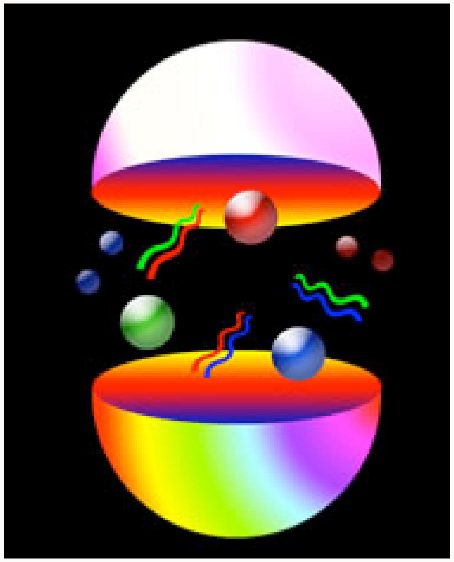
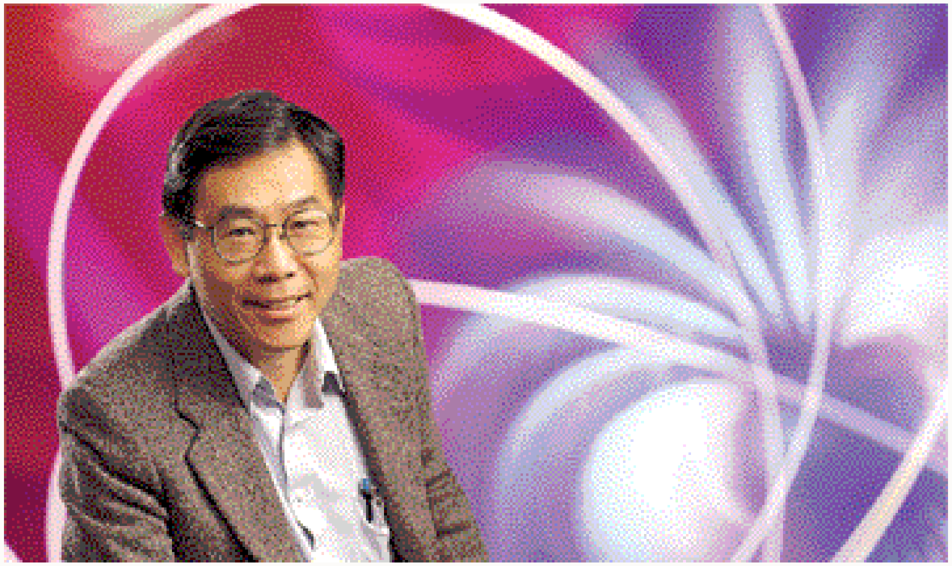
AdS/CFT and Novel Effects in QCD



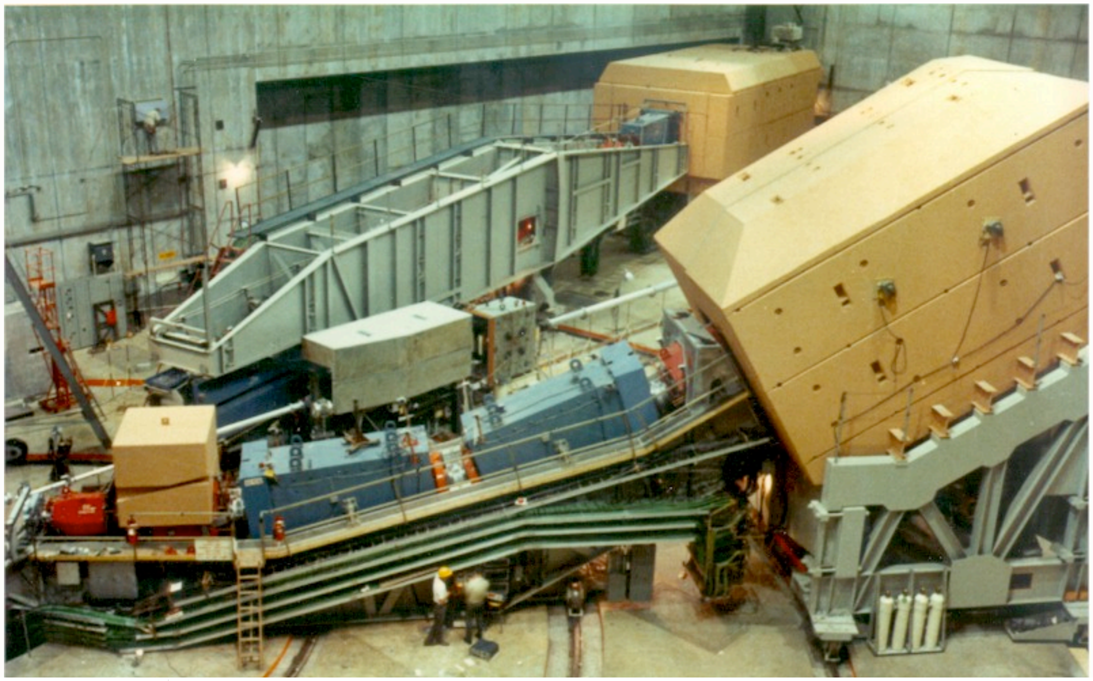


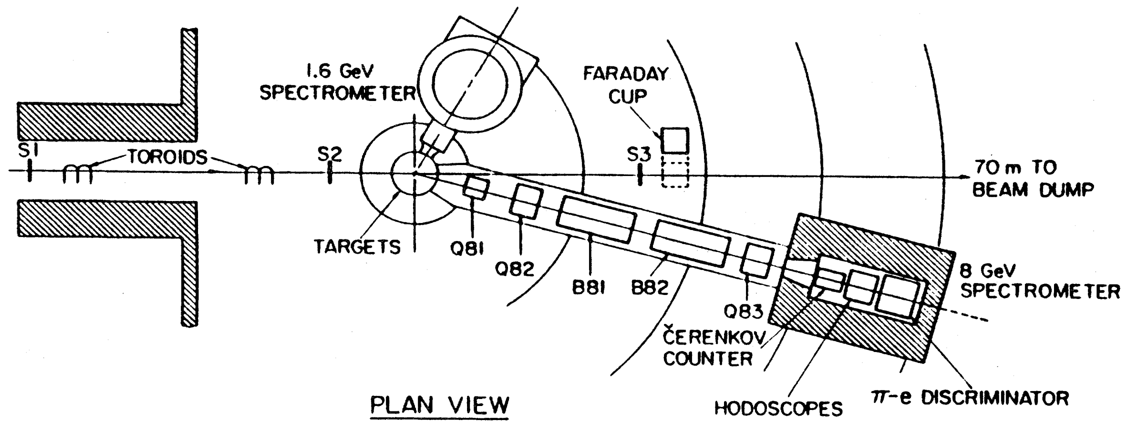
Kei-Fei Liu Symposium

University of Kentucky
April 19, 2007

Stan Brodsky, SLAC

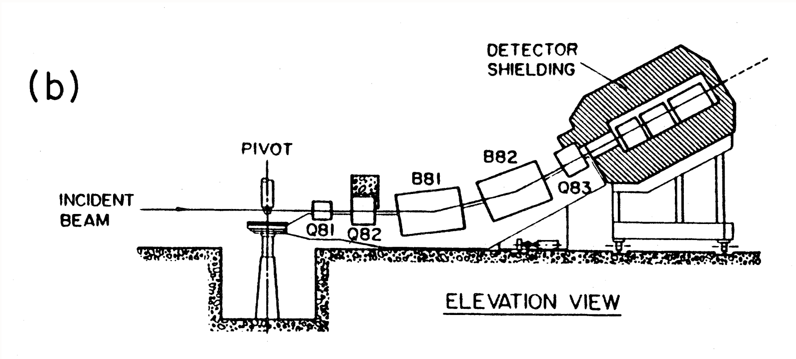
SLAC Two-Mile Linear Accelerator 1967

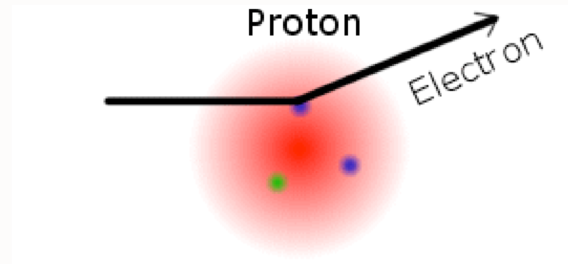




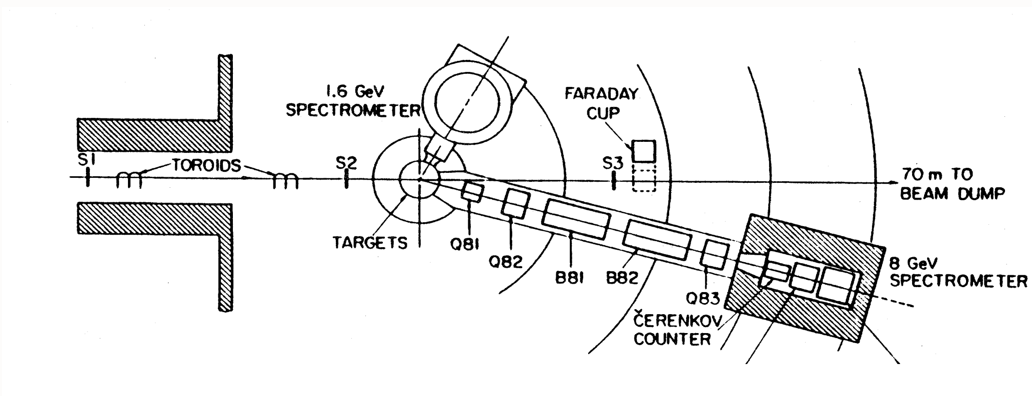
1967 SLAC Experiment:
Scatter Electrons on protons in a Hydrogen Target
Discovery of the Quark Structure of Matter

