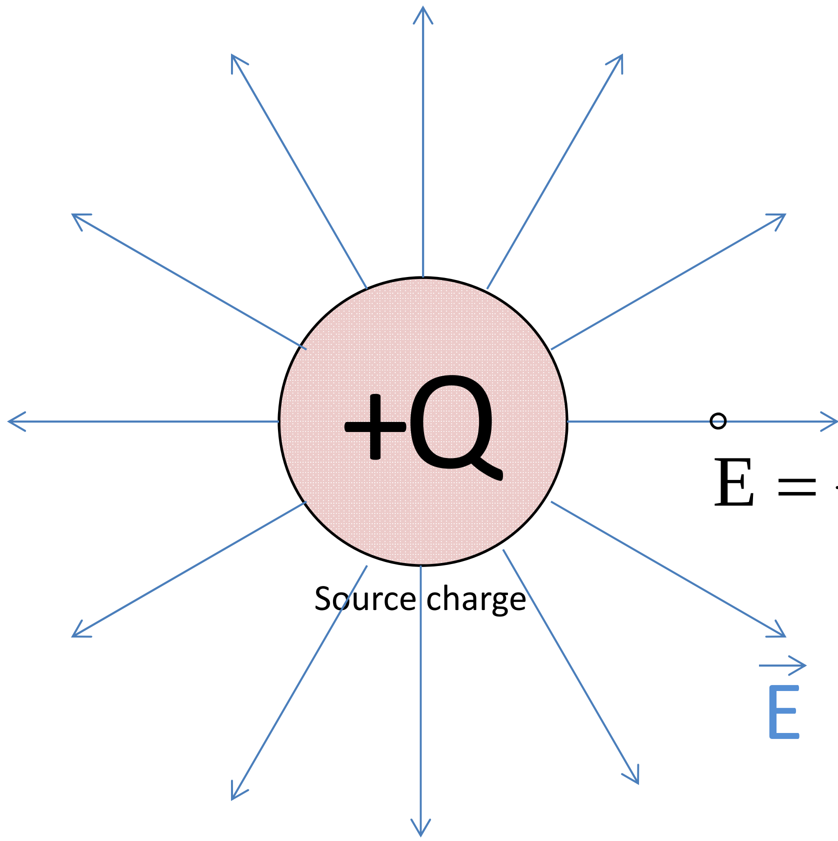
Class 4: Electric Flux and Gauss's Law

Concept of Fields



There are fields attached to a charge. The fields (geometry and intensity) depend on the charge distribution.

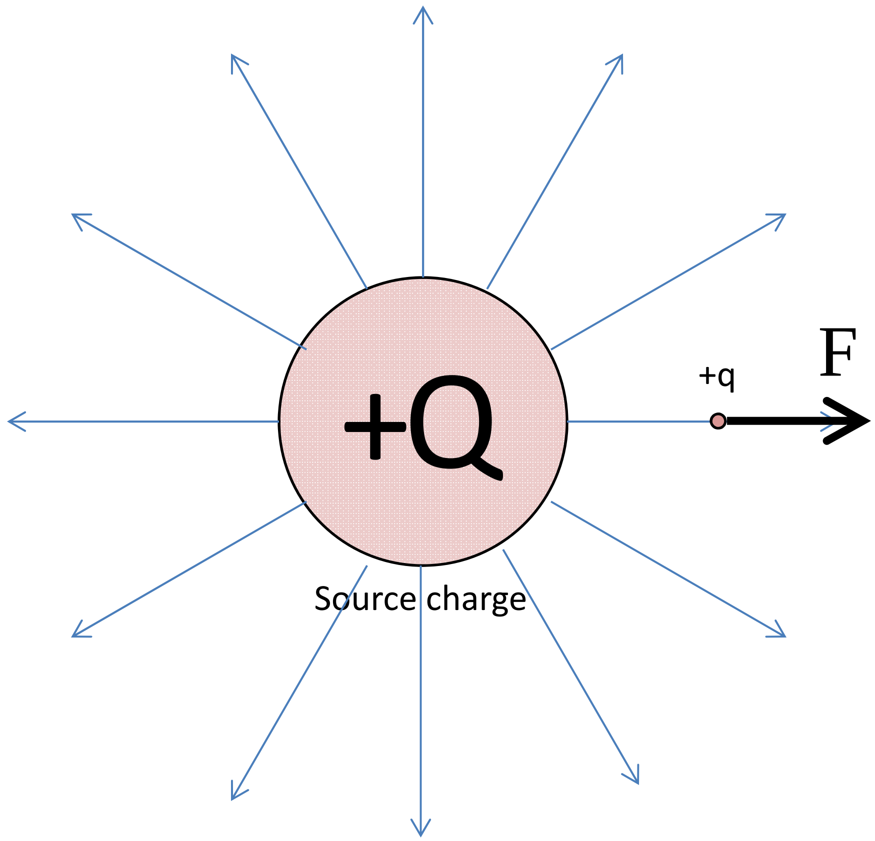
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$ ⟵ Magnitude equation

