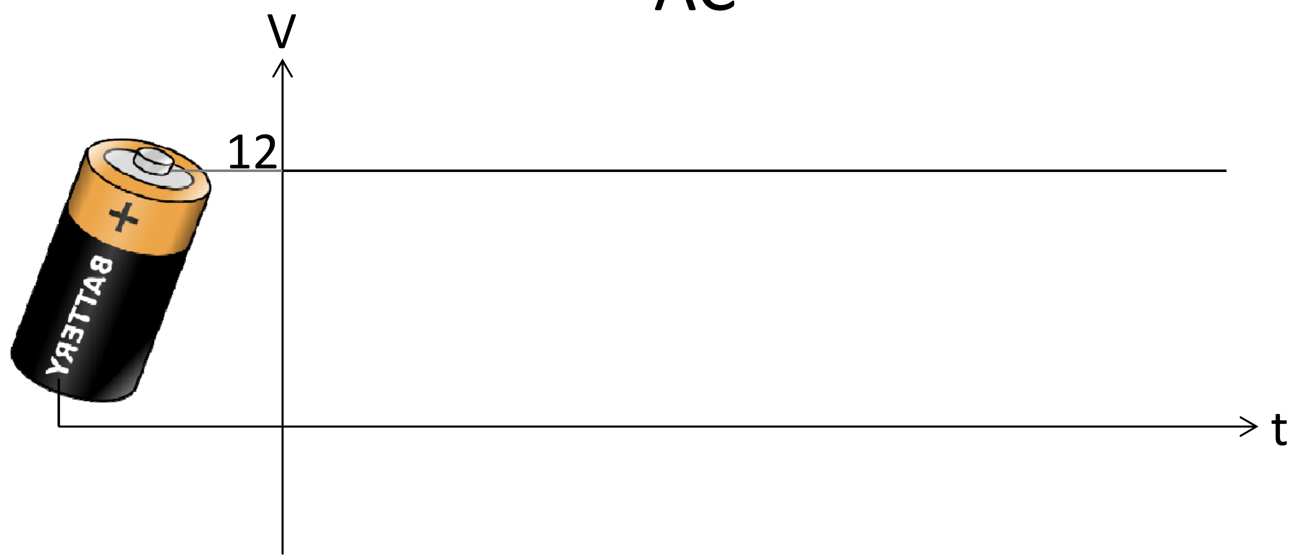
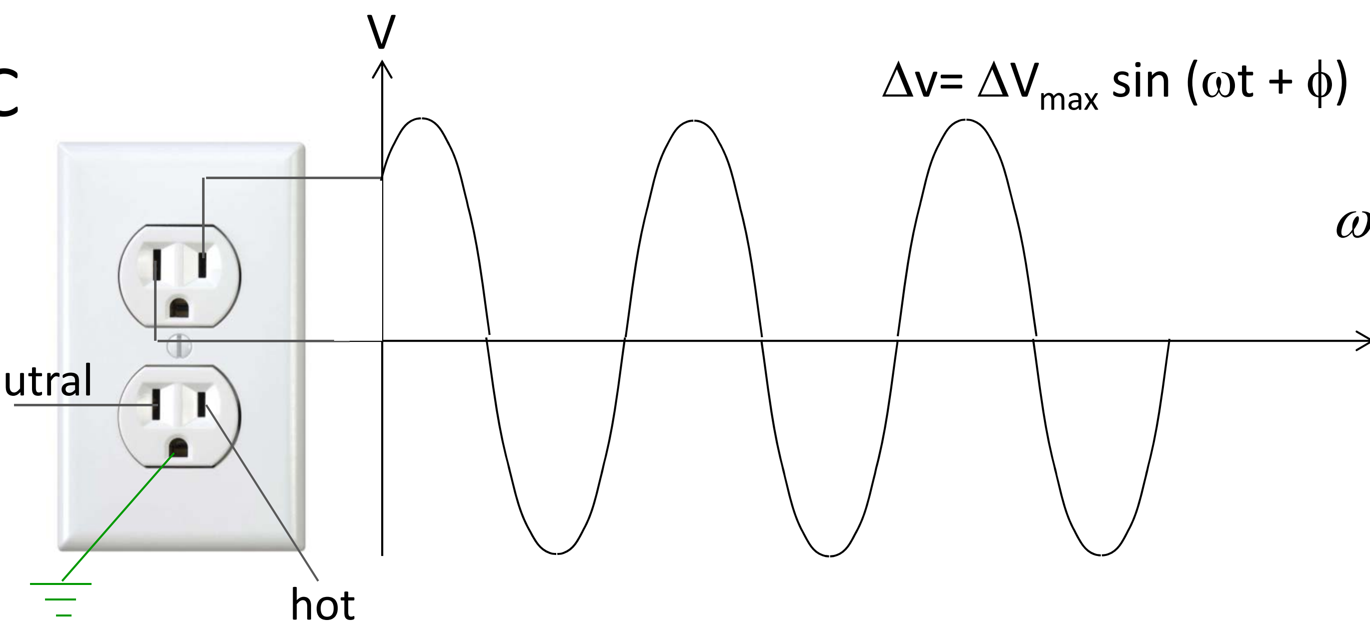
Class 37 Displacement currents

