

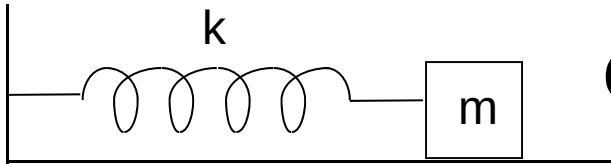
Class 36 LC Oscillation

Oscillation - Spring

Potential energy $\leftrightarrow$ Kinetic energy

$$\frac{1}{2}kx^2$$

$$\frac{1}{2}mv^2$$



Conservation of energy:

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$$

$$= \frac{1}{2}kA^2 \text{ or } \frac{1}{2}mv_{\text{max}}^2$$

Equation of motion: $m \frac{d^2}{dt^2} x = -kx$

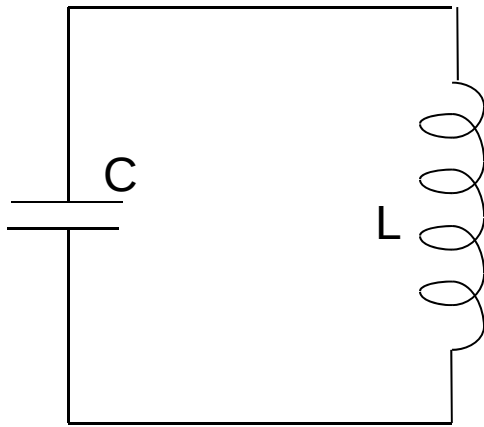
Solution: $x = A \sin(\omega t + \phi)$ with $\omega = \sqrt{\frac{k}{m}}$

Oscillation – LC circuit

Electric energy $\leftrightarrow$ Magnetic energy

$$\frac{1}{2} \frac{Q^2}{C}$$

$$\frac{1}{2} LI^2$$



Conservation of energy:

$$\frac{1}{2} \frac{1}{C} Q^2 + \frac{1}{2} LI^2 = \text{constant}$$

$$= \frac{1}{2} \frac{1}{C} Q_{\text{max}}^2 \text{ or } \frac{1}{2} LI_{\text{max}}^2$$

Kirchhoff's rule : $-L \frac{dI}{dt} = \frac{1}{C} Q \Rightarrow \frac{d^2 Q}{dt^2} = -\frac{1}{C} Q$

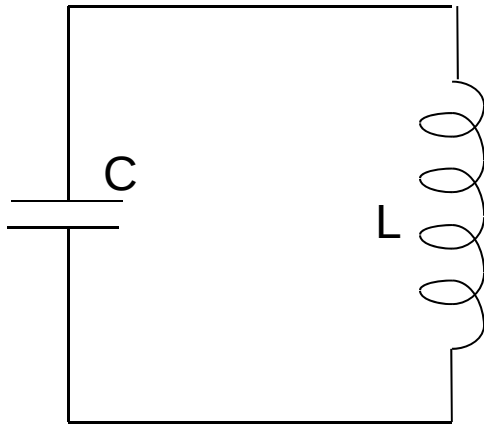
Solution: Solve the differential equation!

Oscillation – LC circuit

Electric energy $\leftrightarrow$ Magnetic energy

$$\frac{1}{2} \frac{Q^2}{C}$$

$$\frac{1}{2} LI^2$$



Conservation of energy:

$$\frac{1}{2} \frac{1}{C} Q^2 + \frac{1}{2} LI^2 = \text{constant}$$

Kirchhoff's rule :

$$-L \frac{dI}{dt} = \frac{1}{C} Q \Rightarrow L \frac{d^2 Q}{dt^2} = -\frac{1}{C} Q$$

