Class 38: Electromagnetic radiation

Applying Maxwell’s Third Equation to Plane Electromagnetic Waves

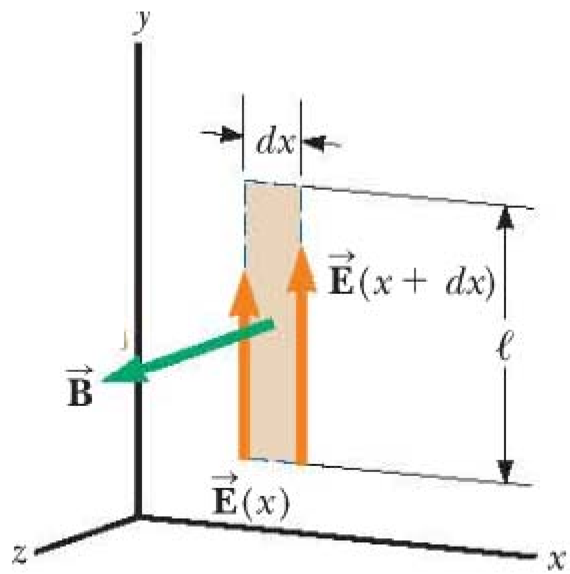


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x + dx)$.

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \oiint \vec{B} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{s} = E(x + dx)\ell + 0 - E(x)\ell + 0$$

$$= \ell[E(x + dx) - E(x)]$$

$$= \ell \frac{\partial E}{\partial x} \cdot dx$$

$$-\frac{d}{dt} \oiint \vec{B} \cdot d\vec{A} = -\frac{\partial}{\partial t}(B \cdot \ell\, dx)$$

$$= -\ell \frac{\partial B}{\partial t} \cdot dx$$

$$\therefore \quad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Applying Maxwell's Fourth Equation to Plane Electromagnetic Waves

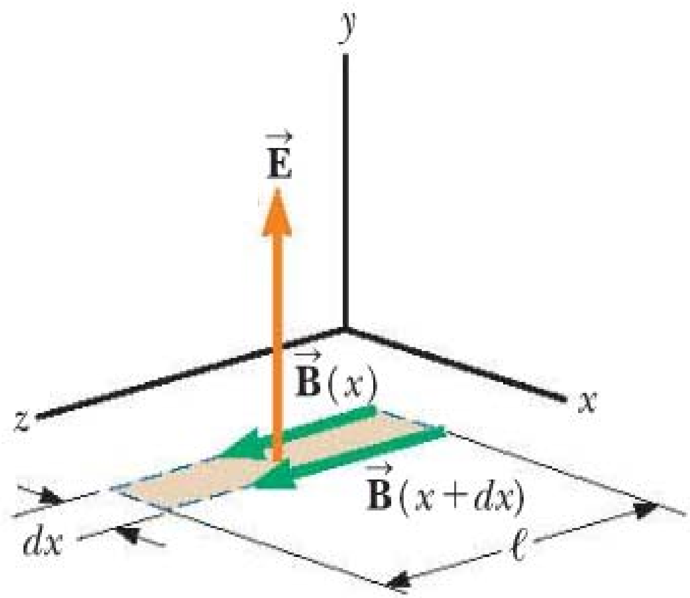


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{B}(x)$ to $\vec{B}(x + dx)$.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 (I_{in} + \varepsilon_0 \frac{d}{dt}\oiint \vec{E}\cdot d\vec{A}) \qquad (I_{in} = 0)$$

$$\oint \vec{B}\cdot d\vec{s} = B(x)\ell + 0 - B(x+dx)\ell + 0$$

$$= -\ell[B(x+dx) - B(x)]$$

$$= -\ell\frac{\partial B}{\partial x}\cdot dx$$

$$\mu_0\varepsilon_0 \frac{d}{d\,t}\oiint \vec{E}\cdot d\vec{A} = \varepsilon_0\mu_0 \frac{\partial}{\partial\, t}(E\cdot \ell\, dx)$$

$$= \varepsilon_0\mu_0\ell\frac{\partial\, E}{\partial\, t}\cdot dx$$

$$\therefore\ -\frac{\partial B}{\partial x} = \varepsilon_0\mu_0\frac{\partial\, E}{\partial\, t}$$

