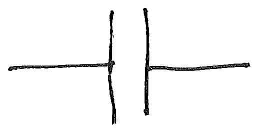
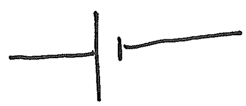
$$C = \frac{Q}{V} \quad \text{or} \quad Q = CV \quad — (1).$$

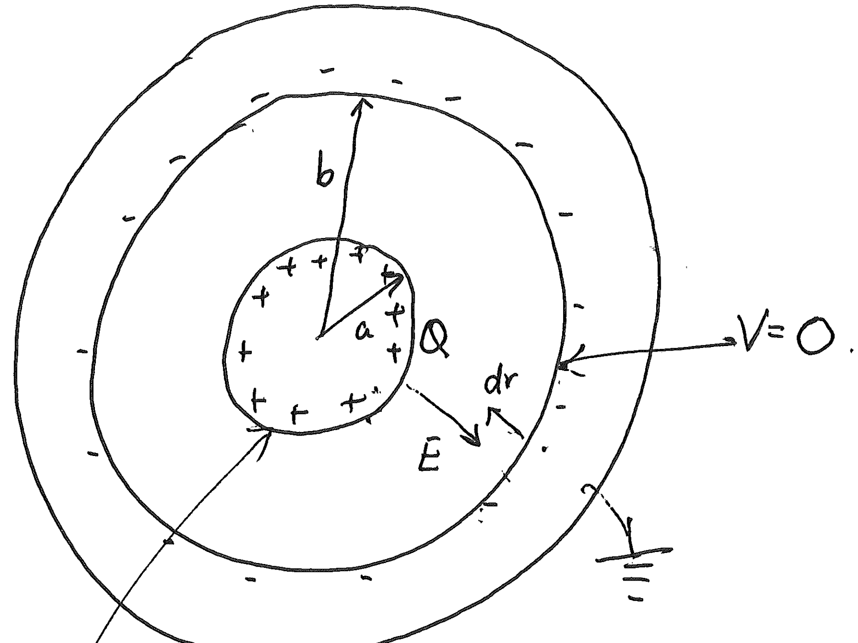
Symbol:



capacitor



battery



Too careless!

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}. \Rightarrow \cancel{V = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}.$$

only work for $V=0$ at $r=\infty$.

$$V = \int_b^a -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,dr$$

$$= \left[+\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\right]_b^a$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a} - \frac{Q}{b}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right).$$

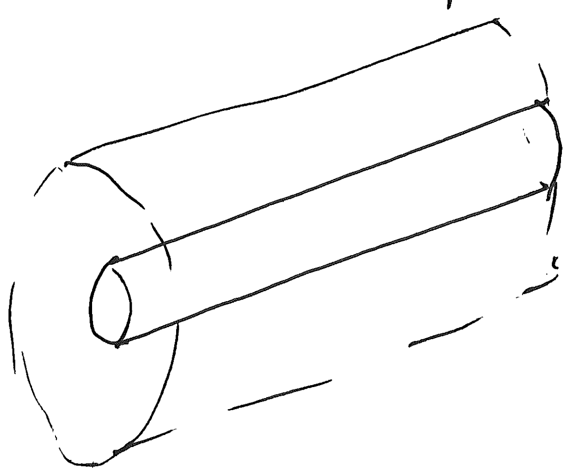
$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}.$$

$$= \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}.$$

$$= \frac{4\pi\varepsilon_0}{\frac{b-a}{ab}}$$

$$= \frac{4\pi\varepsilon_0 ab}{b-a}.$$

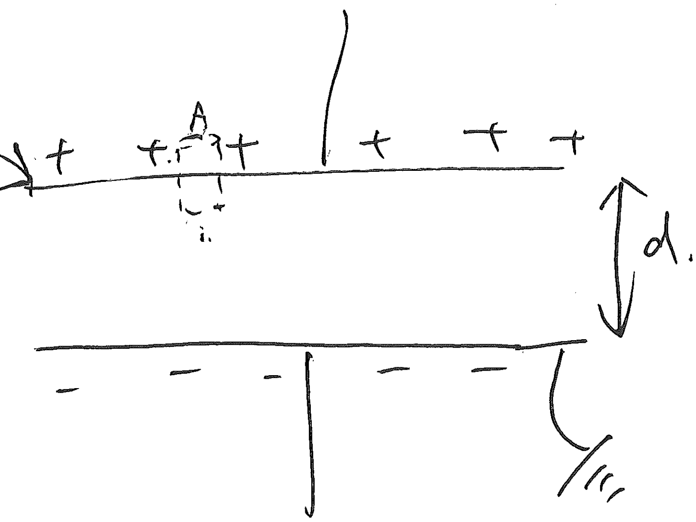
More common and important:



← Concentric cylinders.

