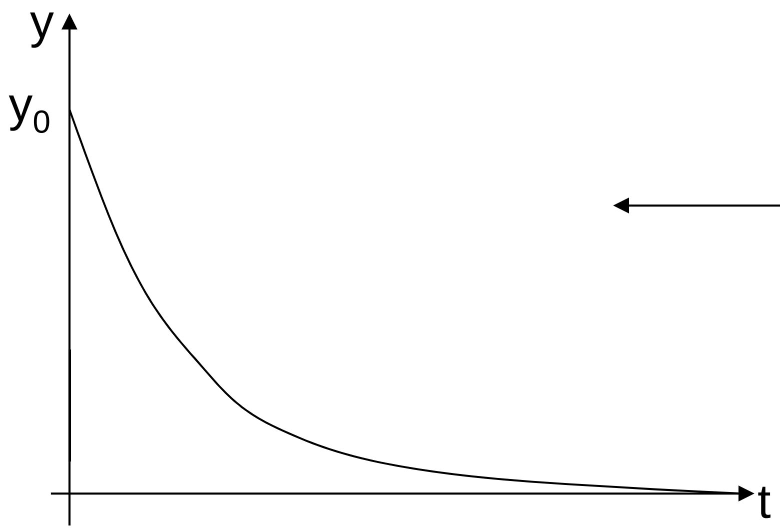
$$\Delta V_R = IR = I_0 R e^{-\frac{R}{L}t}$$

$$\Delta V_L = L\frac{dI}{dt} = -I_0 R e^{-\frac{R}{L}t}$$

$\Delta V_R + \Delta V_L = 0$

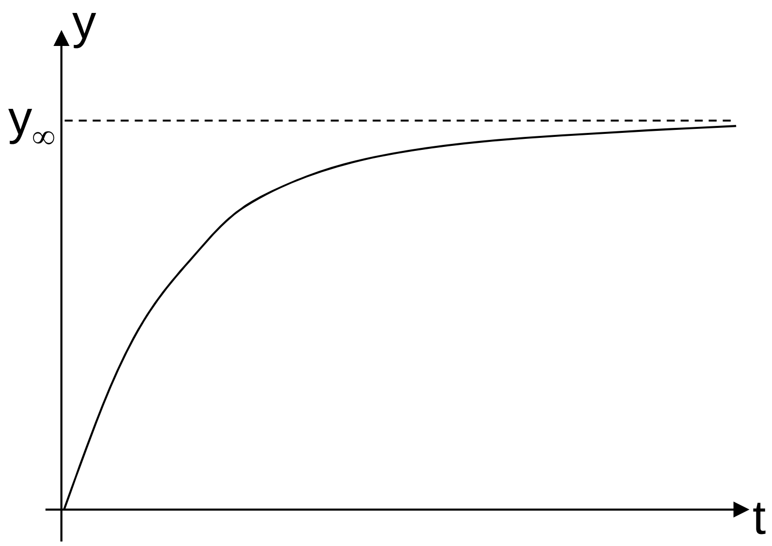
In Summary

For both charge and discharge, Q, I, ΔV_C, and ΔV_R must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

$$y = y_\infty (1 - e^{-\frac{t}{RC}})$$



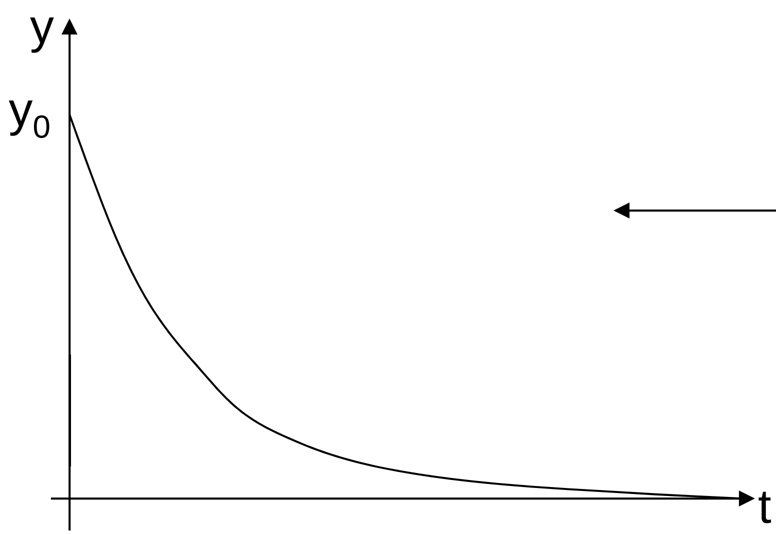
y can be Q, I, ΔV_C, or ΔV_R

From Class 21

Slide #7

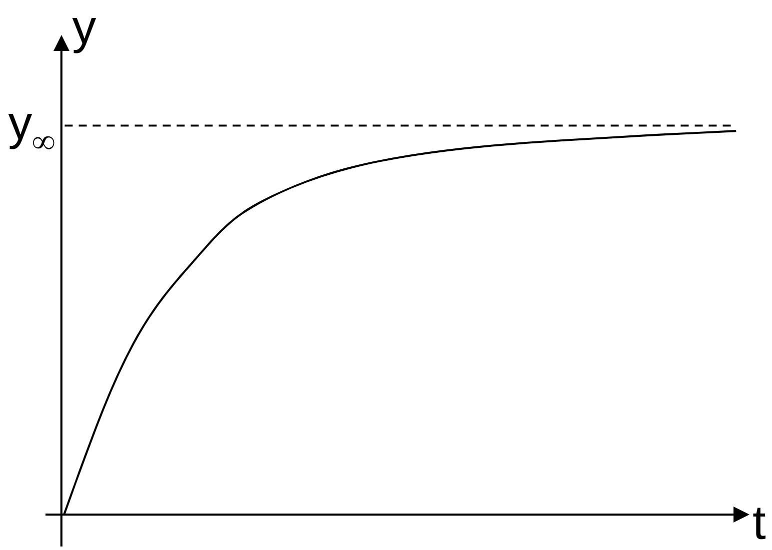
In Summary

For both charge and discharge, I, dI/dt, ΔV_L, and ΔV_R must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

$$y = y_\infty (1 - e^{-\frac{t}{RC}})$$



y can be I, dI/dt, ΔV_L, or ΔV_R