Classical wave equation

$$\frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2}$$

Electric and magnetic fields in wave motion

Third equation $\Rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \Rightarrow \begin{cases} \frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t} & \text{---(1a)} \\ \frac{\partial^2 E}{\partial t \partial x} = -\frac{\partial^2 B}{\partial t^2} & \text{---(1b)} \end{cases}$

Fourth equation $\Rightarrow -\frac{\partial B}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \Rightarrow \begin{cases} \frac{\partial^2 B}{\partial x^2} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial x \partial t} & \text{---(2a)} \\ \frac{\partial^2 B}{\partial t \partial x} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} & \text{---(2b)} \end{cases}$

(1a) and (2b) $\Rightarrow \frac{\partial^2 E}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \Rightarrow \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2}$

(1b) and (2a) $\Rightarrow -\frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = -\frac{\partial^2 B}{\partial t^2} \Rightarrow \varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2} = \frac{\partial^2 B}{\partial x^2}$

Both E and B follow the classical wave equation :

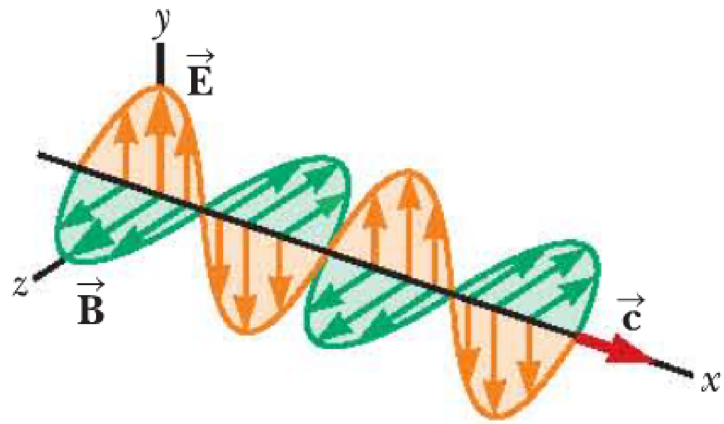
$$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial x^2}$$

with $v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ ←

$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$

Electromagnetic plane waves



ACTIVE FIGURE 34.8

A sinusoidal electromagnetic wave moves in the positive *x* direction with a speed *c*.

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} \quad \text{and} \quad \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = \frac{\partial^2 B}{\partial x^2}$$

Sinusoidal solution :

$$E = E_{max} \cos(kx - \omega t)$$

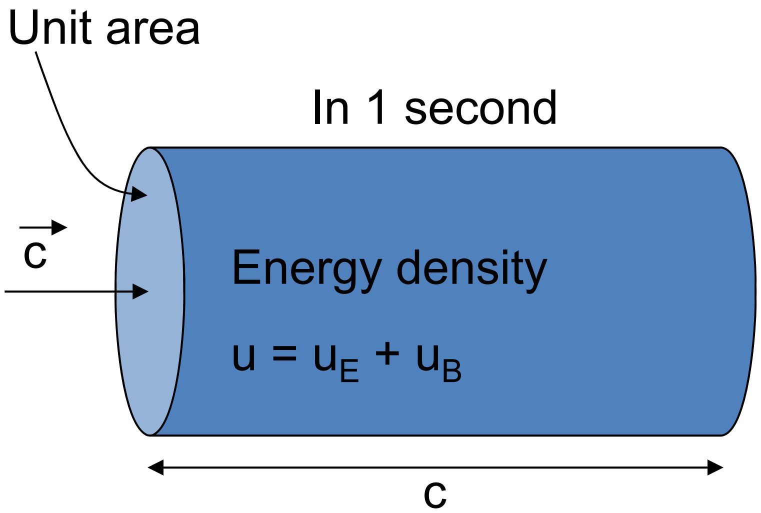
$$B = B_{max} \cos(kx - \omega t)$$

In here : $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f$

$$\omega = ck, \quad E_{max} = cB_{max}, \text{ and } \quad E = cB$$

$$\Downarrow$$

$$c = f\lambda$$



Intensity

Intensity I is defined as:

