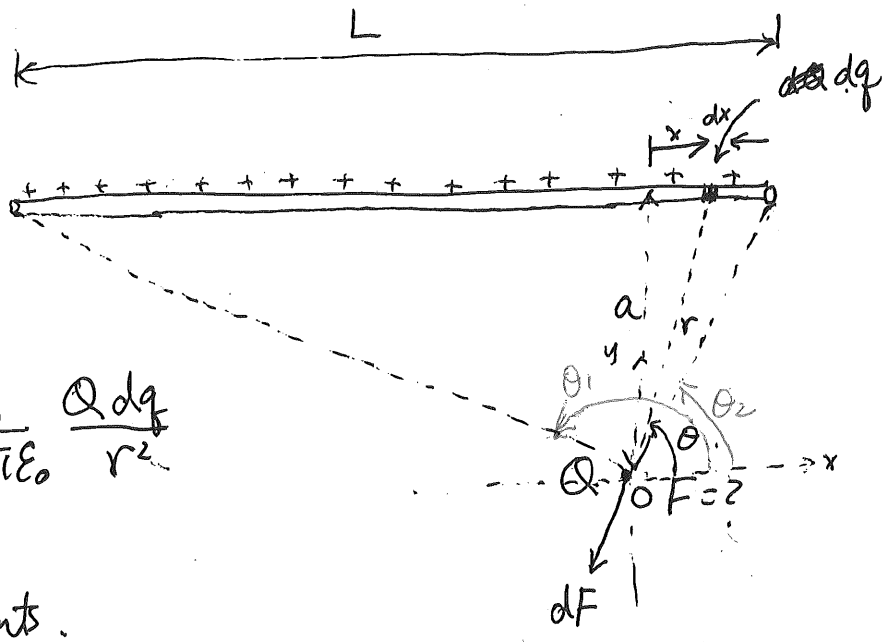
$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2} \hat{r}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \quad N m^2/C^2.$$



Step 1 $\vec{dF} = \frac{1}{4\pi\varepsilon_0}\frac{Q\,dq}{r^2}$

Step 2 Components.

$$dF_x = -dF\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{Q\,dq}{r^2}\cos\theta.$$

$$dF_y = -dF\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{Q\,dq}{r^2}\sin\theta.$$

Step 3. "Unifying" variables:

Linear charge density $= \lambda$ ← Charge per unit length.

$\therefore dQ = \lambda\,dx$

$$dF_x = -\frac{1}{4\pi\varepsilon_0}\frac{Q\lambda\,dx}{r^2}\cos\theta. \leftarrow$$

Write x and r in terms of θ.

$r = \frac{a}{\cos\theta}$ ~~$r = \sqrt{\frac{a}{\cos\theta}}$~~ $r = \frac{a}{\sin\theta}$ $\left(\sin\theta = \frac{a}{r}\right)$ ←

$\tan\theta = \frac{a}{x} \Rightarrow x = \frac{a}{\tan\theta}$ or $a\cot\theta$ ←

$$x = a\cot\theta = a\frac{\cos\theta}{\sin\theta}$$

$$dx = a\, d\left(\frac{\cos\theta}{\sin\theta}\right) = a\,\frac{\cos\theta\cdot\cos\theta - \sin\theta(-\sin\theta)}{\sin^2\theta}$$

$$= \frac{a}{\sin^2\theta}.$$

$$dx = a\cdot d\left(\frac{\cos\theta}{\sin\theta}\right) = a\cdot\frac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta}\,d\theta$$

$$= -\frac{a}{\sin^2\theta}\,d\theta$$

$$dF_x = -\frac{Q\lambda}{4\pi\varepsilon_0}\,\frac{\left(-\frac{a}{\sin^2\theta}\right)d\theta\,\cos\theta}{\left(\frac{a}{\sin\theta}\right)^2}$$

$$= +\frac{Q\lambda}{4\pi\varepsilon_0 a}\cos\theta\, d\theta$$

$$F_x = \int dF_x = \int_{\theta_1}^{\theta_2}\frac{Q\lambda}{4\pi\varepsilon_0 a}\cos\theta\, d\theta.$$

$$= \frac{Q\lambda}{4\pi\varepsilon_0 a}\left[\sin\theta_1 - \sin\theta_2\right].$$

$dE_y = -\frac{Q\lambda}{4\pi\varepsilon_0} d\theta$

$$dF_y = -\frac{1}{4\pi\varepsilon_0}\frac{Q\lambda\, dx}{r^2}\sin\theta.$$

$$= +\frac{Q\lambda}{4\pi\varepsilon_0 a}\sin\theta\, d\theta$$

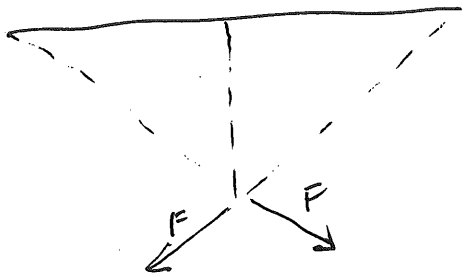
$$F_y = \int dF_y = \int_{\theta_1}^{\theta_2} \frac{Q\lambda}{4\pi\varepsilon_0 a}\sin\theta\, d\theta$$

$$= \frac{Q\lambda}{4\pi\varepsilon_0 a}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \frac{Q\lambda}{4\pi\varepsilon_0 a}\left[\cos\theta_1 - \cos\theta_2\right]$$

Ha Ha

Sign problem!?

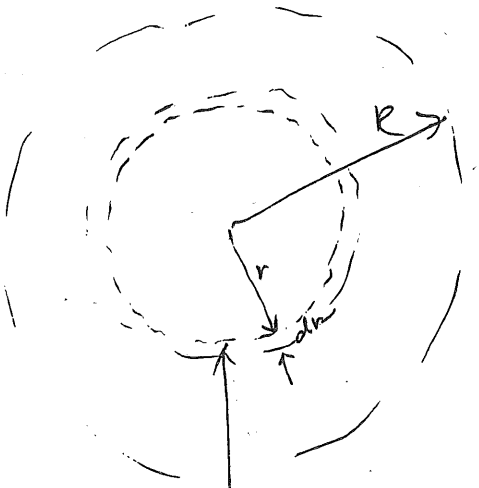


$\theta_1 = 180° - \theta_2$

$$F_x = \frac{Q\lambda}{4\pi\varepsilon_0 a}\left[\sin(180° - \theta_2) - \sin\theta_2\right]$$

($\sin(180° - \theta_2) = \sin\theta_2$)

$$= 0$$



$dV = 4\pi r^2 dr$

$$V = \int_0^R 4\pi r^2 dr = \left[\frac{4\pi}{3} r^3\right]_0^R$$

$$= \frac{4}{3}\pi R^3.$$