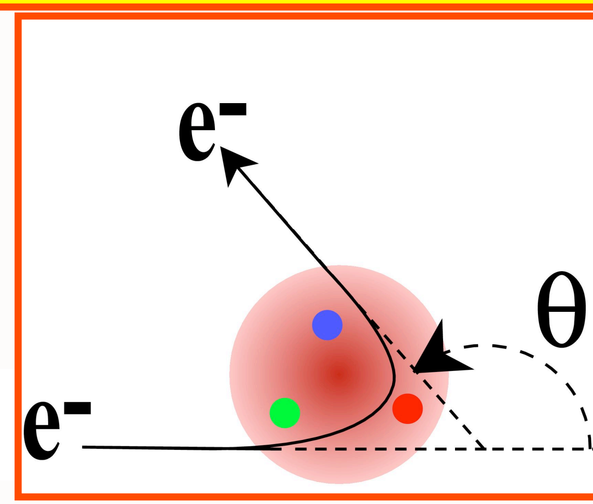
$$ep \to e' X$$





Discovery of quarks!

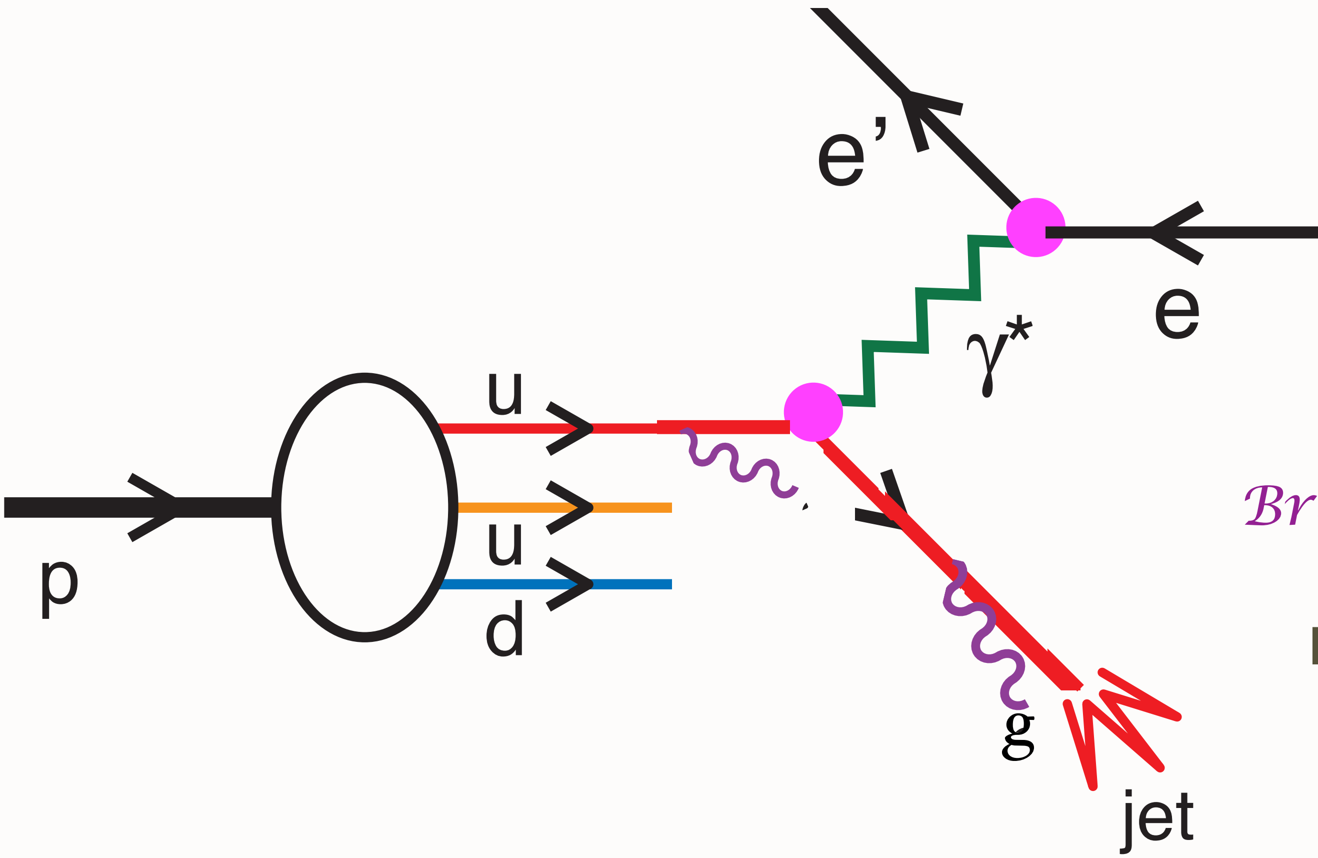




Deep inelastic scattering: Experiments on the proton and the observation of scaling*

Friedman, Kendall, Taylor: Nobel Prize

First Evidence for Quark Structure of Matter

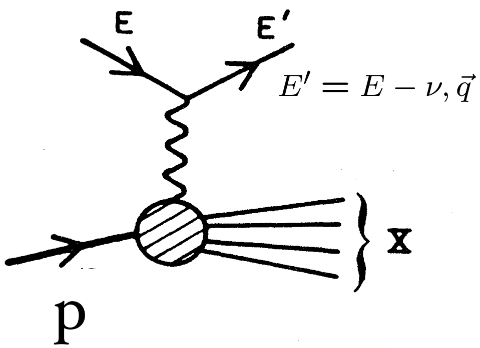


Gluonic Bremmstrahlung

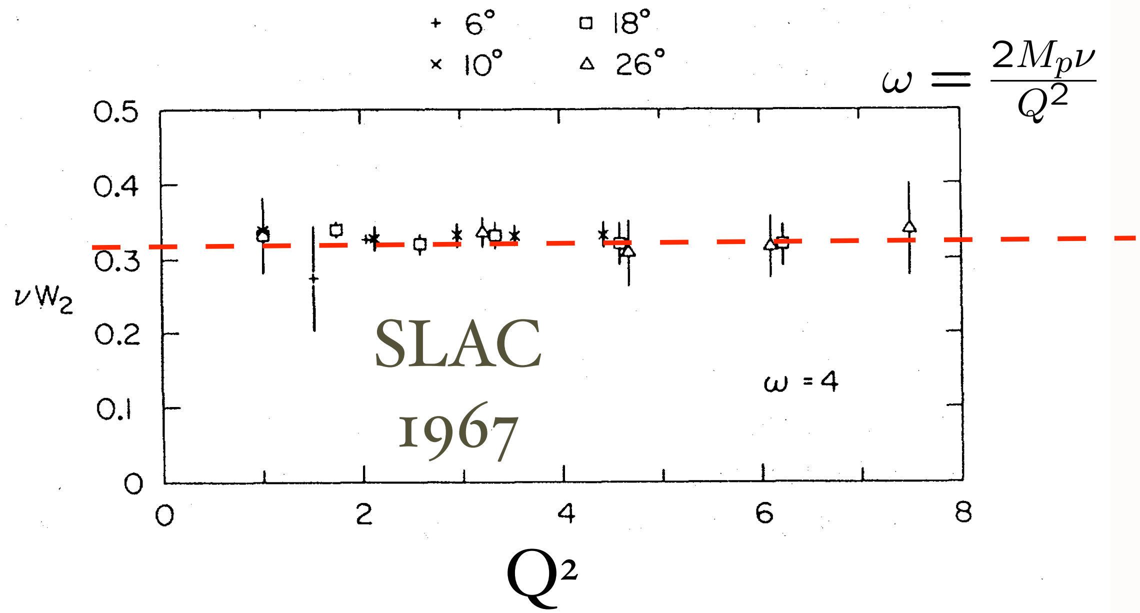
DGLAP Evolution

Deep Inelastic Electron-Proton Scattering

$$ep \to e'X$$



$$Q^2 = \vec{q}^2 - \nu^2$$



No intrinsic length scale !

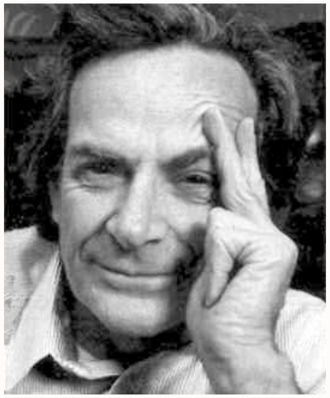
Measure rate as a function of energy loss ν and momentum transfer Q

Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

Discovery of Bjorken Scaling

Electron scatters on point-like quarks!