AC

DC



AC

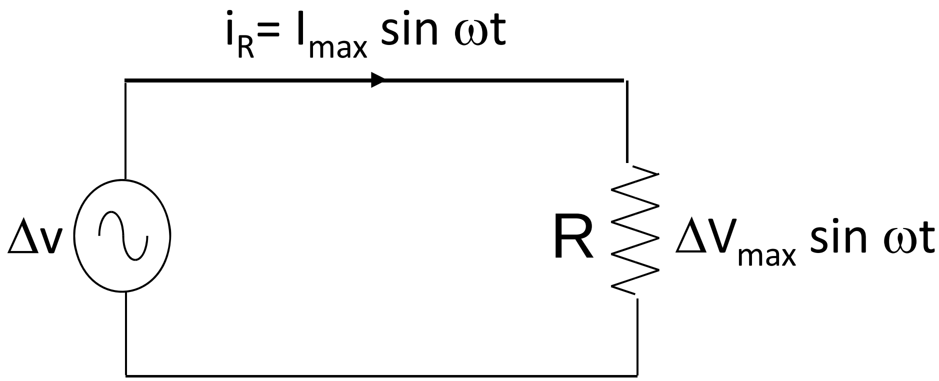


$$\Delta v = \Delta V_{max} \sin(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

For US, $f = 60Hz$ and $V_{max} \sim 155V$

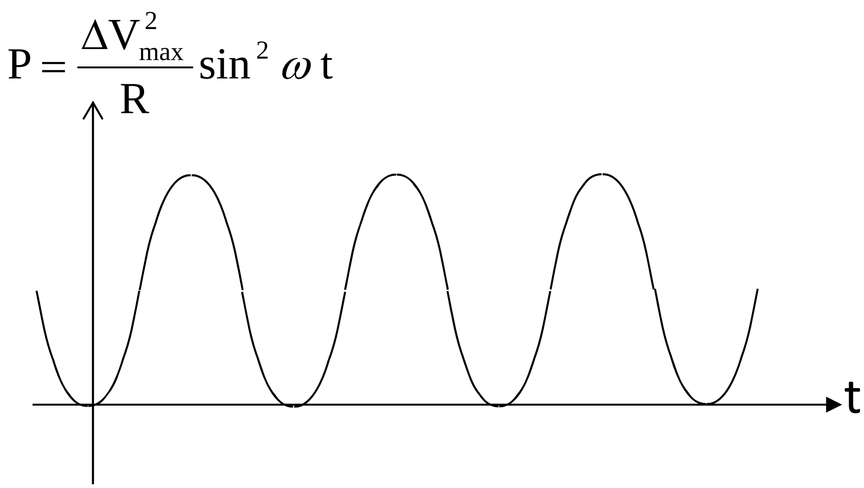
Power dissipated in a Resistor in an AC circuit



