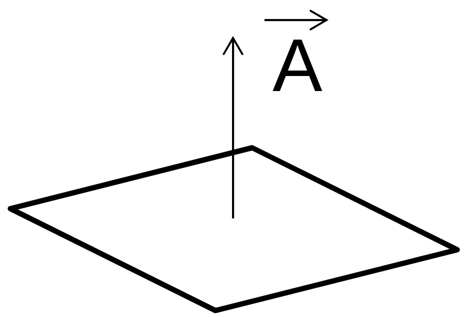
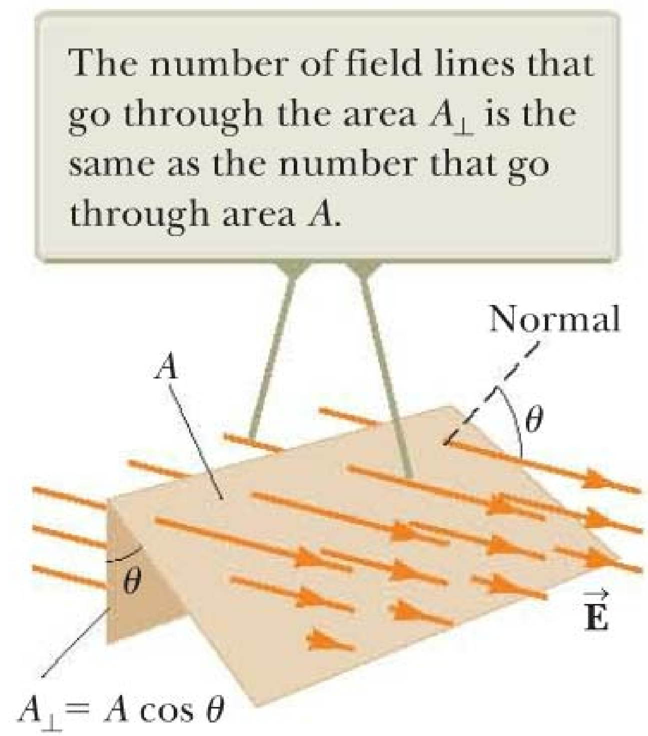
Class 5: Gauss’s Law

Area vector and Flux

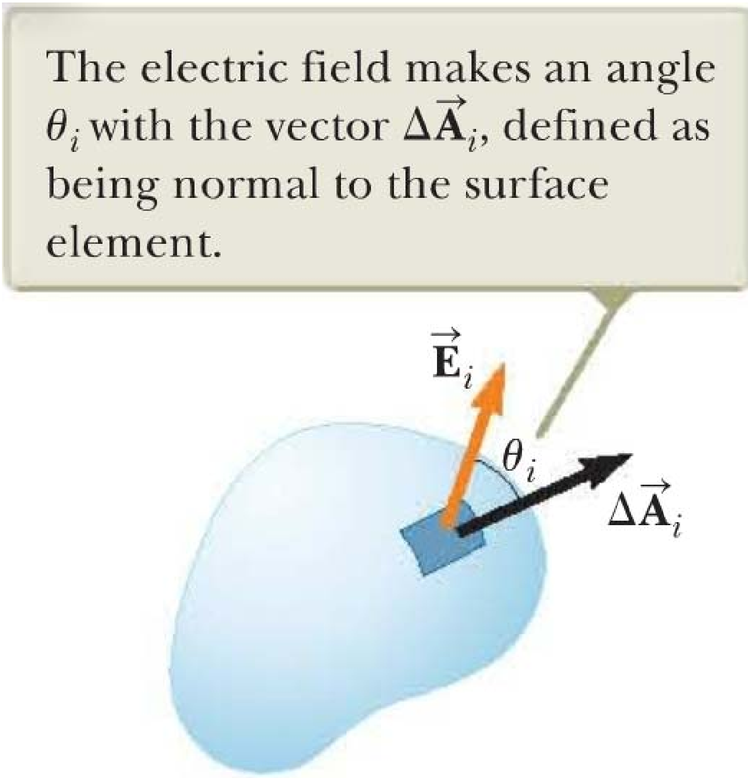


Area vector is a vector perpendicular to a plane surface with magnitude equals to the area of the plane.

