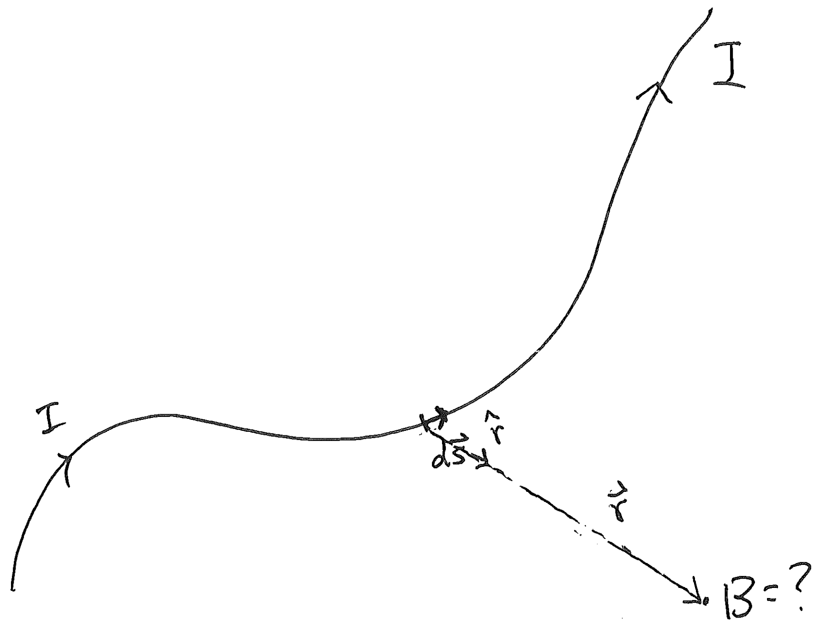
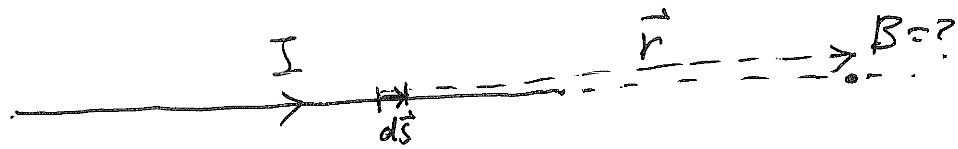


$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}.$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^3}.$$

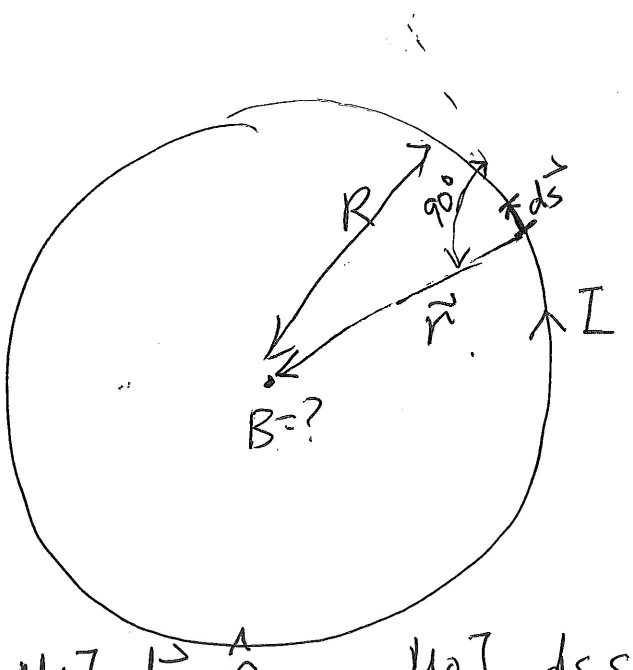
Ex. 1



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = 0$$ since $d\vec{s} \parallel \vec{r}$ for all infintesimal Elements.

$$\Rightarrow \vec{B} = 0.$$

Ex. 2



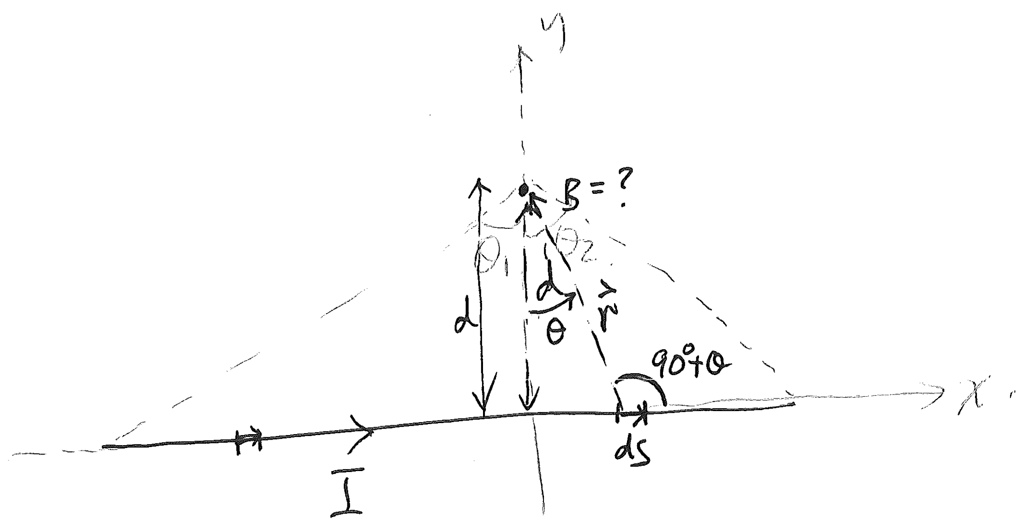
For center only.