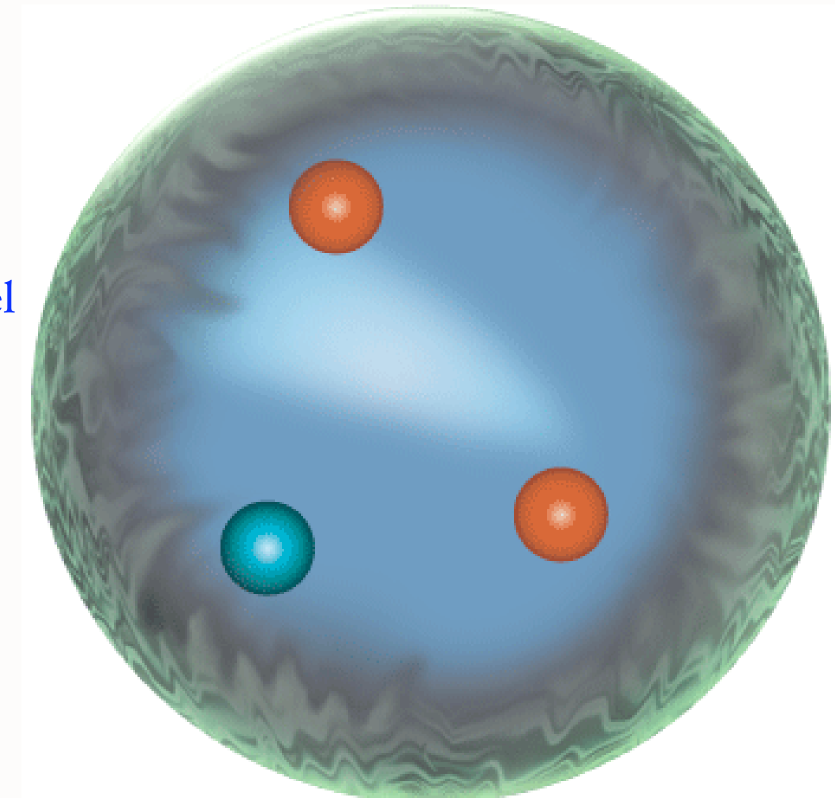
Quarks in the Proton



Feynman: "Parton" model

Bjorken Scaling:
Pointlike Quarks

p = (u u d)

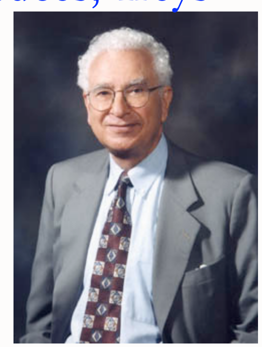


$1\,fm$

$10^{-15}m = 10^{-13}cm$

Ne'eman: $SU(3)_F$

Zweig: "Aces, Deuces, Treys"



Gell Mann: "Three Quarks for Mr. Mark"

QCD Lagrangian

gluon dynamics

quark kinetic energy + quark-gluon dynamics

mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \overline{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi}_f \psi_f$$

QCD color charge

field strength tensor

covariant derivative

quark field

Yang-Mills Gauge Principle: Invariance under Color Rotation and Phase Change at Every Point of Space and Time

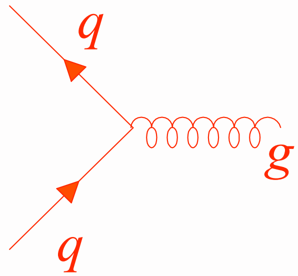
Dimensionless Coupling

Renormalizable

Asymptotic Freedom

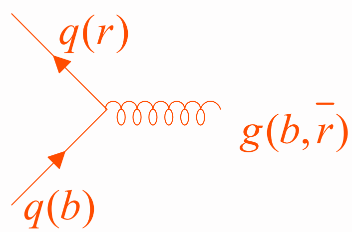
Color Confinement

Fundamental QCD Couplings

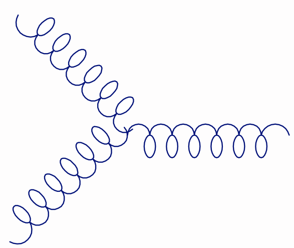


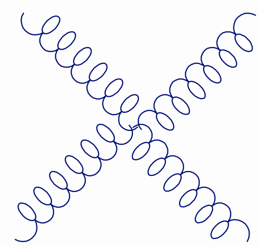
Similar to QED

More exactly

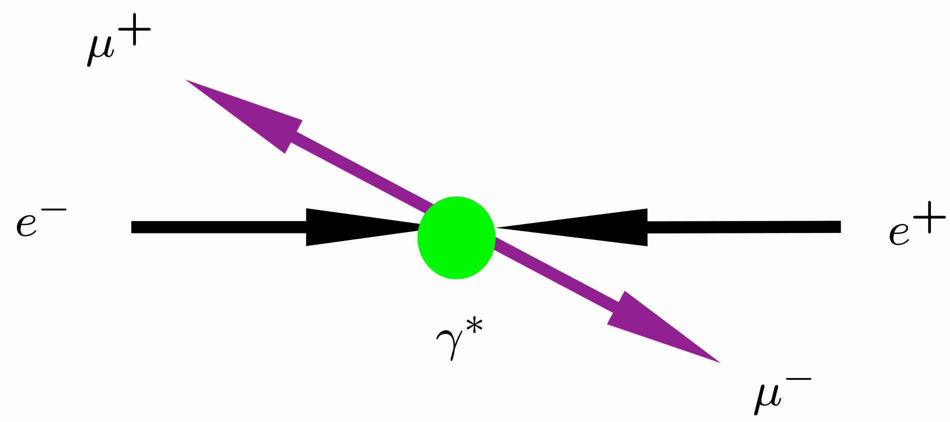


Gluon vertices



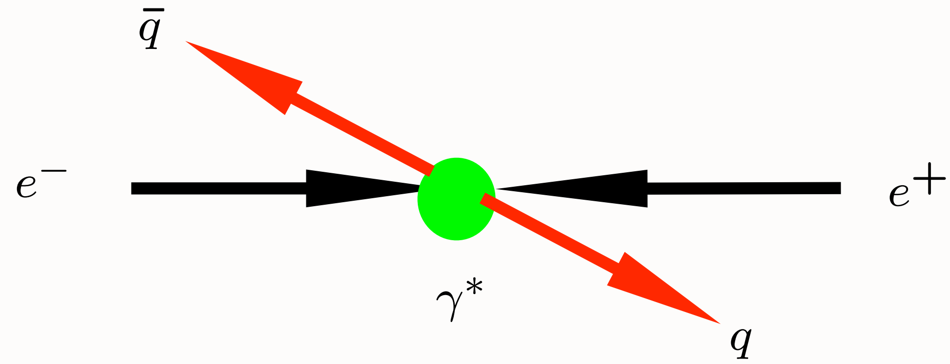


Electron-Positron Annihilation



$$e^+e^- \to \gamma^* \to \mu^+\mu^-$$

Electron-Positron Annihilation



$$e^+e^- \to \gamma^* \to q\bar{q}$$

Rate proportional to quark charge squared
and number of colors

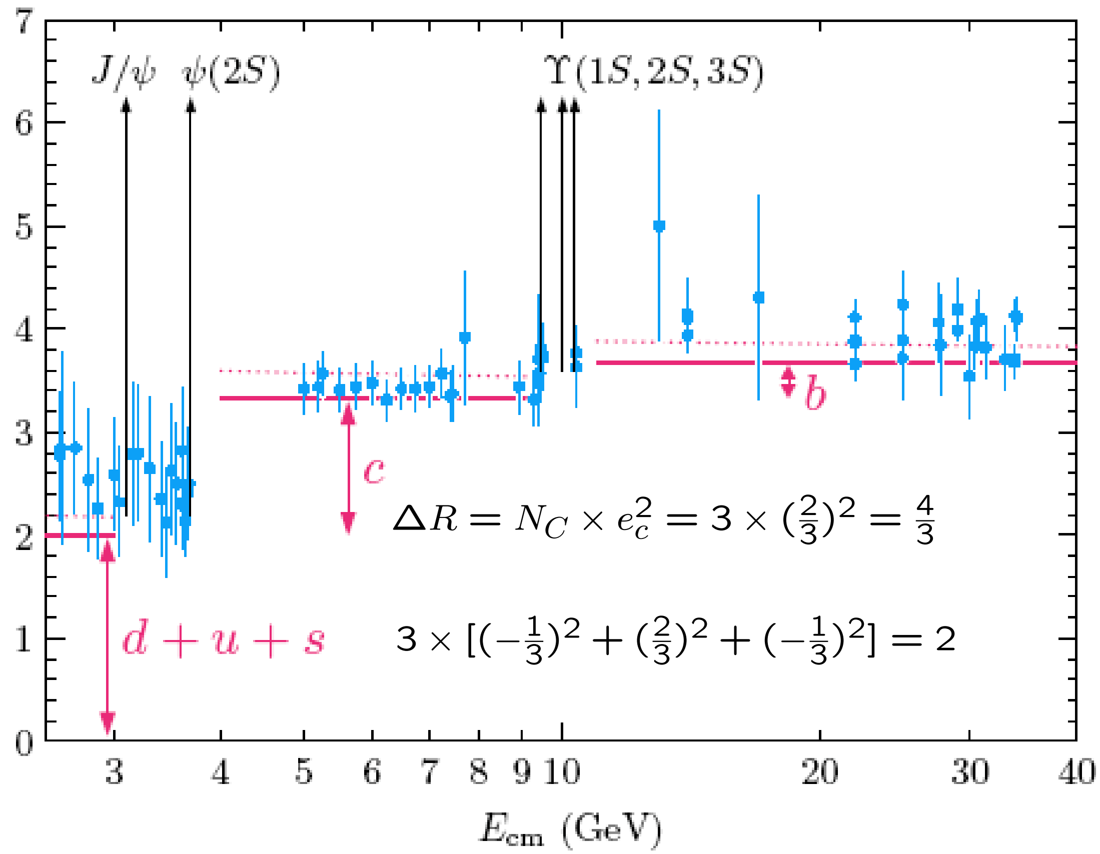
$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

How to Count Quarks and Color

$$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$$

$J/\psi = (c\bar{c})_{1S}$

$\Upsilon = (b\bar{b})_{1S}$



$3 \times (-\frac{1}{3})^2 = \frac{1}{3}$

$N_C = 3$

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing ...