$$I = S_{avg}$$

From pointing vector:

$$I = S_{avg}$$

$$\text{But } S = c(\frac{1}{\mu_0}B^2) \text{ or } c(\varepsilon_0 E^2)$$

$$\therefore I = c(\frac{1}{\mu_0}B^2) \text{ or } c(\varepsilon_0 E^2)$$

$$= c(\frac{1}{\mu_0} < B^2 >) \text{ or } c(\varepsilon_0 < E^2 >)$$

$$= c(\frac{1}{2\mu_0}B^2_{max}) \text{ or } c(\frac{\varepsilon_0}{2}E^2_{max})$$

From above picture:

$$I = < \text{energy in the blue cylinder} >$$

$$= c(< u_E > + < u_B >)$$

$$= c(\frac{1}{2}\varepsilon_0 < E^2 > + \frac{1}{2\mu_0} < B^2 >)$$

$$= c(\frac{1}{4}\varepsilon_0 E^2_{max} + \frac{1}{4\mu_0} B^2_{max})$$

$$= c(\frac{1}{4}\varepsilon_0 (cB)^2_{max} + \frac{1}{4\mu_0} B^2_{max}) \text{ or } c(\frac{1}{4}\varepsilon_0 E^2_{max} + \frac{1}{4\mu_0}(\frac{E_{max}}{c})^2_{max})$$

$$= c(\frac{1}{4\mu_0}B^2_{max} + \frac{1}{4\mu_0}B^2_{max}) \text{ or } c(\frac{1}{4}\varepsilon_0 E^2_{max} + \frac{1}{4}\varepsilon_0 E^2_{max})$$

$$= c(\frac{1}{2\mu_0}B^2_{max}) \text{ or } c(\frac{1}{2}\varepsilon_0 E^2_{max})$$

Poynting vector

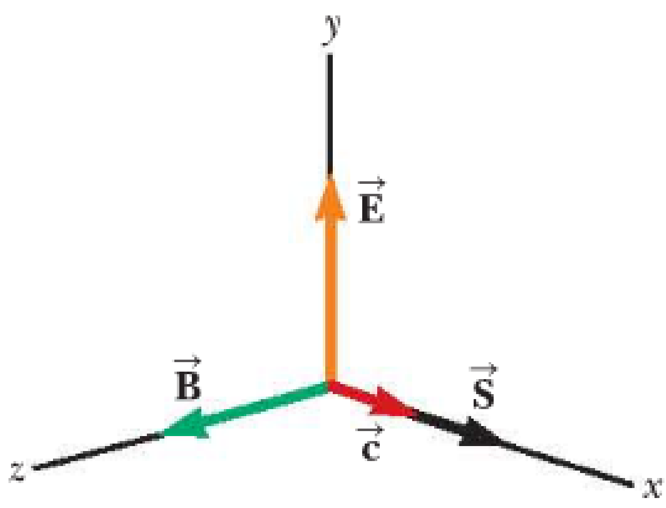


Figure 34.10 The Poynting vector $\vec{S}$ for a plane electromagnetic wave is along the direction of wave propagation.