$$\therefore \boxed{\vec{B} = \frac{\mu_0 I}{2R} \odot}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\cdot\frac{ds\sin 90°}{R^2} \odot$$

$$= \frac{\mu_0 I}{4\pi}\frac{ds}{R^2} \odot$$

$$\vec{B} = \int \frac{\mu_0 I}{4\pi}\cdot\frac{ds}{R^2} \odot = \frac{\mu_0 I}{4\pi R^2} \odot \int ds = \frac{\mu_0 I}{4\pi R^2}\cdot 2\pi R \odot$$



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{dx \sin(90°+\theta)}{r^2} \quad \sin(90°+\theta) = \cos\theta$$

$\odot$ only one integration!

$$r = \frac{d}{\cos\theta}$$

$$x = r\sin\theta = d\tan\theta$$

$$d\left(\frac{\sin\theta}{\cos\theta}\right) = \frac{1}{\cos^2\theta}$$

$$dx = \frac{+d}{\cos^2\theta} d\theta$$

$$d\vec{B} = +\frac{\mu_0 I}{4\pi} \cdot \frac{d}{\cos^2\theta} d\theta \cdot \frac{\cos\theta}{\left(\frac{d}{\cos\theta}\right)^2} \quad \odot$$

$$d\vec{B} = -\frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta \quad \odot$$

$$= \frac{\mu_0 I}{4\pi d}\left[+\sin\theta\right]_{\theta_1}^{\theta_2} \quad \odot$$

$$d\left(\frac{\sin\theta}{\cos\theta}\right) = \frac{\cos\theta \, d\sin\theta - \sin\theta \, d\cos\theta}{\cos^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$= \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right) \quad \odot$$

For ∞ Long wire, $\theta_2 = 90°$, $\theta_1 = -90°$.

$$\therefore B = \frac{\mu_0 I}{4\pi d}(1-(-1)) = \frac{\mu_0 I}{2\pi d}$$

Puppy $\longleftrightarrow$ Dog

Kitten $\longleftrightarrow$ Cat

~~Cat~~ Coulomb's Law $\longleftrightarrow$ Gauss' Law

Biot-Savart Law $\longleftrightarrow$ Ampere's Law.

$$-E \cdot d\vec{s} = \Delta U \Rightarrow \oint \vec{E} \cdot d\vec{s} = 0$$

$\vec{E}$ is a conservative field. (NOT ALWAYS CORRECT!!)

For any conservative field $\vec{X}$:

$$\oint \vec{X}\cdot d\vec{s} = 0.$$

$$\Downarrow$$

$$\oint (\nabla \times \vec{X})\cdot d\vec{A} = 0.$$

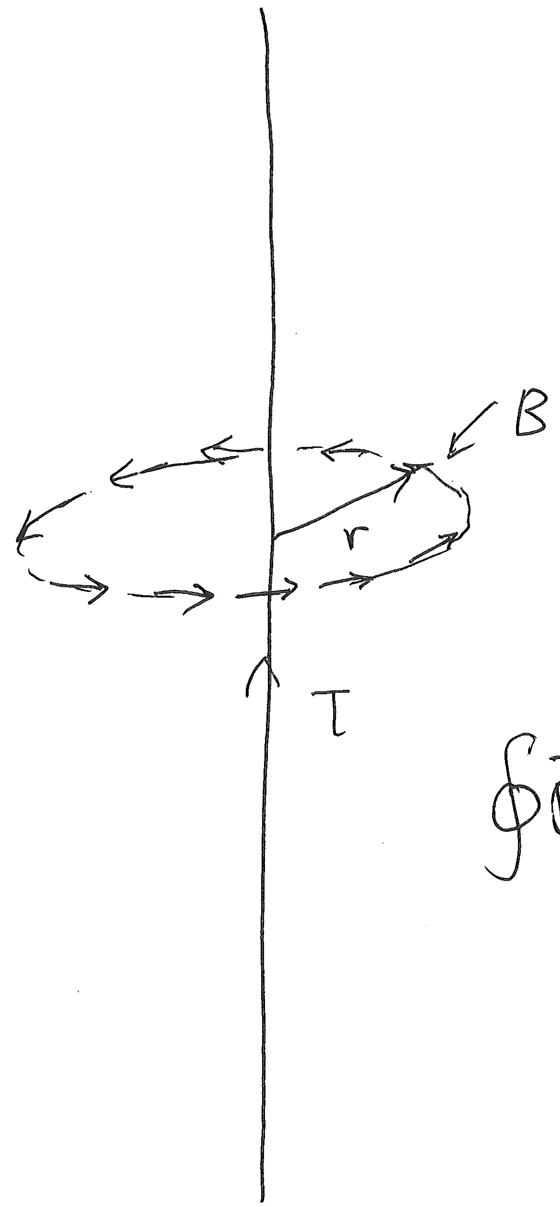
$$\Downarrow$$

$$\nabla \times \vec{X} = 0. \qquad (\nabla \times \nabla f = 0).$$

$$\Downarrow$$

$$\vec{X} = -\nabla \psi.$$

Potential.



$$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds\cos 0^\circ$$
$$= \oint B\,ds$$
$$= B\cdot\oint ds$$
$$= B\cdot 2\pi r$$

Long wire.

Ampere's Law: $B\cdot 2\pi r = \mu_0 I$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$