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities.

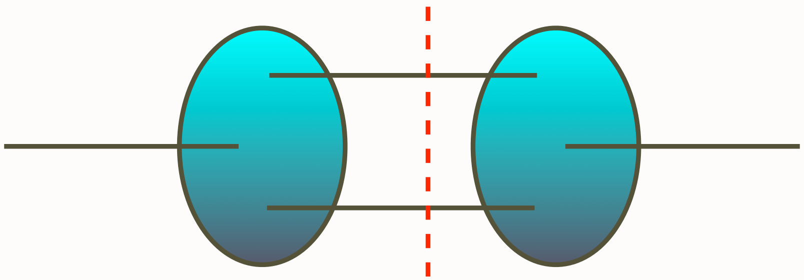
—Mark Twain

Kei-Fei Liu:
Key Fundamental Questions Concerning Hadron Structure

- How can we describe the dynamics of a hadron in terms of confined quark and gluon constituents?
- How is the proton's momentum and spin carried by its quark and gluon constituents?
- What is the analog of the atomic Schrodinger equation and wavefunction for hadrons?
- Quantum Fluctuations: What is the role of the quark-antiquark sea?

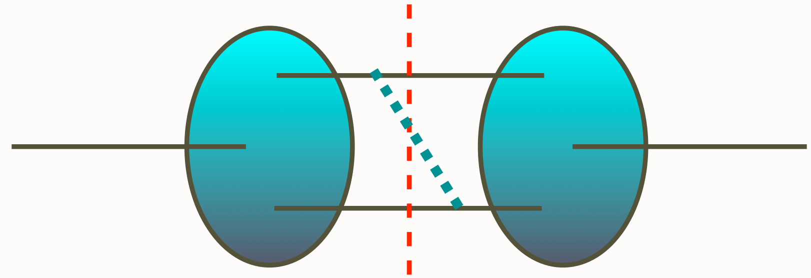
Quantum Mechanics: Uncertainty in p, x, spin

Relativistic Quantum Field Theory: Uncertainty in particle number n



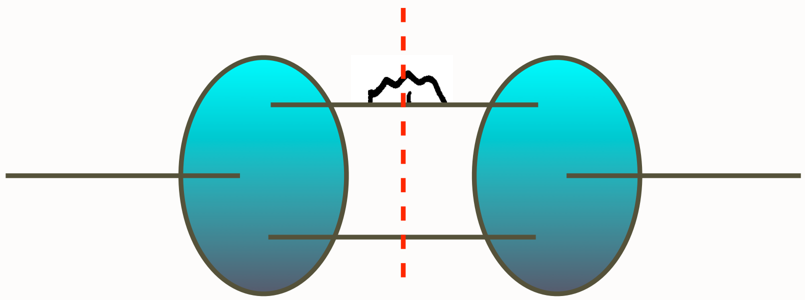
Positronium n=2

e^+e^-



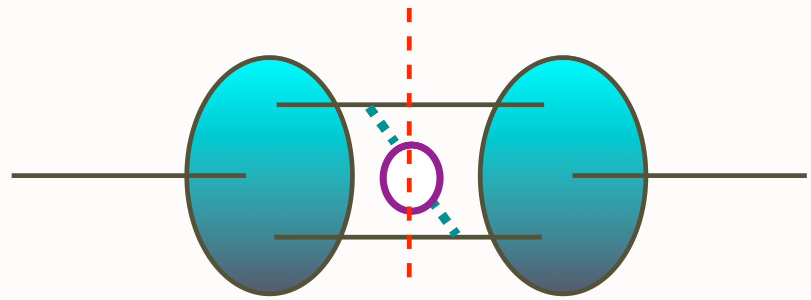
Hyperfine splitting n=3

$e^+e^-\gamma$



Lamb Shift n=3

$e^+e^-\gamma$



Vacuum Polarization n=4

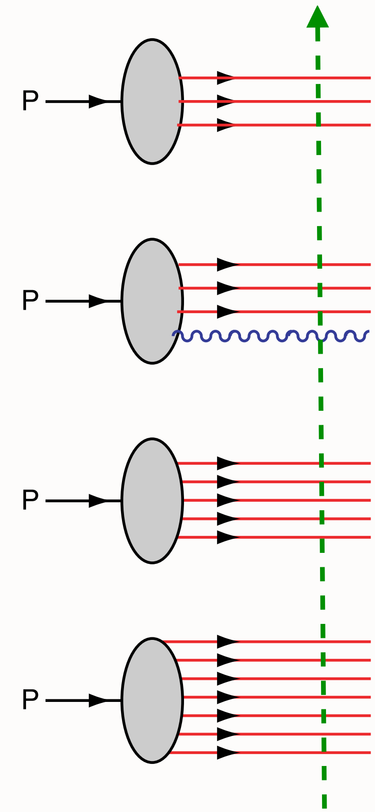
$e^+e^-e^+e^-$

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$



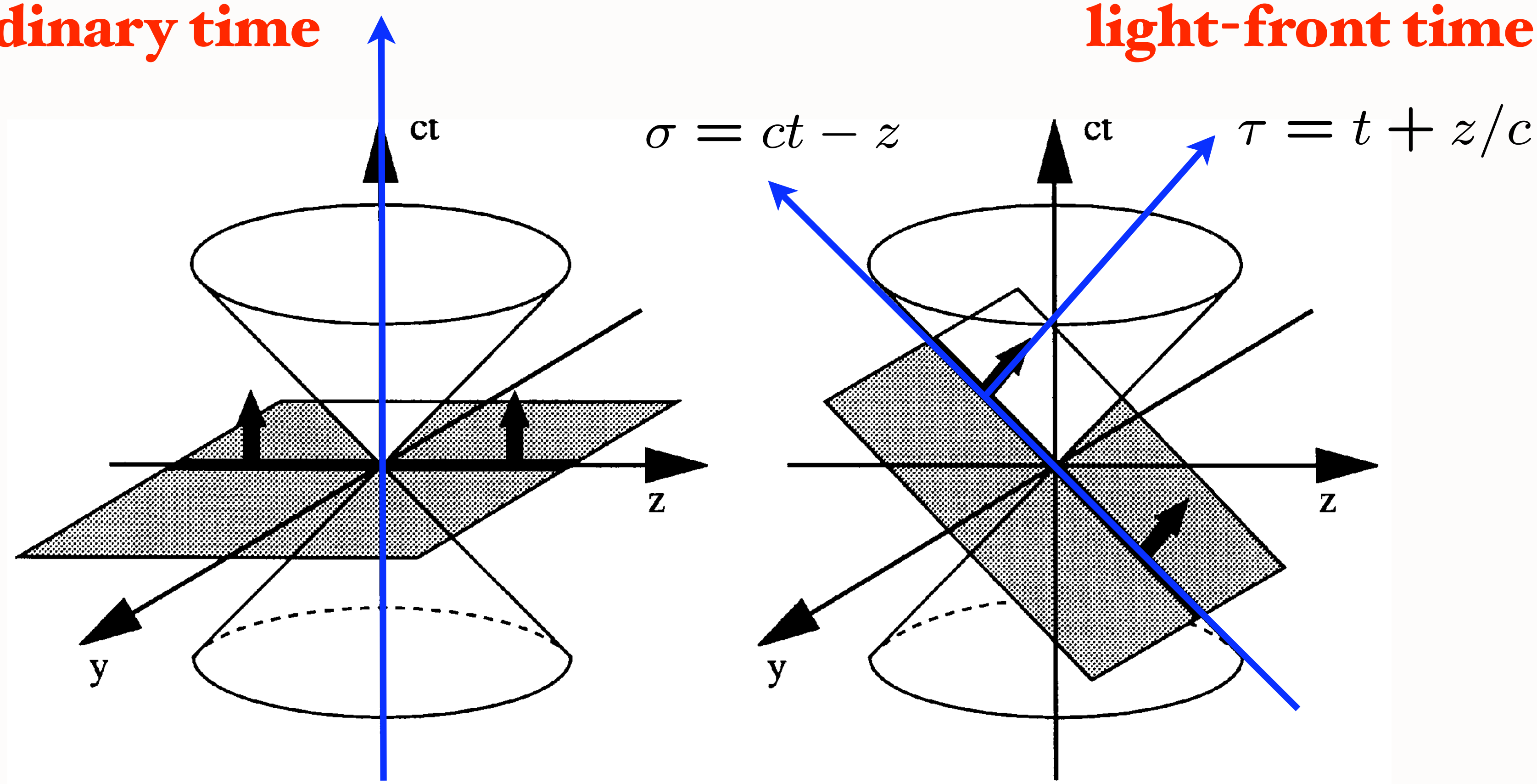
$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Physics of intrinsic gluons, sea quarks, asymmetries

Dirac's Amazing Idea:
The "Front Form"

Evolve in ordinary time

Evolve in light-front time!