Electric field is a vector.
Unit of electric field: N/C

The charges that give rise to the electric fields are called the *source* charges.

Charge	$\Rightarrow$	Electric field
Mass	$\Rightarrow$	Gravitational field

Concept of Fields



$$F = qE = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

When another charge is put in the field, it will experience an electric force. The force depends on the charge and the electric field at that point.

Let us call the charges that are being placed in an electric field to experience the force the *external* charges.
In the above figure, Q is the source charge and q is the external charge.

The problem is now split into two:

1. Calculate the electric field due to the source charges.

2. Calculate the force acting on the external charges.

General advice:

From the setting of the problem figure out which are the source charges and which are the external charges.

source charge → Q external charge → q

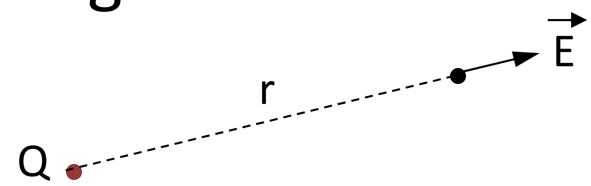
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$F = qE$$

1. Calculate the electric field due to the source charges

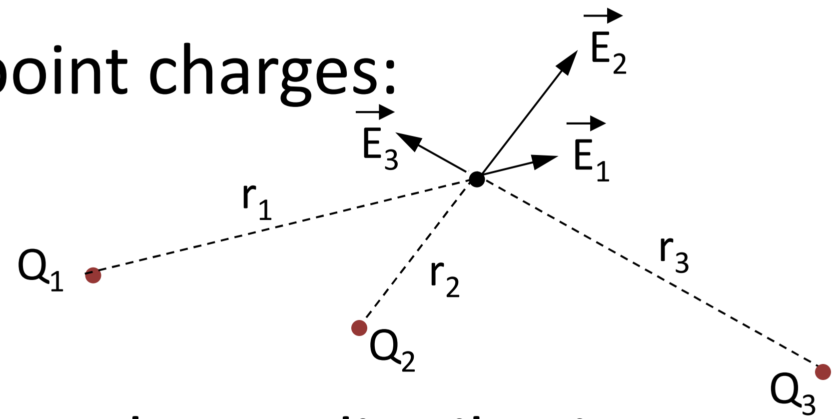
Electric field due to a point charge:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



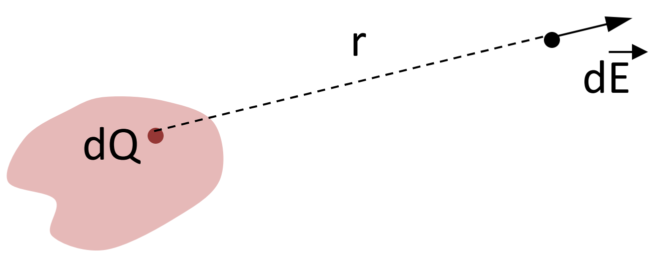
Electric field due to several point charges:

$$\vec{E} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{Q_i}{r_i^2}\hat{r}_i$$



Electric field due to continuous charge distribution:

$$\vec{E} = \int d\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\hat{r}$$