$$I_{max} = \frac{\Delta V_{max}}{R}$$

$$\Delta v = \Delta V_{max} \sin \omega t$$

$$i_R = I_{max} \sin \omega t$$



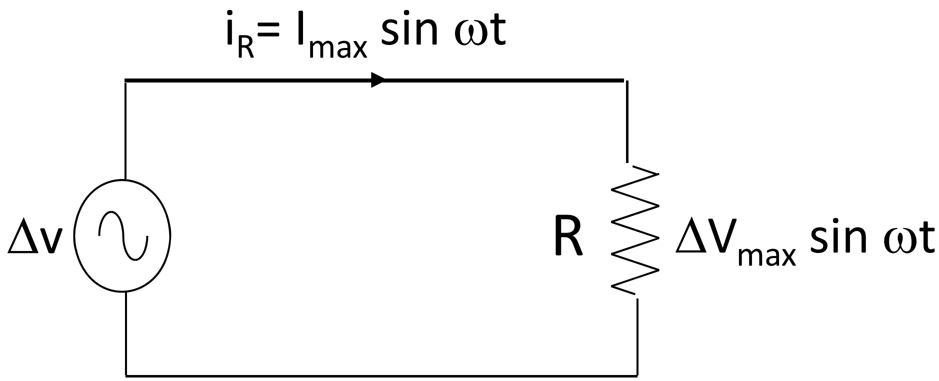
$$P = i_R \Delta v = \frac{\Delta V_{max}^2}{R} \sin^2 \omega t$$

Average power dissipated in one period (T):

$$\langle P \rangle = \frac{1}{T} \int_0^T \frac{\Delta V_{max}^2}{R} \sin^2 \omega t \; dt$$

$$= \frac{1}{2} \frac{\Delta V_{max}^2}{R}$$

Root Mean Square Voltage and Current

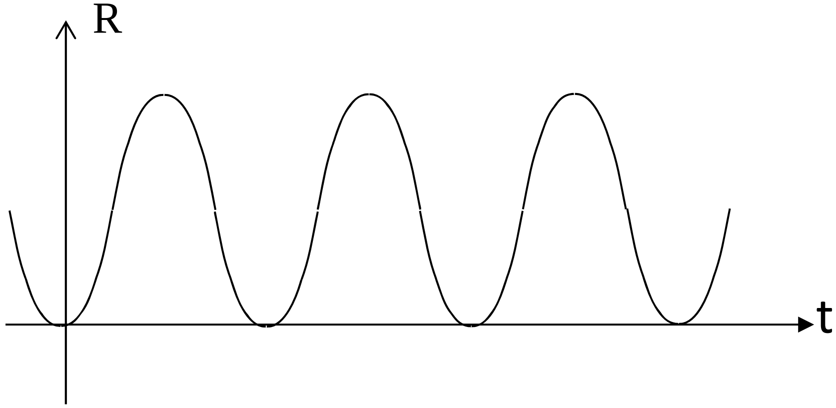


$$< P > = \frac{1}{2}\frac{\Delta V_{max}^2}{R}$$

$$\text{Define } \Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} \sim 0.707\ V_{max}$$