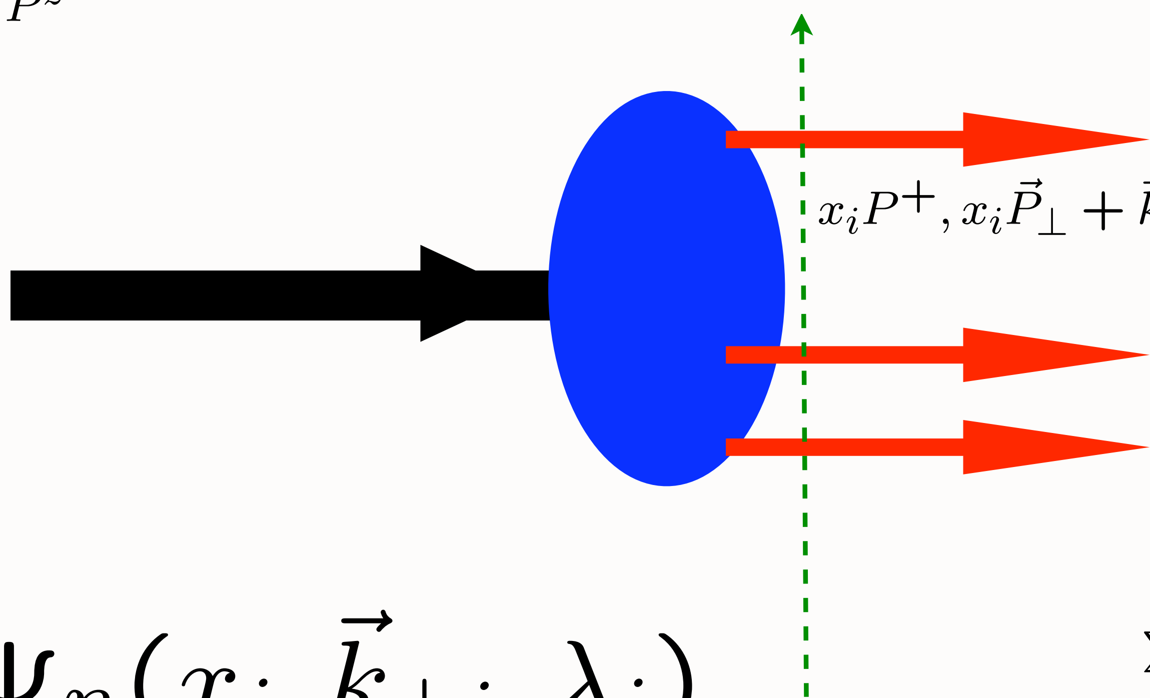
Instant Form

Front Form

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$P^+, \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

'Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
'tis an immortal thing

Montgomery

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

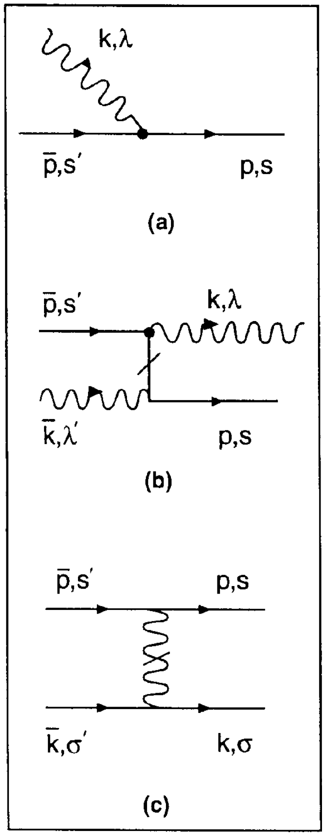
Invariant under boosts. Independent of P^μ

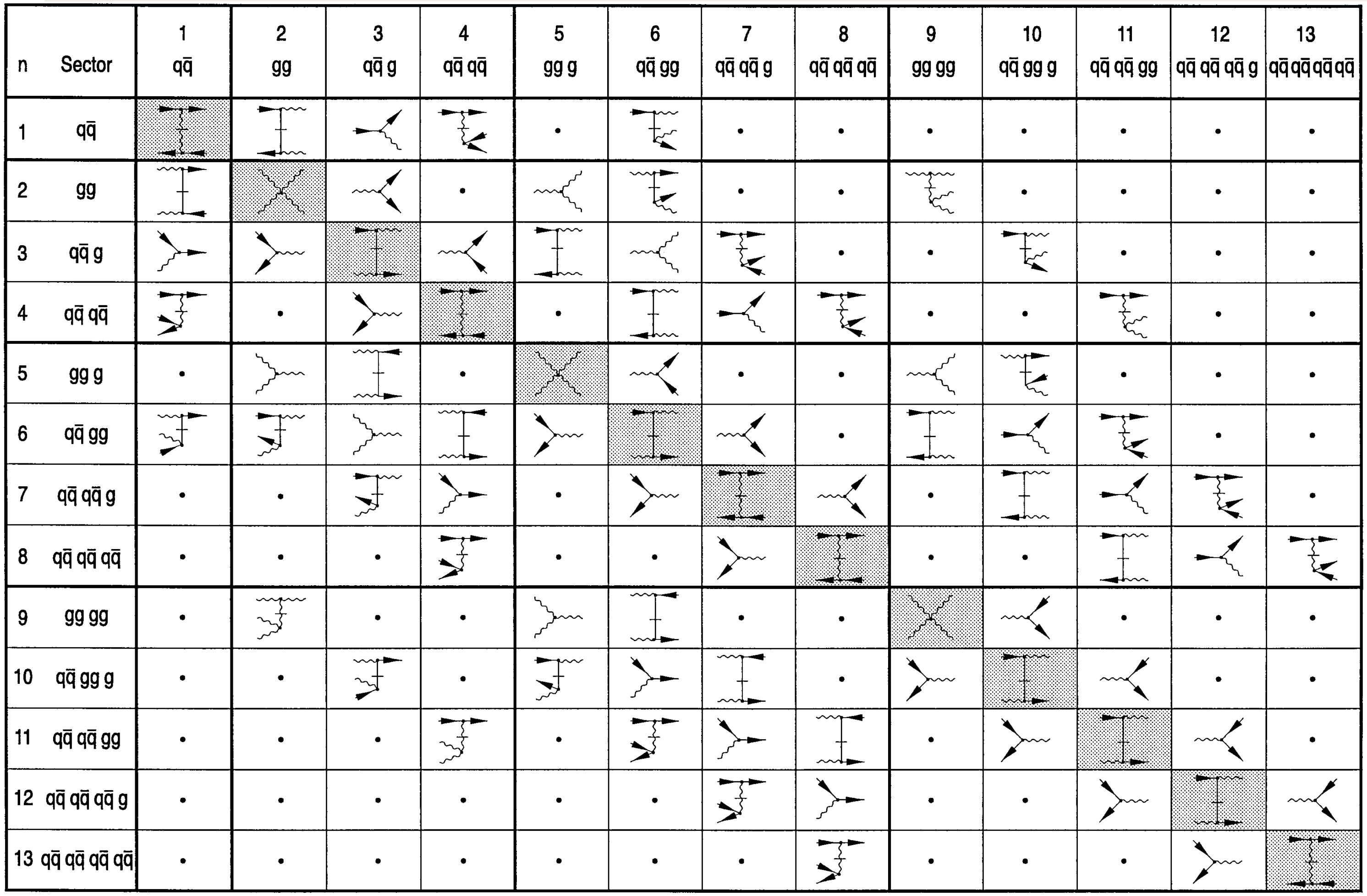
$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC} |\Psi_h\rangle = \mathcal{M}^2_h \; |\Psi_h\rangle$$

