Class 6: Applications of Gauss's Law

Gauss's Law (Maxwell's first equation)

For *any* closed surface,

$$\varepsilon_0 \Phi_E = q_{in} \quad \text{or} \quad \varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{in}$$

Two types of problems that involve Gauss's Law:

1. Give you left hand side (i.e. flux through a given surface), calculate the right hand side (i.e. charge enclosed by that surface).

$$\varepsilon_0 \Phi_E \quad \Rightarrow \quad q_{in}$$

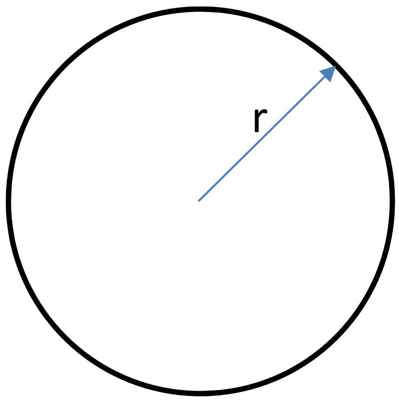
2. Give you right hand side (i.e. a charge distribution) , calculate the left hand side (i.e. flux).

$$\varepsilon_0 \Phi_E \quad \Leftarrow \quad q_{in}$$

In some simple (but important) cases, we can then calculate electric field from the flux.

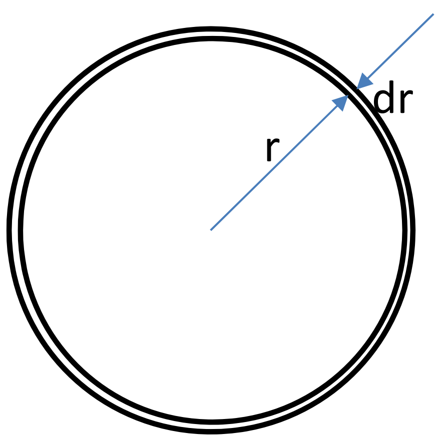
$$E \underset{\text{If } \Phi_E = EA}{\leftarrow} \varepsilon_0 \Phi_E \quad \Leftarrow \quad q_{in}$$

Important formulae for a sphere



$$\text{Volume } V = \frac{4}{3}\pi r^3$$

$$\text{Surface area} = 4\pi r^2$$



Volume of infinitesimal spherical shell

$$dV = 4\pi r^2 \, dr$$

Application of Gauss's Law

Gauss's Law can be used to calculate electric field. Practically, there are three common cases in which Gauss's Law can be applied effectively for this purpose:

1) Uniform spherical distribution of source charges
2) Uniform cylindrical distribution of source charges
3) Uniform distribution of source charges in an infinite plane.

Strategy

Step 1. Construct a Gaussian surface passing through the point you want to calculate the E field. Most likely the Gaussian surface is parallel to source charge distribution.

Step 2. For this method to work, the magnitude of the E field has to be constant on the Gaussian surface. Most likely the flux through the Gaussian surface is just Φ_E=EA, with E as the unknown. (If this is not the case, Gauss's Law is still correct, but this method just does not work.)

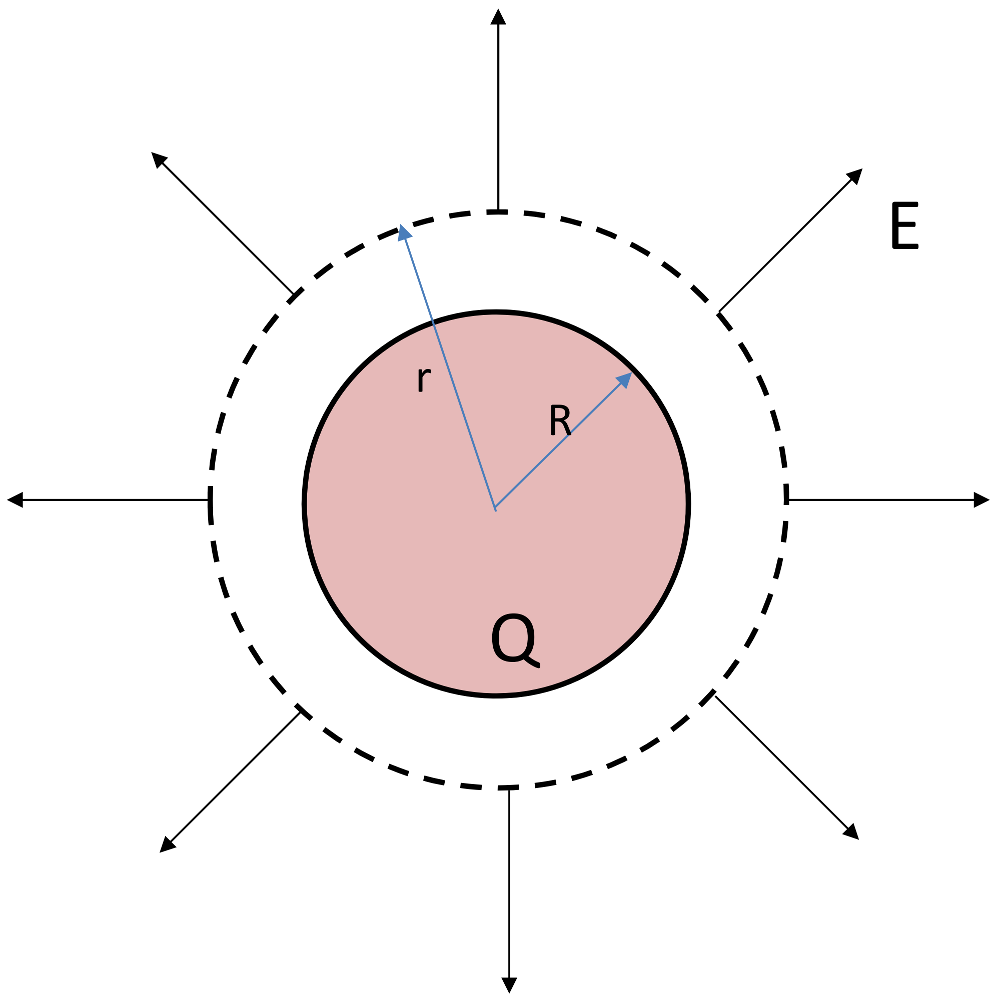
Step 3. Calculate the charge enclosed by the Gaussian surface, q_{in}.

Step 4. Now apply Gauss's Law:

$$\varepsilon_0 \Phi_E = q_{in} \Rightarrow \varepsilon_0 EA = q_{in}$$

and E can be solved.

Uniform spherical distribution



For r>R

$$\varepsilon_0 \Phi_E = Q \Rightarrow \varepsilon_0 \cdot E \cdot 4\pi r^2 = Q$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Note that point charge belongs to this case.

For r<R

Depends on the actual charge distribution.