$$< P > = \frac{\Delta V_{rms}^2}{R}$$

$$P = \frac{\Delta V_{max}^2}{R}\sin^2 \omega t$$

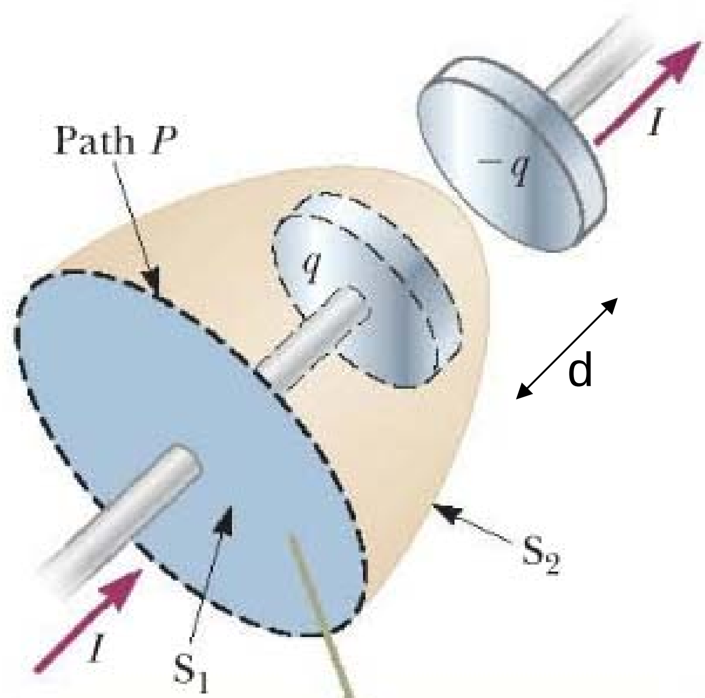


$$i_{max} = \frac{V_{max}}{R} \Rightarrow i_{rms} = \frac{V_{rms}}{R}$$

$$\text{with } i_{rms} = \frac{i_{max}}{\sqrt{2}} \sim 0.707 i_{max}$$

$$< P > = i_{rms}^2 R$$

Revisit Ampere's Law



The conduction current I in the wire passes only through S_1, which leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

For DC, I=0 and B=0, so there is no problem.

If I is changing with time, I $\neq$0 (except at the gap) and there will be a magnetic field (changing with time also).

If the gap d is very small (d $\rightarrow$0), there should be magnetic field everywhere surrounding the wire even though there is no physical current through the gap.

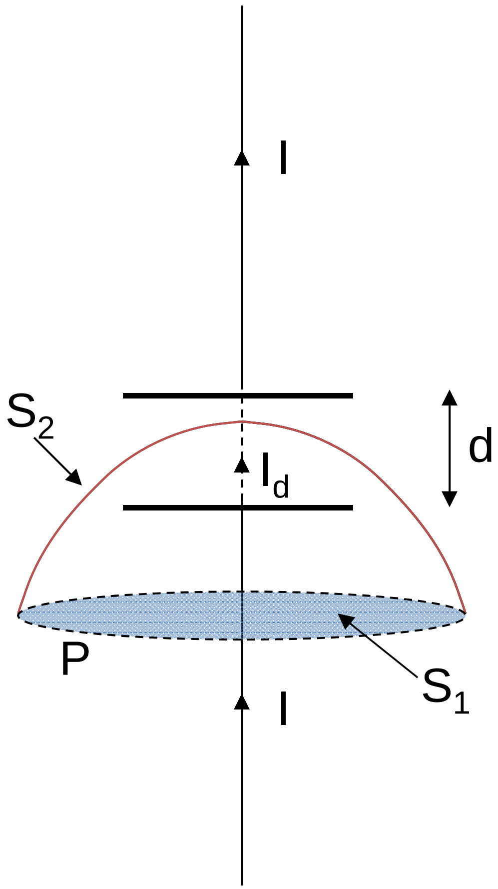
The problem now is:

$$\text{For surface } S_1: \oint_{\text{Path P}} \vec{B}\cdot d\vec{s} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_1}} = \mu_0 I$$

$$\text{For surface } S_2: \oint_{\text{Path P}} \vec{B}\cdot d\vec{s} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_2}} = 0$$

How to reconcile the difference?

Maxwell’s proposal



We can introduce an imaginary current, called displacement current, I_d within the gap so the current now looks like continuous.

