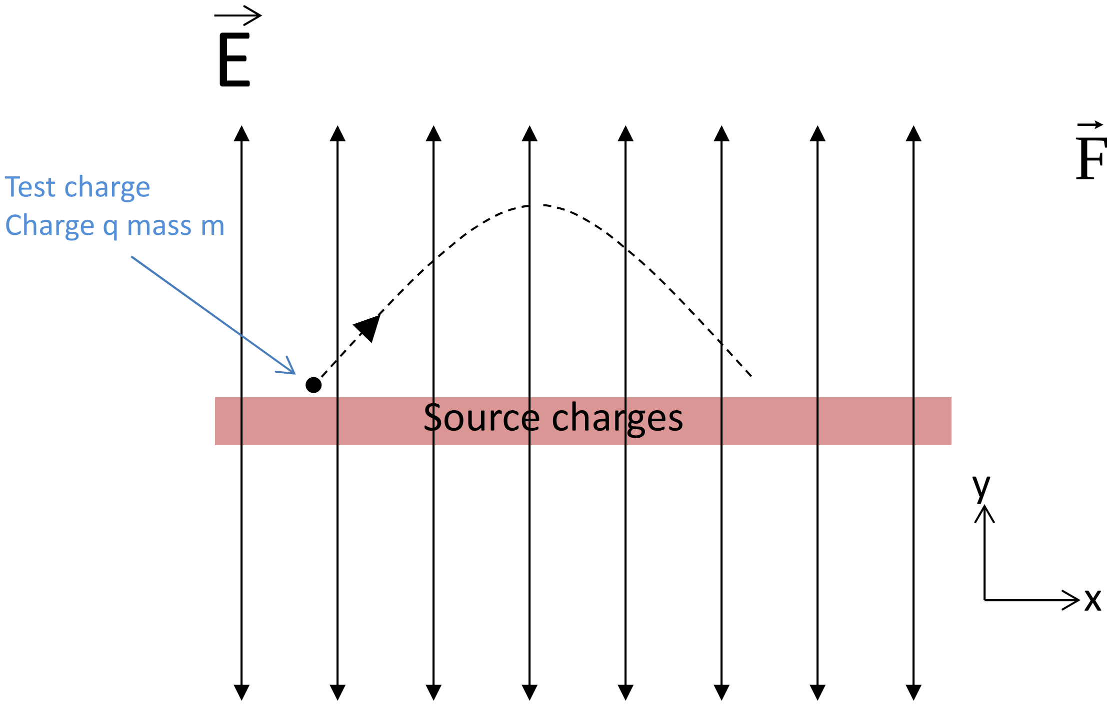
Class 9: Motion of Charged Particles in an Electric Field

Dynamics in a Constant Electric Field

This is essentially a projectile problem because the acceleration is constant.



$$\vec{F} = q\vec{E} \Rightarrow m\vec{a} = q\vec{E}$$

$$\Rightarrow \vec{a} = \frac{q}{m}\vec{E}$$

$$a_x = \frac{q}{m}E$$

$$a_y = 0$$

Projectile problem:

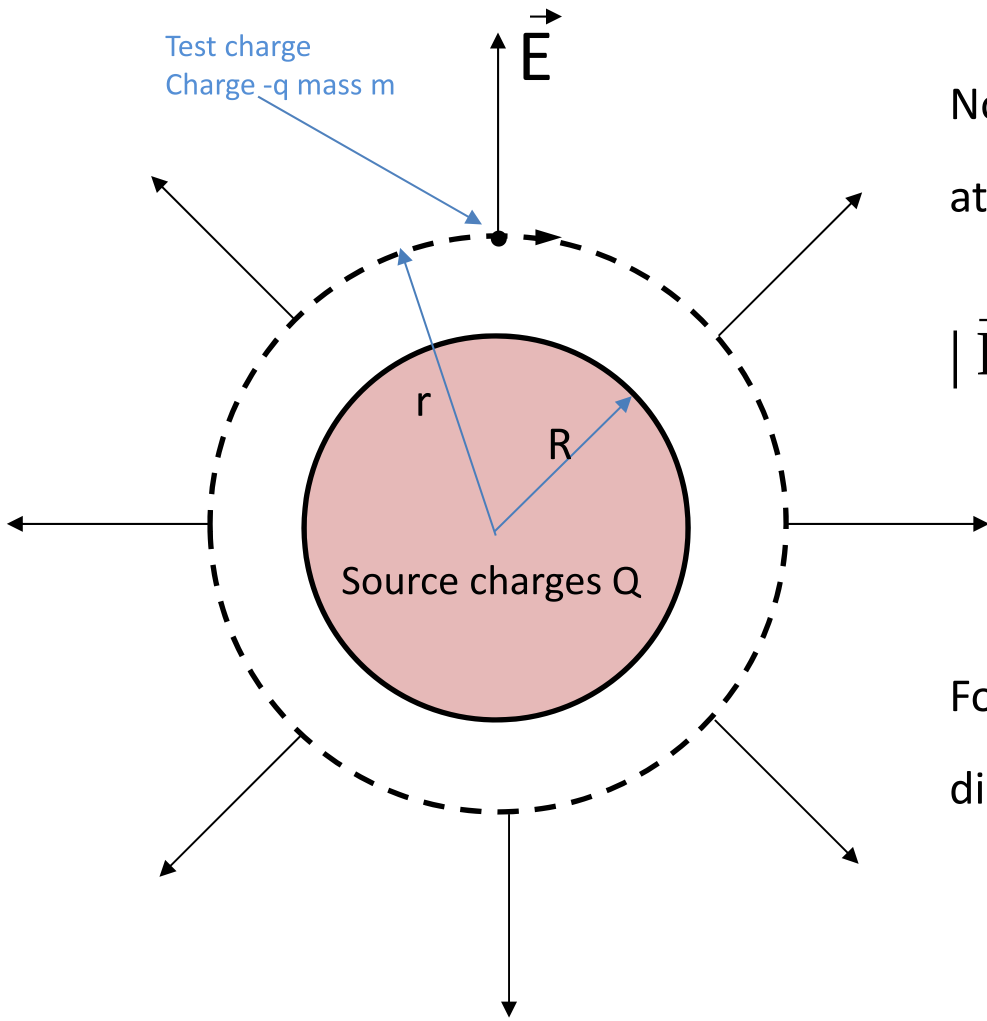
$$\text{Initial condition}: \vec{r}(0) = (x_0, y_0) \text{ and } \vec{v}(0) = v_{x0}\,\hat{i} + v_{y0}\,\hat{j}$$