DLCQ





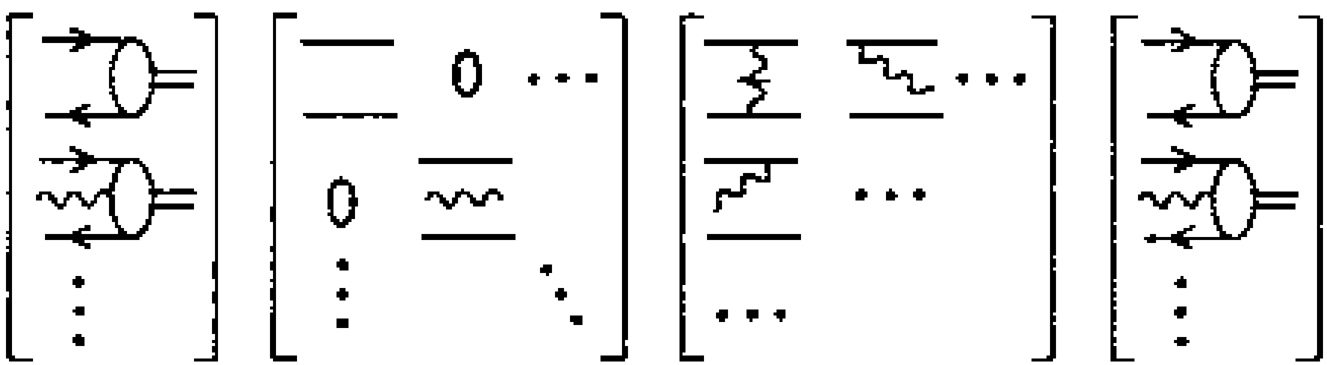
$A^+ = 0$

Discretized Light-Cone Quantization

Pauli, Pinsky, sjb

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

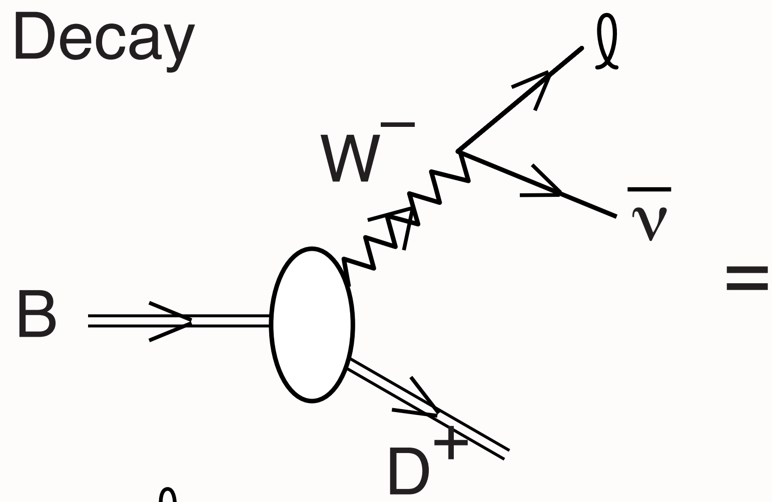


$A^+ = 0$

G. P. Lepage, sjb

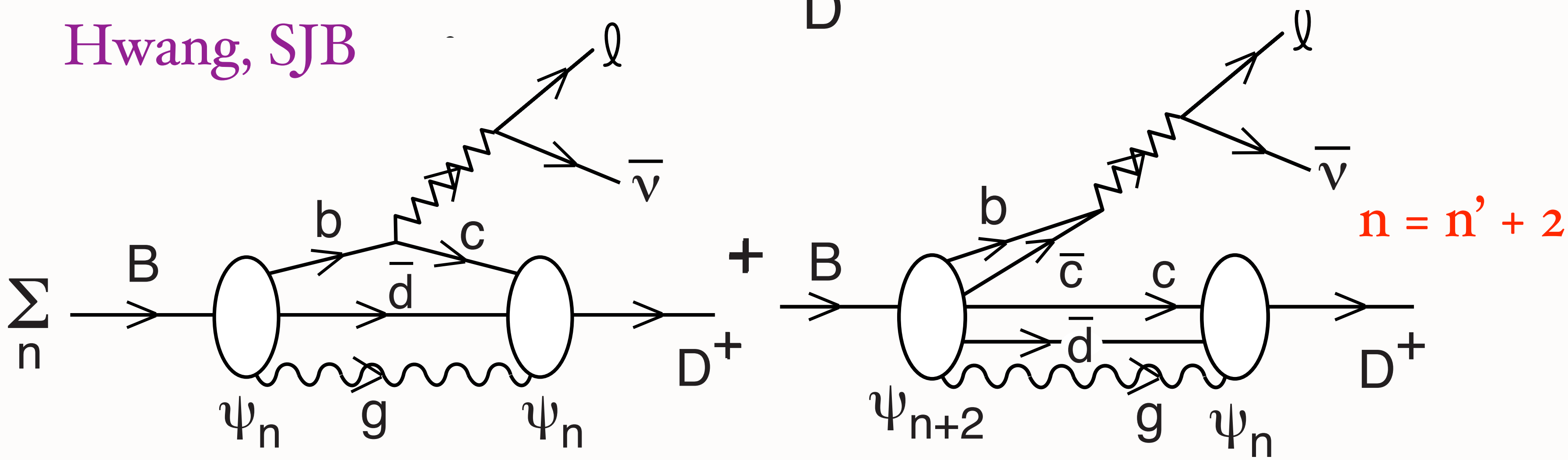
Weak Exclusive Decay

$$\langle D \,|\, J^{+}(0) \,|\, B \rangle$$



Exact Formula!

Hwang, SJB



Annihilation amplitude needed for Lorentz Invariance

Remarkable Features of Hadron Structure

Kei-Fei Liu

- Valence quarks carry less than half of the proton's spin and momentum
- Non-zero quark orbital angular momentum
- Asymmetric sea: $\bar{u}(x) \neq \bar{d}(x)$ relation to meson cloud
- Non-symmetric strange and antistrange sea $\bar{s}(x) \neq s(x)$
- Intrinsic charm and bottom at high x
- Hidden-Color Fock states of the Deuteron

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

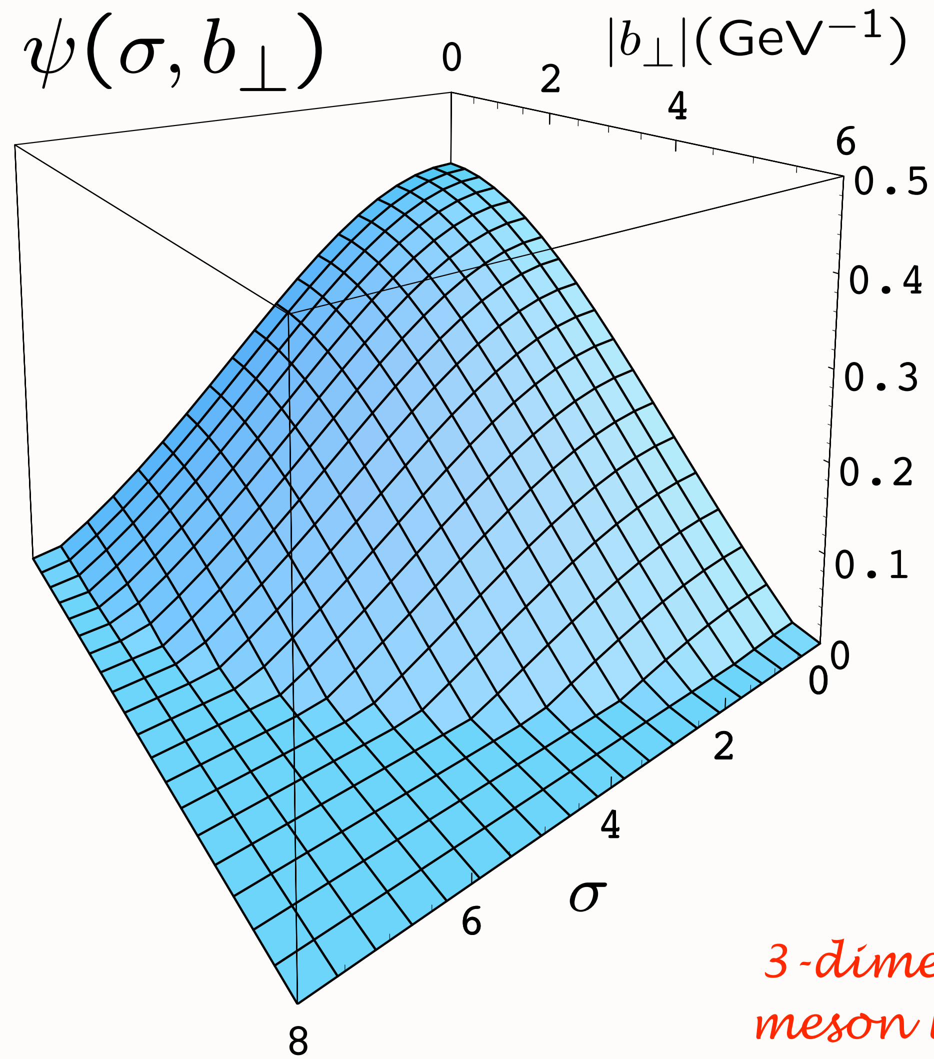
Invariant under boosts. Independent of P^μ

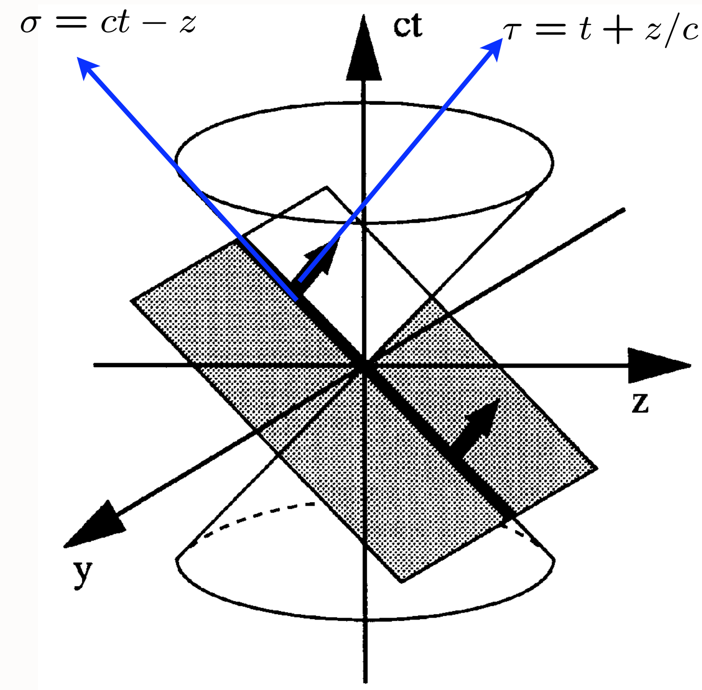
$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