With this displacement current:

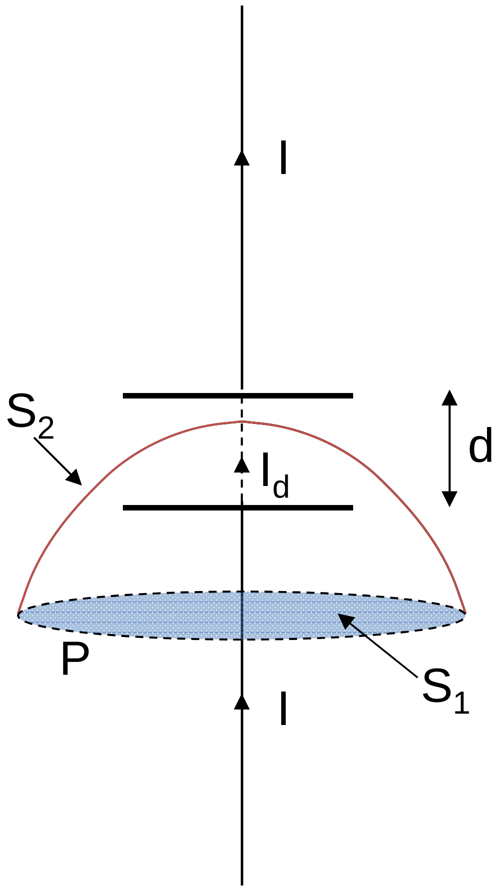
$$\text{For surface } S_1 : \oint_P \vec{B} \cdot d\vec{S} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_1}} = \mu_0 I$$

$$\text{For surface } S_2 : \oint_P \vec{B} \cdot d\vec{S} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_2}} = \mu_0 I_d = \mu_0 I$$

Ampere’s Law now becomes:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 (I_{\text{Enclosed}_1} + I_d)$$

Displacement current



But at the end what is a displacement current?

It is not a real current due to motion of charges within the gap, so we have to relate it to something that really exists in the gap: electric field.

$$I_d = I = \frac{dq}{dt} = C\frac{dV}{dt} \qquad (q = CV)$$

$$= Cd\frac{dE}{dt} \qquad (V = Ed)$$

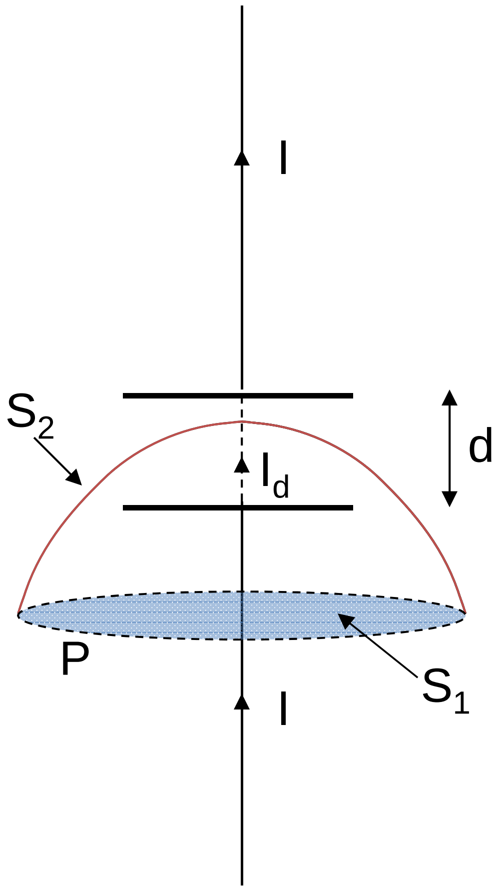
$$= \frac{\varepsilon_0 A}{d} \cdot d\frac{dE}{dt} \qquad (C = \frac{\varepsilon_0 A}{d})$$

$$= \varepsilon_0 \frac{d(EA)}{dt}$$

$$= \varepsilon_0 \frac{d\Phi_E}{dt}$$

Abstraction

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$



We got this idea from parallel plate capacitor. We expand this and say this is generally true for any geometry and Ampere's Law now becomes:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 (I_{Enclosed} + \varepsilon_0 \frac{d\Phi_E}{dt})$$

$$= \mu_0 (I_{Enclosed} + \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A})$$

Maxwell's Equations

Maxwell's 1st equation: (Gauss's Law foe electric field)

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{in}$$

Maxwell's 2nd equation: (Gauss's Law for magnetic field)

$$\oiint \vec{B} \cdot d\vec{A} = 0$$

Maxwell's 3rd equation: (Faraday's Law)

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{d\,t}\int \vec{B} \cdot d\vec{A}$$

Maxwell's 4rd equation: (Ampere's Law – Now complete)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{in} + \varepsilon_0 \frac{d}{dt}\int \vec{E} \cdot d\vec{A})$$

Note the symmetry between these two equations.