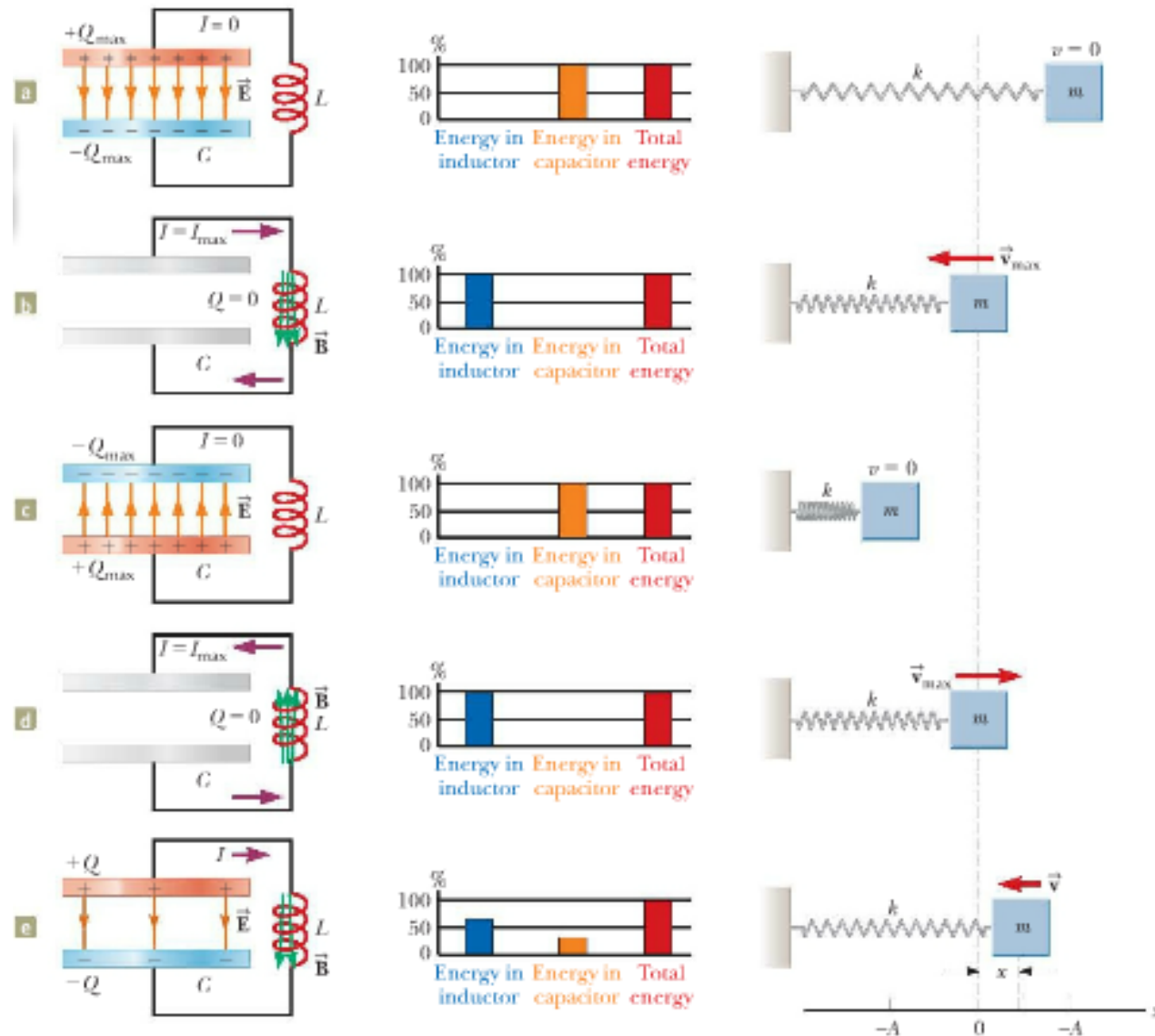
Solution:

$$x = A \sin(\omega t + \phi) \text{ with } \omega = \sqrt{\frac{k}{m}}$$

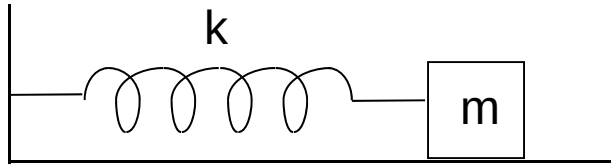
↓

$$Q = A \sin(\omega t + \phi) \text{ with } \omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

Similarity between Spring Oscillation and LC Oscillation I



Similarity between Spring Oscillation and LC Oscillation II



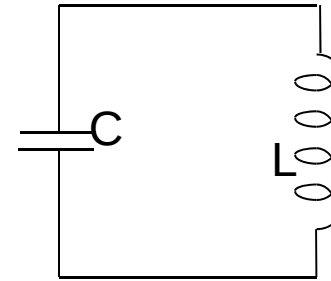
Potential energy $\leftrightarrow$ Kinetic energy

$$\frac{1}{2} kx^2$$

$$\frac{1}{2} mv^2$$

Newton's Law

$$m \frac{d^2}{dt^2} x = -kx$$



Electric energy $\leftrightarrow$ Magnetic energy

$$\frac{1}{2} \frac{Q^2}{C}$$

$$\frac{1}{2} LI^2$$

Kirchhoff's rule:

$$-L \frac{dI}{dt} = \frac{1}{C} Q \Rightarrow \frac{d^2 Q}{dt^2} = -\frac{1}{C} Q$$