$$v_y = v_{y_0} + a_y t$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \qquad v_x = v_{x0}$$

$$v_y{}^2 = v_{y0}{}^2 + 2a_y(y - y_0) \qquad x = x_0 + v_{x0}t$$

Circular Motion in an Electric Field II



Note that the electric force has to be attractive for this to work

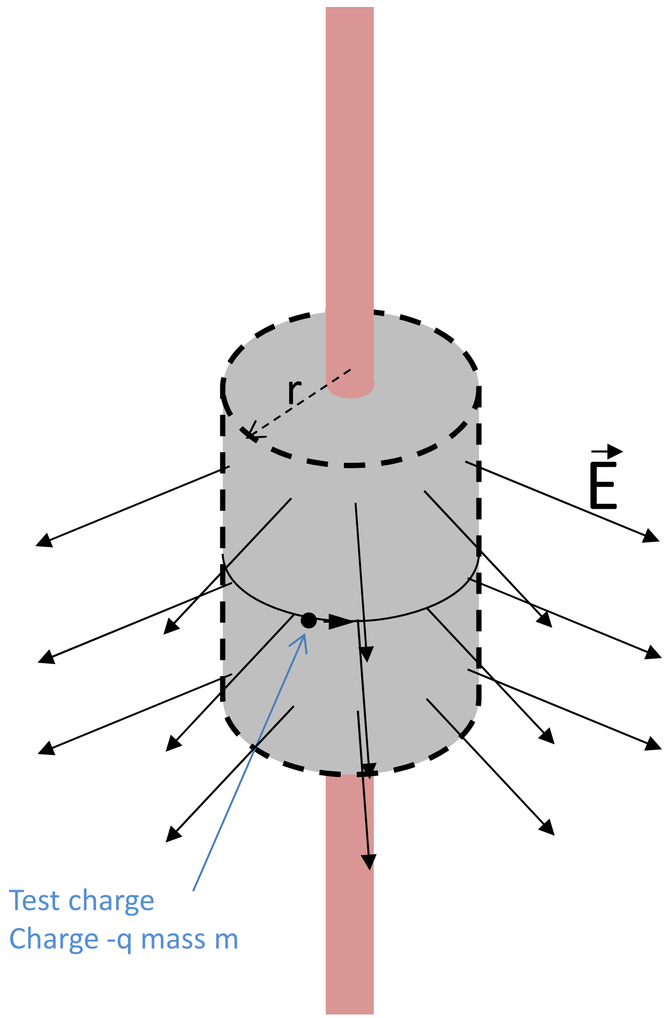
$$|\vec{F}| = q\,|\vec{E}| \Rightarrow ma_c = qE$$

$$\Rightarrow m\frac{v^2}{r} = \frac{q}{m}E$$

For uniform spherical (source) charge distribution:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

Circular Motion in an Electric Field II



Source charges
Linear charge density λ

Note that the electric force has to be attractive for this to work

$$|\vec{F}| = q|\vec{E}| \Rightarrow ma_c = qE$$

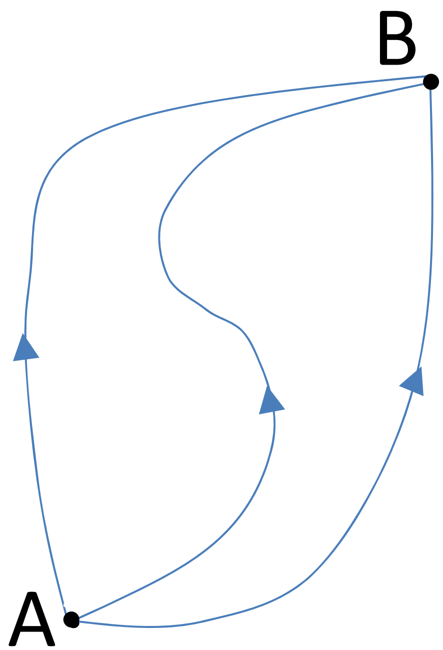
$$\Rightarrow m\frac{v^2}{r} = \frac{q}{m}E$$

For uniform cylindrical (source) charge distribution:

$$E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}$$

Potential Energy U

If $\vec{F}(\vec{r})$ is conservative, the potential energy change ΔU is defined as the *negative* work done by the force $\vec{F}(\vec{r})$, which is path independent.



$$\Delta U = -\int_i^f \vec{F}(\vec{r}) \cdot d\vec{r}$$

Pay attention to the negative sign