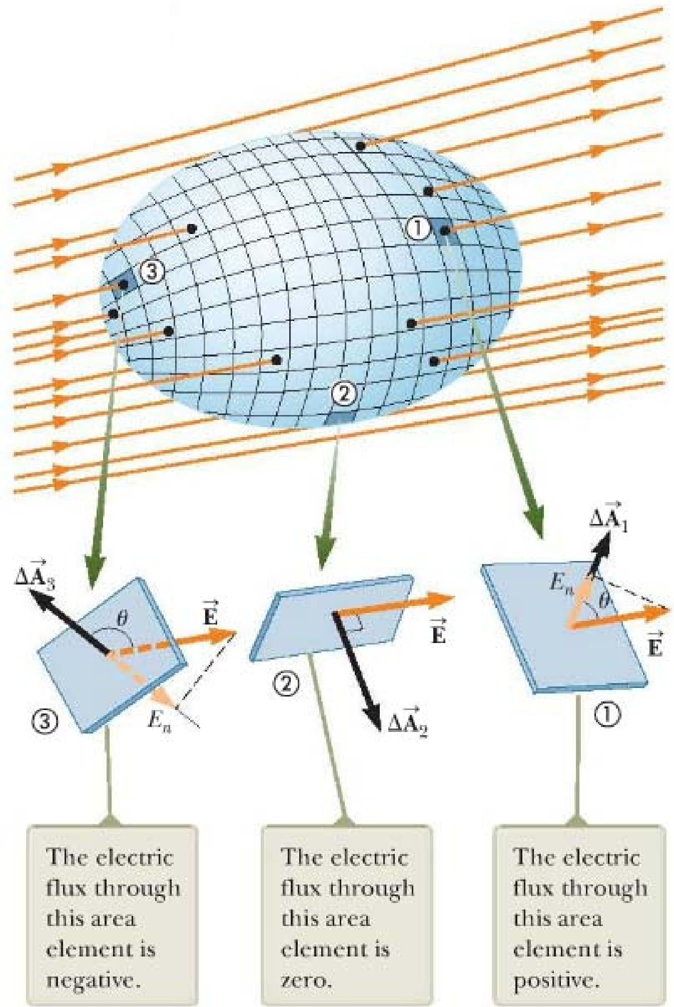


$$\text{Flux } \Phi_E = E\,A\cos\theta = \vec{E}\cdot\vec{A}$$

Flux for curve surface



$$d\Phi_E = \vec{E}\cdot d\vec{A} \quad \Rightarrow \quad \boxed{\Phi_E = \int \vec{E}\cdot d\vec{A}}$$



For closed surface, surface vector is positive when it is pointing outward.

Gauss's Law (Maxwell's first equation)

For *any* closed surface,

$$\varepsilon_0 \Phi_E = q_{in} \quad \text{or} \quad \varepsilon_0 \oint\!\!\oint \vec{E} \cdot d\vec{A} = q_{in}$$

Two types of problems that involve Gauss's Law:

1. Give you left hand side (i.e. flux through a given surface), calculate the right hand side (i.e. charge enclosed by that surface).

$$\varepsilon_0 \Phi_E \quad \Rightarrow \quad q_{in}$$

2. Give you right hand side (i.e. a charge distribution) , calculate the left hand side (i.e. flux).

$$\varepsilon_0 \Phi_E \quad \Leftarrow \quad q_{in}$$

In some simple (but important) cases, we can then calculate electric field from the flux.

$$E \quad \underset{\text{If } \Phi_E = EA}{\leftarrow} \quad \varepsilon_0 \Phi_E \quad \Leftarrow \quad q_{in}$$