Potential energy $\frac{1}{2} kx^2$

Kinetic energy $\frac{1}{2} mv^2$

Spring constant k

Mass m

Displacement x

Velocity v $v = \frac{dx}{dt}$

Electrical energy $\frac{1}{2} \frac{Q^2}{C}$

Magnetic energy $\frac{1}{2} LI^2$

1/Capacitance $\frac{1}{C}$

Inductance L

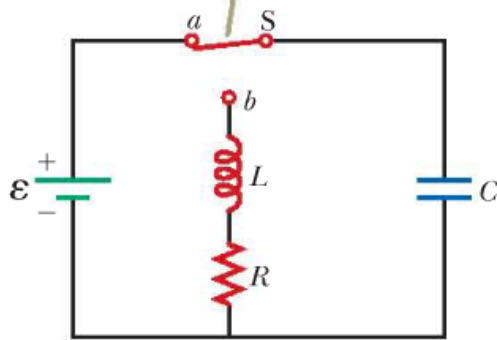
Charge Q

Current I $I = \frac{dQ}{dt}$

RLC circuit

RLC circuit

The switch is set first to position *a*, and the capacitor is charged. The switch is then thrown to position *b*.

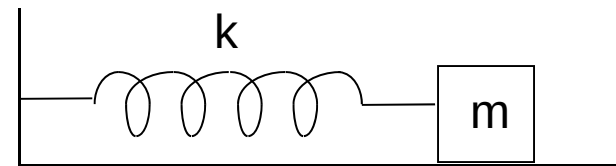


ACTIVE FIGURE 32.15



A series *RLC* circuit.

Damped Oscillation



$$\text{Friction} = -bv$$

Kirchhoff's rule :

$$0 = \frac{Q}{C} + IR + L \frac{dI}{dt} \quad (I = \frac{d}{dt}Q)$$

$$\Rightarrow L \frac{d^2}{dt^2}Q + R \frac{d}{dt}Q + \frac{Q}{C} = 0$$

Equation of motion :

$$m \frac{d^2}{dt^2}x = -bv - kx \quad (v = \frac{d}{dt}x)$$

$$\Rightarrow m \frac{d^2}{dt^2}x + b \frac{d}{dt}x + kx = 0$$

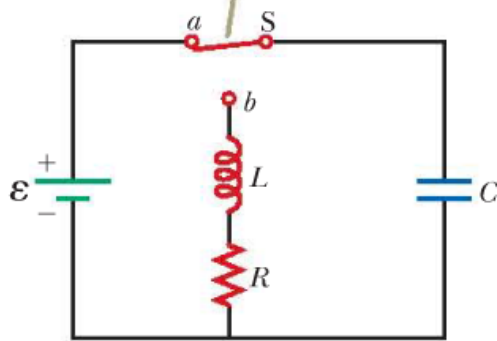
RLC circuit and Mechanical Oscillation

RLC circuit	Mechanical
Q	x
$I = dQ/dt$	$v = dx/dt$
C	$1/k$
R	b
L	m
Magnetic energy $\frac{1}{2}LI^2$	Kinetic energy $\frac{1}{2}mv^2$
Electrical energy $\frac{1}{2}(1/C)Q^2$	Potential energy $\frac{1}{2}kx^2$

RLC circuit

RLC circuit

The switch is set first to position *a*, and the capacitor is charged. The switch is then thrown to position *b*.



ACTIVE FIGURE 32.15

A series *RLC* circuit.



Solution:

$$Q(t) = Q_0 e^{-\frac{R}{2L}t} \cos \omega_d t$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

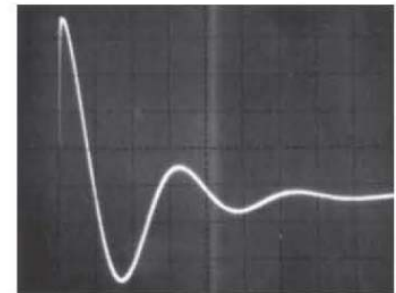
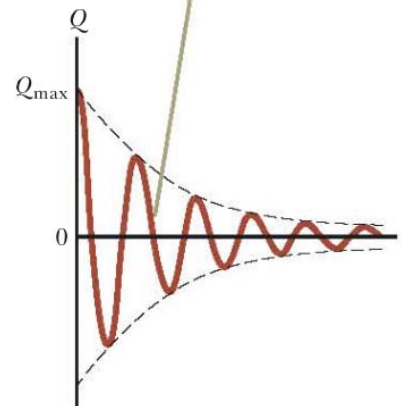
under damped
critically damped
over damped

Kirchhoff's rule :

$$0 = \frac{Q}{C} + IR + L \frac{dI}{dt} \quad (I = \frac{d}{dt}Q)$$

$$\Rightarrow L \frac{d^2}{dt^2}Q + R \frac{d}{dt}Q + \frac{Q}{C} = 0$$

The *Q*-versus-*t* curve represents a plot of Equation 32.31.



Damping

$$Q(t) = Q_0 e^{-\frac{R}{2L}t} \cos \omega_d t$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

ω_d real: under damped

$\omega_d = 0$: critically damped

ω_d imaginary: overdamped