Maxwell's equations describe only the fields, it does not include the effect of the field on charges or currents:

$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad (\text{or } d\vec{F}_B = Id\vec{s} \times \vec{B})$$

Not included in Maxwell's equations

Maxwell's Equations

PHY 232:

$$\varepsilon_0 \oint\!\!\oint \vec{E}\cdot d\vec{S} = Q_{in}$$

$$\oint\!\!\oint \vec{B}\cdot d\vec{S} = 0$$

$$\oint \vec{E}\cdot d\vec{\ell} + \frac{\partial}{\partial t}\oint\!\!\oint \vec{B}\cdot d\vec{S} = 0$$

$$\oint\!\!\oint \vec{B}\cdot d\vec{\ell} - \varepsilon_0\mu_0 \frac{\partial}{\partial t}\oint\!\!\oint \vec{E}\cdot d\vec{S} = \mu_0 I_{in}$$

For most physicists / engineers:

$$\varepsilon_0 \nabla\cdot\vec{E} = \rho$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} + \frac{\partial}{\partial t}\vec{B} = 0$$

$$\frac{1}{\mu_0}\cdot\nabla\times\vec{B} - \varepsilon_0\frac{\partial}{\partial t}\vec{E} = J$$

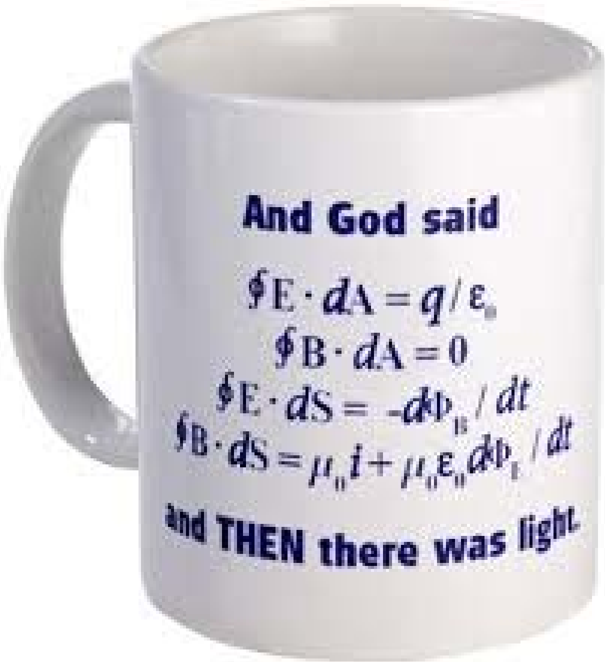
For theoretical physicists:

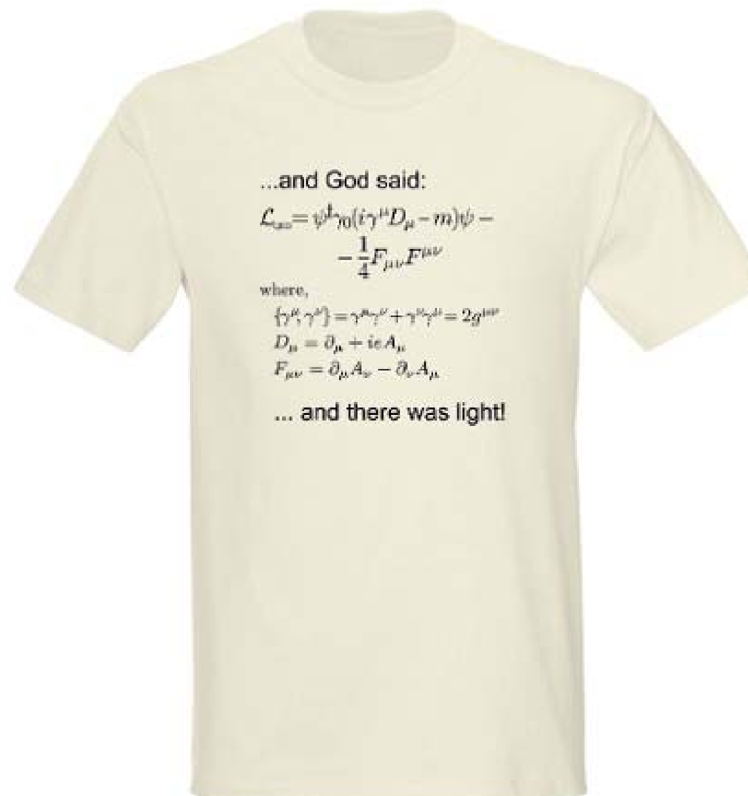
$$\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} j^\beta$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$$

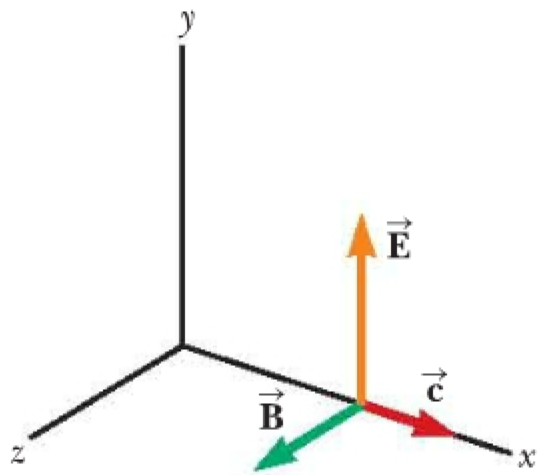
(Gaussian units)

Three different forms of Maxwell's Equations





Linearly polarized electromagnetic Waves



ACTIVE FIGURE 34.5

Electric and magnetic fields of an electromagnetic wave traveling at velocity $\vec{\mathbf{c}}$ in the positive x direction. The field vectors are shown at one instant of time and at one position in space. These fields depend on x and t.

Linearly polarized waves

The wave is traveling in the $\vec{E} \times \vec{B}$ direction.

Applying Maxwell's Third Equation to Plane Electromagnetic Waves

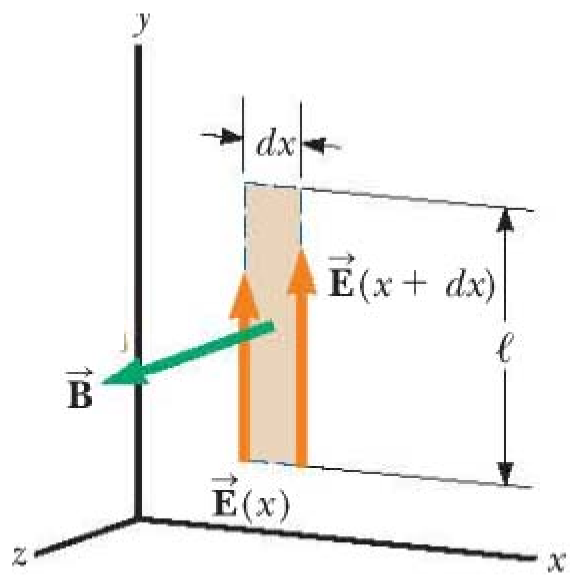


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x+dx)$.

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{s} = E(x+dx)\ell + 0 - E(x)\ell + 0$$

$$= \ell[E(x+dx) - E(x)]$$

$$= \ell\frac{\partial E}{\partial x}\cdot dx$$

$$-\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A} = -\frac{\partial}{\partial t}(B\cdot \ell\, dx)$$

$$= -\ell\frac{\partial B}{\partial t}\cdot dx$$

$$\therefore\quad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$