At any point, knowing $\vec{E}$ and $\vec{B}$, we can define Poynting vector $\vec{S}$ as:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

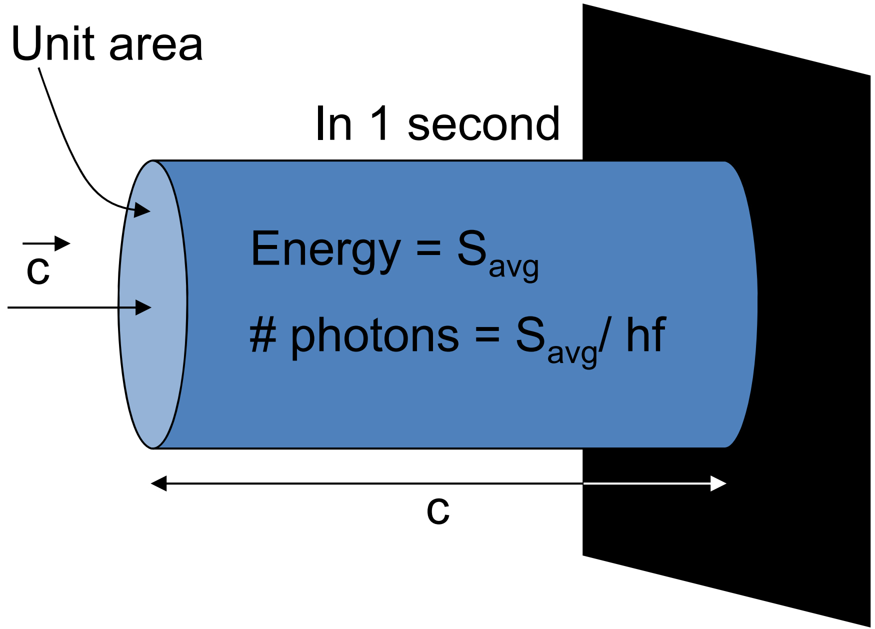
Poynting vector gives the energy passes through a unit surface area perpendicular to the direction of wave propagation. $\vec{S}$ is along the direction of wave propagation and has unit W/m^2.

For plane wave:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \Rightarrow S = \frac{EB}{\mu_0} = \frac{c}{\mu_0}B^2 \text{ or } \frac{1}{c\mu_0}E^2$$

$$\Rightarrow S = c(\frac{1}{\mu_0}B^2) \text{ or } c(\varepsilon_0 E^2)$$

Momentum



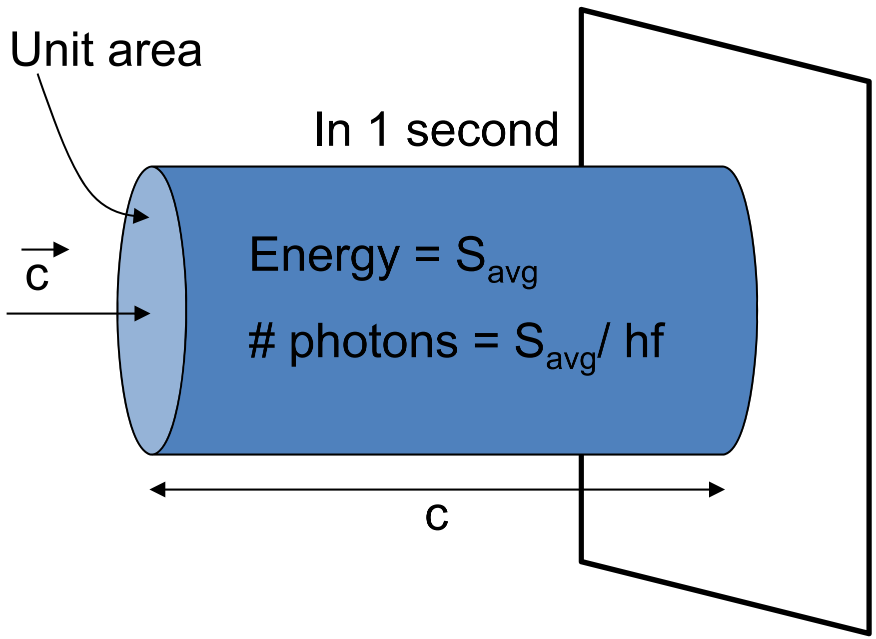
If all these photons are absorbed by the surface:

$$\text{In one second,}\ \Delta p = \frac{S_{avg}}{c}$$

$$\therefore F_{\text{unit area}} = \frac{\Delta p}{\Delta t} = \frac{S_{avg}}{c}$$