AdS/CFT Holographic Model

G. de Teramond
SJB

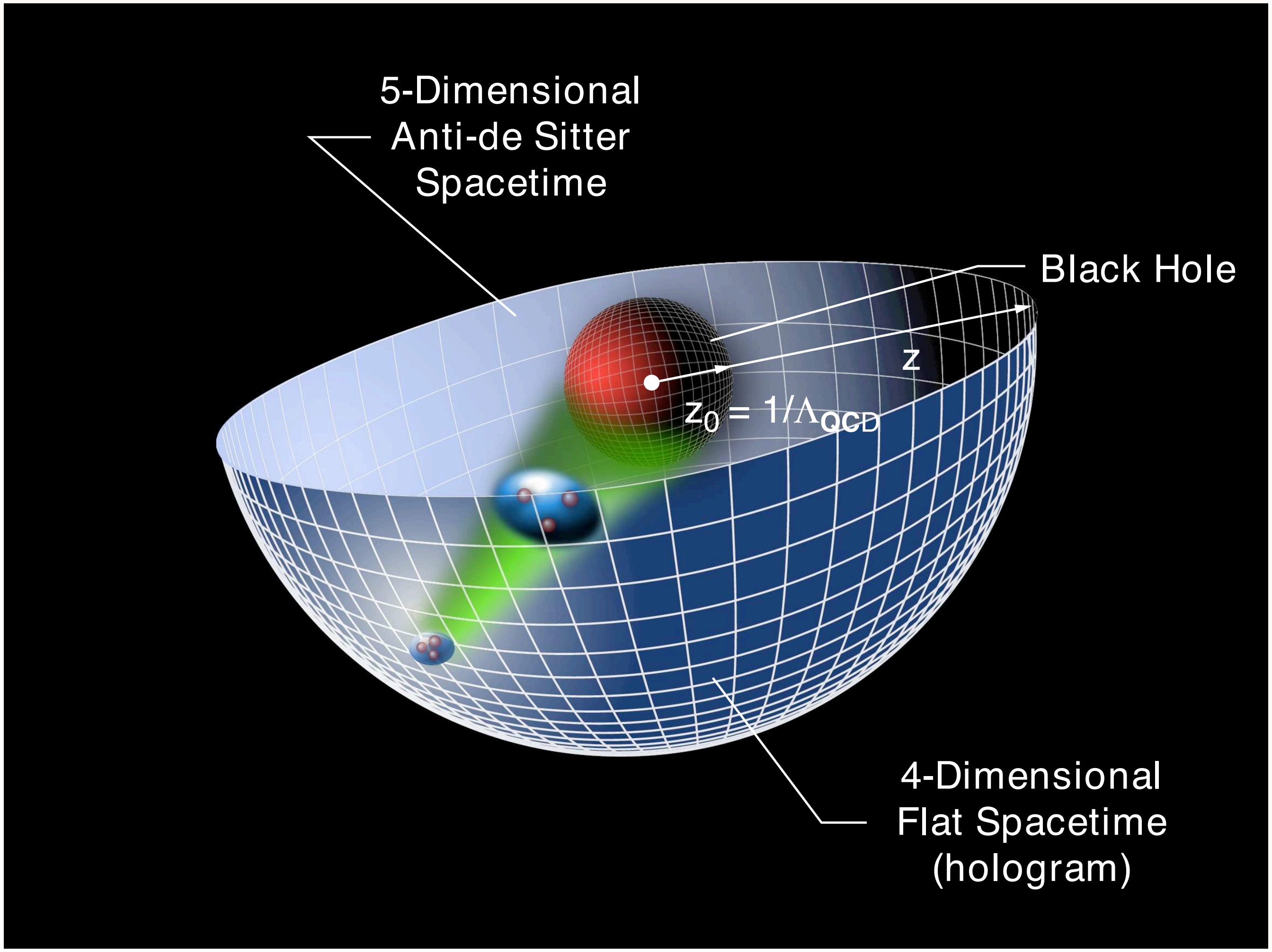




The front form

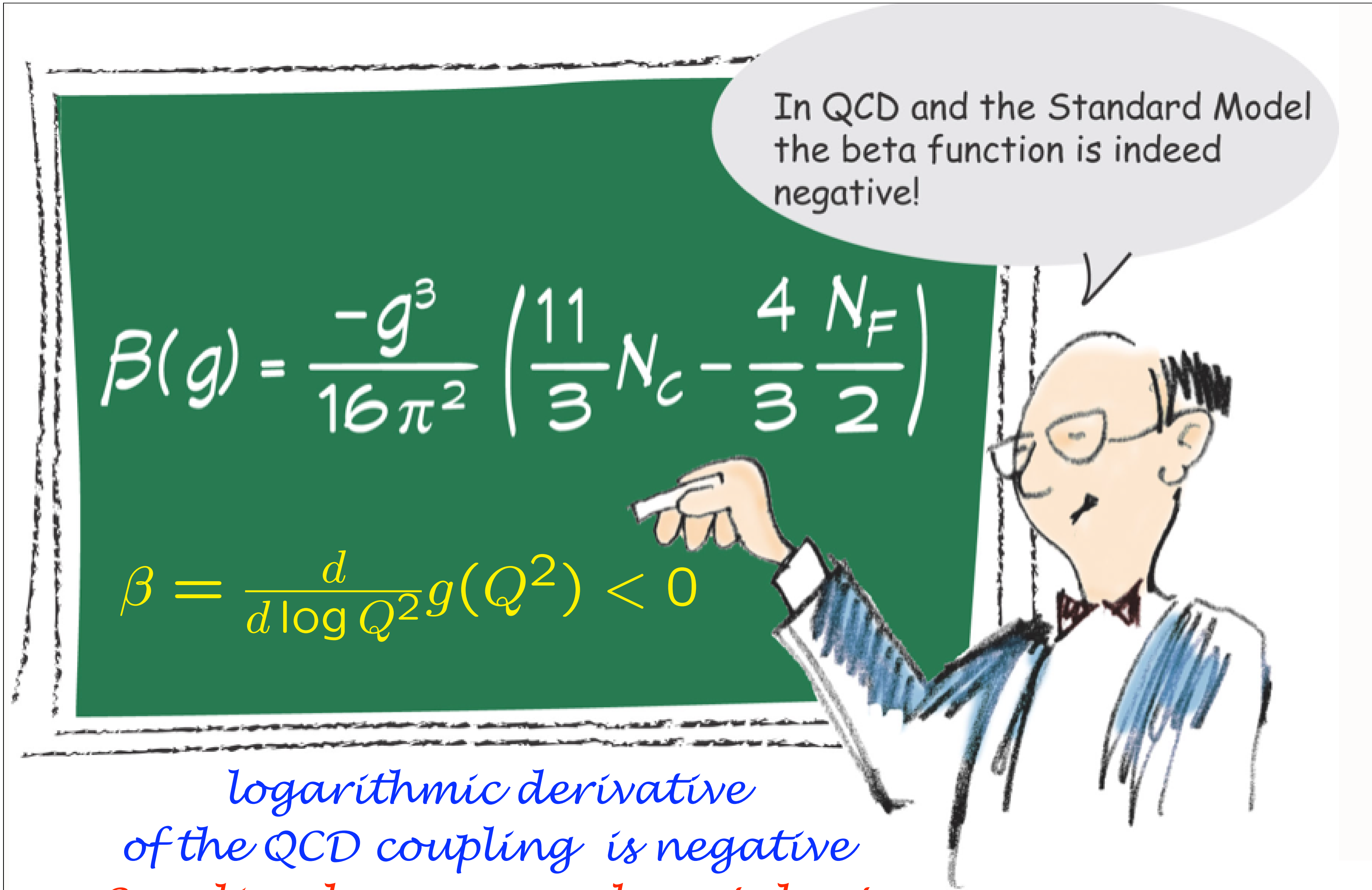
3-dimensional photograph: meson LFWF at fixed LF Time