2. Calculate the force acting on the external charges

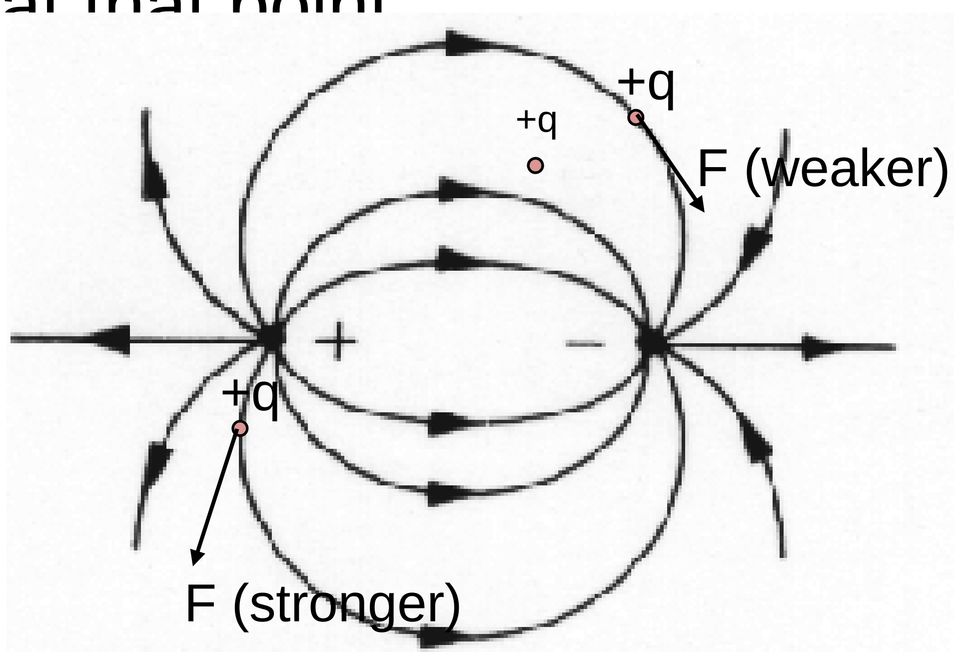
If the external charge is a point charge:

$$\vec{F} = q\,\vec{E}$$

Complications may arise if the source charge or external charge redistribute because of the electric force between them. To make sure this does not happen, we assume the external charge is a point charge and very small in magnitude. If that is the case, the external charge is also known as a test charge.

Properties of field lines I

1. To visualize the electric field, we draw field lines. When we put a *positive* test charge in the electric field, the force acting on it will be tangent to the field line at that point. The magnitude of the force will be proportional to the density of field lines at that point.

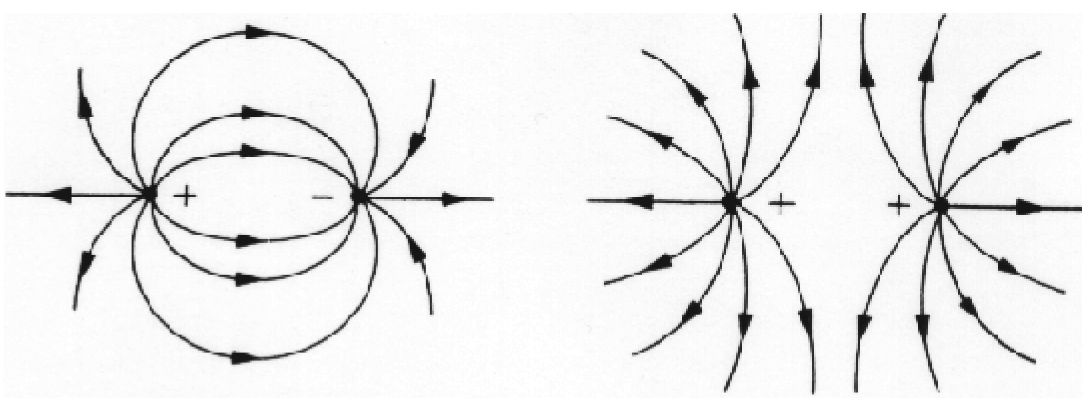


Properties of field lines II

2. Field lines are continuous lines only terminate at charges or at infinity.

3. When a field line terminate at charges, it always comes out from a positive charge, or getting into a negative charge.