But this is on unit area

$$\therefore P = \frac{S_{avg}}{c}$$



If all these photons are reflected by the surface:

$$\text{In one second,}\ \Delta p = 2 \cdot \frac{S_{avg}}{c}$$

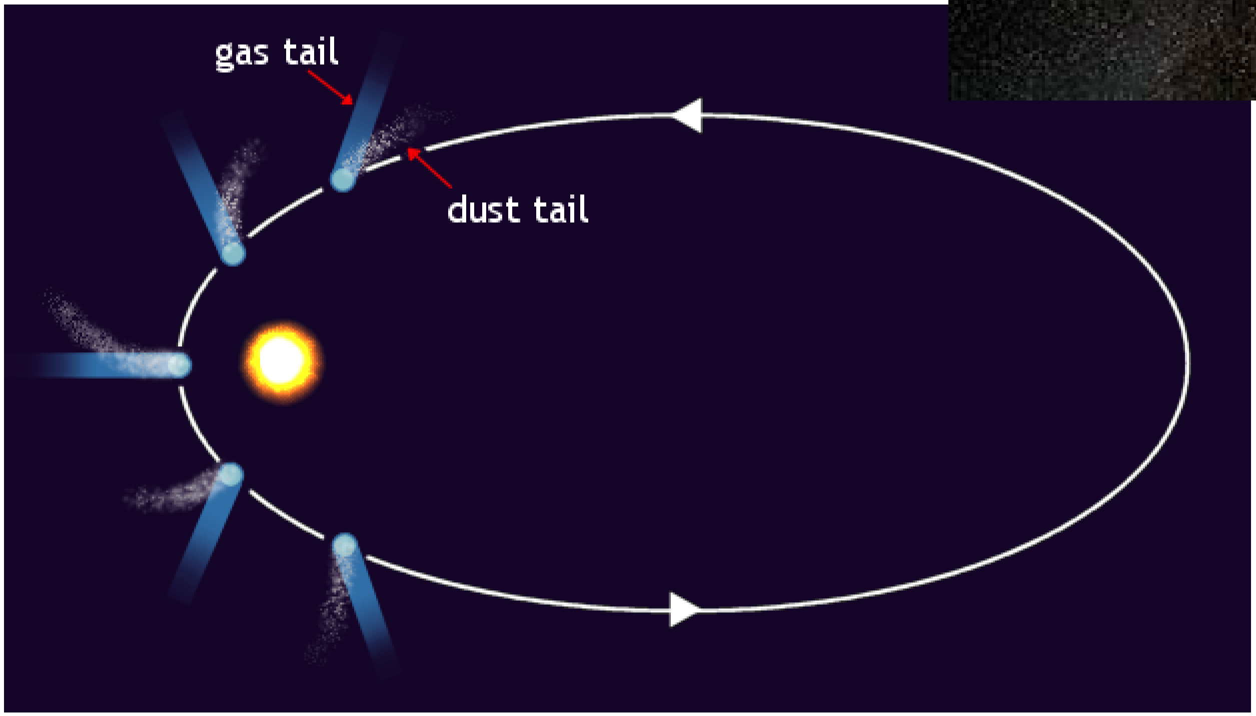
$$\therefore F_{\text{unit area}} = \frac{\Delta p}{\Delta t} = \frac{2S_{avg}}{c}$$

But this is on unit area

$$\therefore P = \frac{2S_{avg}}{c}$$

Radiation Pressure

IKAROS



Comet

Electromagnetic spectrum