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} \qquad b \to \infty$$

$$C = \frac{4\pi\varepsilon_0 a\cancel{b}}{\cancel{b}} = 4\pi\varepsilon_0 a.$$



$$\varepsilon_0 \Phi = Q_{in}$$

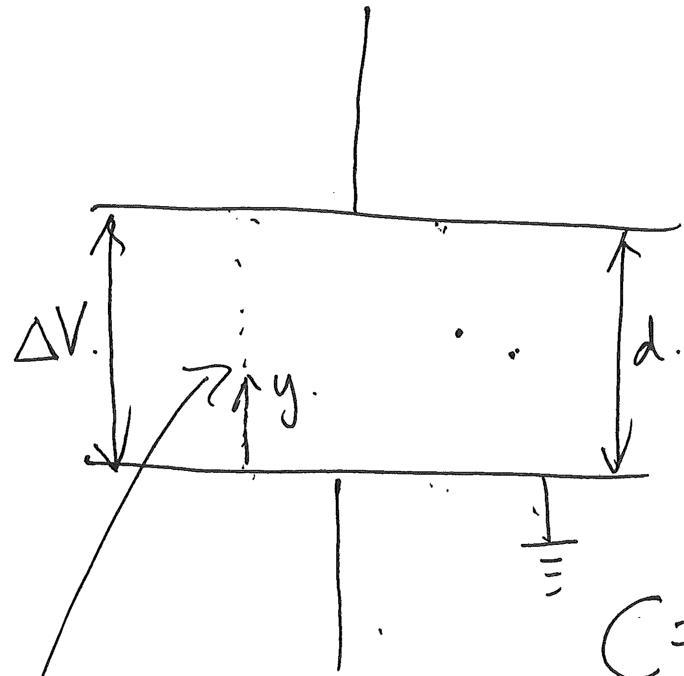
$$\varepsilon_0 \cdot AE = A\sigma$$

$$\Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

$$V = Ed. = \frac{\sigma d}{\varepsilon_0}.$$ but $\sigma = \frac{Q}{A}.$

$$\therefore \; V = \frac{Qd}{A\varepsilon_0}$$

$$\Rightarrow C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} = \frac{\varepsilon_0 A}{d} \,//$$



$C = \frac{\varepsilon_0 A}{d}$

$\Delta V = E d$

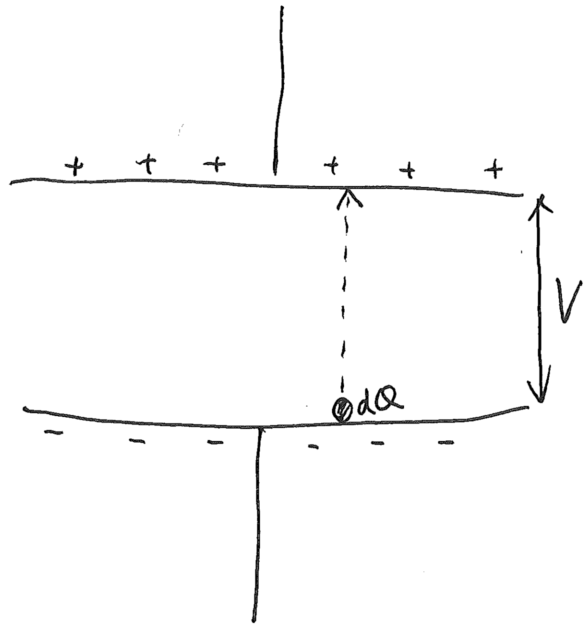
parallel plates only

E field inside the gap is constant.

$C = \frac{Q}{\Delta V}$ ← Generally correct

BUT NOT V!

$V = E y$



$$dE = V\,dQ.$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}.$$

$$dE = \frac{Q\,dQ}{C}.$$

$$U \rightarrow E = \int_0^{Q_f} \frac{Q\,dQ}{C}.$$

$$= \frac{1}{2}\left[\frac{Q^2}{C}\right]_0^{Q_f}$$

$$= \frac{1}{2}\frac{Q_f^{\,2}}{C}.$$

charge in capacitor.

~~Rewi~~ ~~Rewr~~ Rewriting symbol: $U = \frac{1}{2}\frac{Q^2}{C}.$

$$\underline{\underline{U = \frac{1}{2}\frac{Q^2}{C}}} \qquad C = \frac{Q}{V}$$

$$= \frac{1}{2}\frac{(CV)^2}{C} \qquad (Q = CV)$$

$$= \underline{\underline{\frac{1}{2}CV^2}}$$

$$U = \frac{1}{2}\frac{Q^2}{C} \qquad \text{(replacing } C \text{ with } Q \text{ and } V\text{)}$$

$$= \frac{1}{2}\frac{Q^2}{\frac{Q}{V}}$$

$$= \underline{\underline{\frac{1}{2}QV}}$$

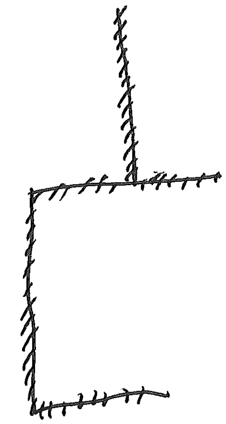
$$U = \frac{1}{2}CV^2 \qquad (V = Ed)$$

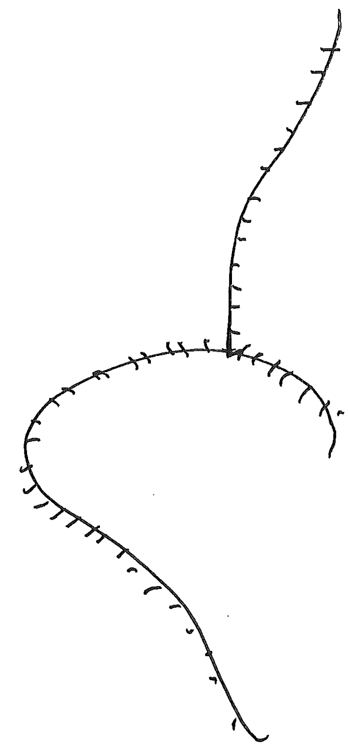
$$= \frac{1}{2}CE^2d^2$$

$$= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)E^2d^2$$

$$= \frac{1}{2}\varepsilon_0 \cdot E^2 \cdot Ad$$

$$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2 \leftarrow \frac{dU}{dVol} = \frac{1}{2}\varepsilon_0 E^2$$





$$C = \frac{\varepsilon_0 A}{d}$$

In parallel,

$$C_p = C_1 + C_2 + C_3 + \cdots + C_N$$