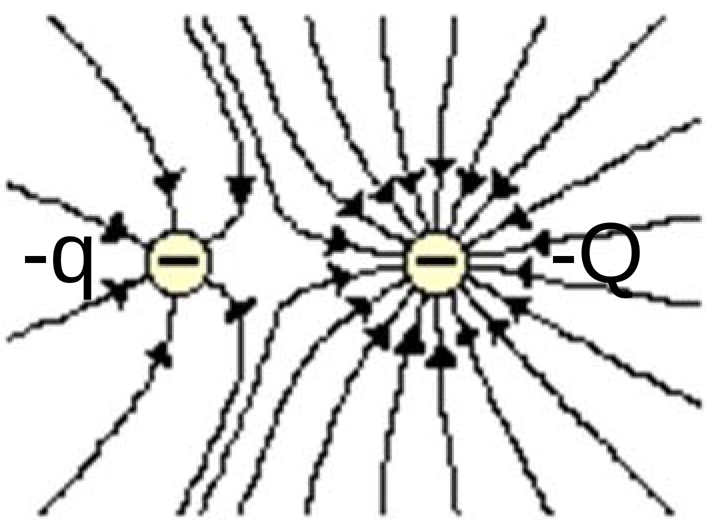
4. Field lines never cross each other.



Properties of field lines III

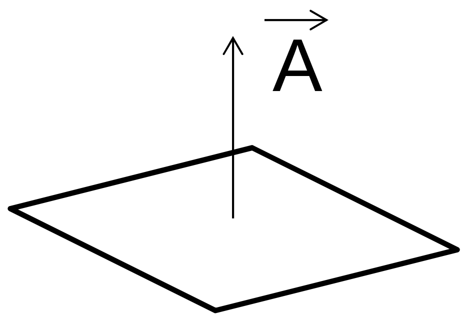
5. Because of property 3 above, a positive charge looks like a sprinkler and a negative charge looks like a drain.

6. The number of lines coming out (or sinking in) from a charge is proportional to the magnitude of that charge.

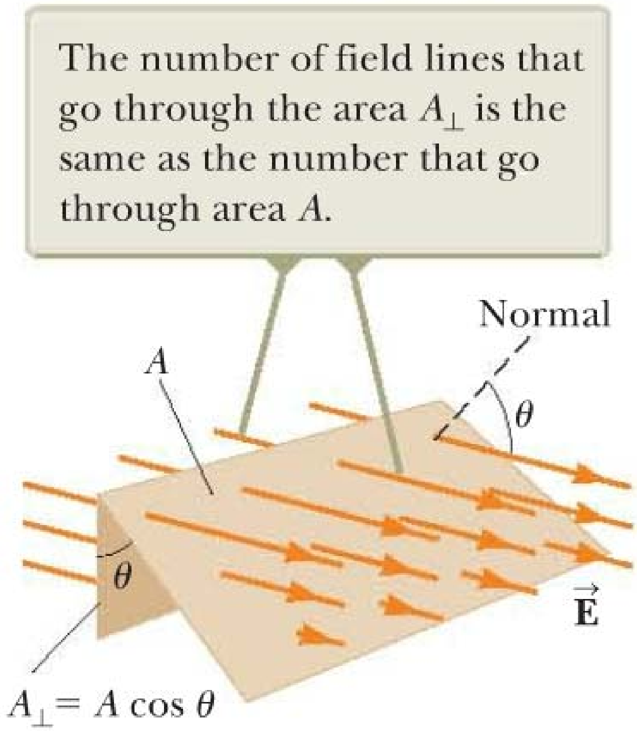


$$\frac{Q}{q} = \frac{18}{6} = 3$$

Area vector and Flux



Area vector is a vector perpendicular to a plane surface with magnitude equals to the area of the plane.



$$\text{Flux } \Phi_E = E\,A\cos\theta = \vec{E}\cdot\vec{A}$$