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond



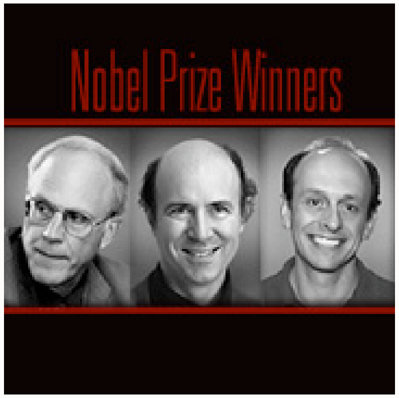
$$\beta = \frac{d}{d\log Q^2} g(Q^2) < 0$$

logarithmic derivative of the QCD coupling is negative

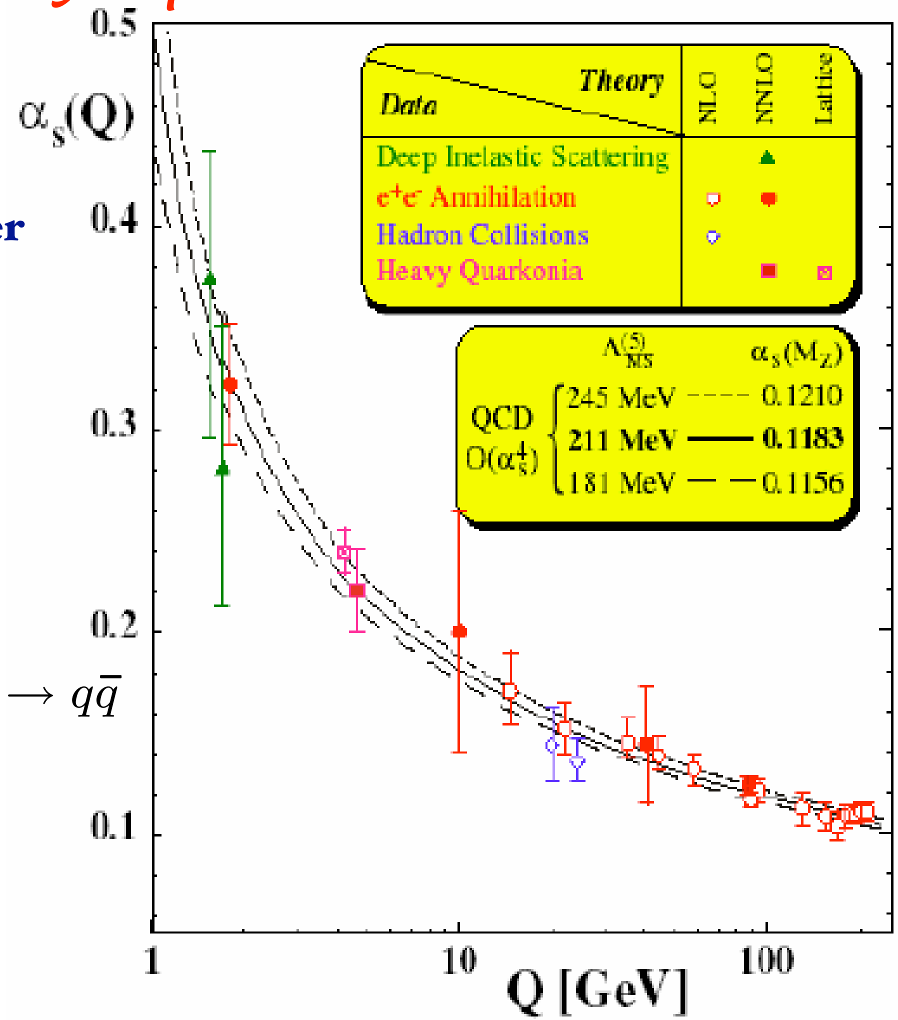
Coupling becomes weaker at short distances or high momentum transfer

Gross, Wilczek, Politzer
Khriplovich, `t Hooft

Verification of Asymptotic Freedom



Gross, Wilczek, Politzer
Khriplovich, `t Hooft



$$\frac{\sigma(e^+e^-\rightarrow \text{three jets})}{\sigma(e^+e^-\rightarrow \text{two jets})}$$

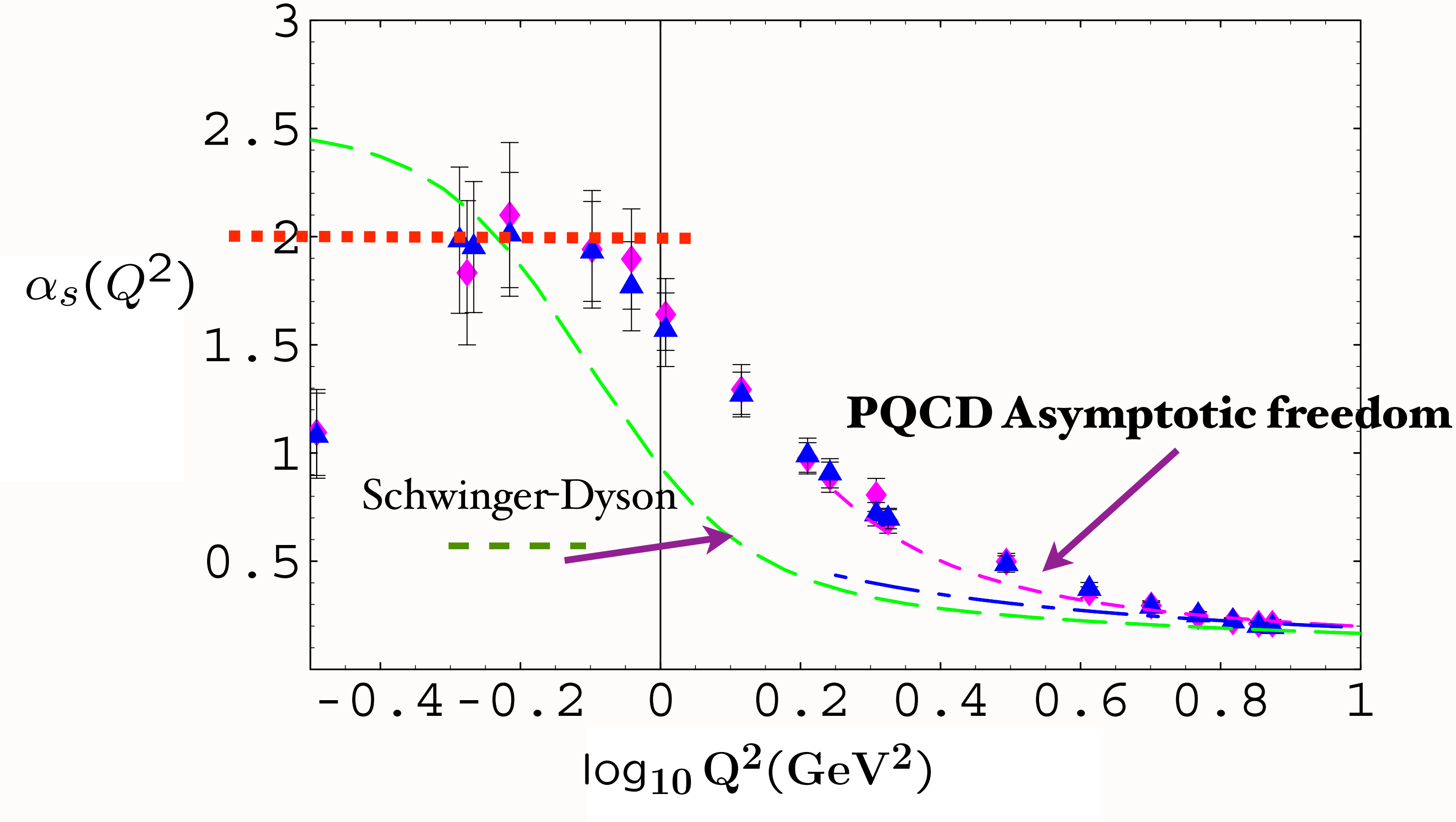
Ratio of rate for $e^+e^- \rightarrow q\bar{q}g$ to $e^+e^- \rightarrow q\bar{q}$
at $Q = E_{CM} = E_{e^-} + E_{e^+}$

proportional to $\alpha_s(Q)$

$$\alpha(Q^2) \simeq \frac{4\pi}{\beta_0} \frac{1}{\log Q^2/\Lambda^2_{QCD}}$$

Conformal window
Infrared fixed-point

$$\beta(Q^2) = \frac{d\alpha_s(Q^2)}{d\log Q^2} \to 0$$



Shirkov
Gribov
Dokshitser
Siminov
Maxwell
Cornwall

lattice: Furui, Nakajima (MILC)

DSE: Alkofer, Fischer, von Smekal et al.

IR Fixed Point for QCD?

- *Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point!* *Alkofer, Fischer, von Smekal et al.*
- *Lattice Gauge Theory*
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small Q^2
- Justifies application of AdS/CFT in strong-coupling conformal window

IR Fixed-Point for QCD?

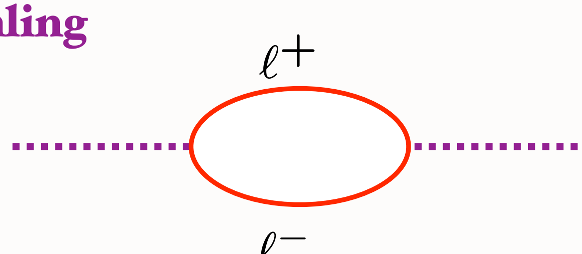
- *Dyson-Schwinger Analysis:* **QCD Coupling has IR Fixed Point**
  Alkofer, Fischer, von Smekal et al.

- *Evidence from Lattice Gauge Theory* Furui, Nakajima

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

- Confined or massive gluons: **Decoupling of QCD vacuum polarization at small Q²** **Serber-Uehling**

$$\Pi(Q^2) \to \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



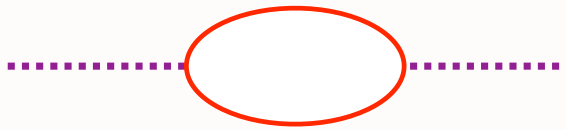
- **Justifies application of AdS/CFT in strong-coupling conformal window**

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge for QED

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}[\frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2})\sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|}]$$

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi} \frac{\log Q^2}{m^2} \qquad Q^2 >> 4M^2$$

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d \log Q^2} = \frac{4}{3}(\frac{\alpha}{4\pi})^2 n_\ell > 0$$

$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi} \frac{Q^2}{m^2} \qquad Q^2 << 4M^2$$

Serber-Uehling

$\beta \propto \frac{Q^2}{m^2}$ **vanishes at small momentum transfer**

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Define QCD Coupling from Observable

Grunberg

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

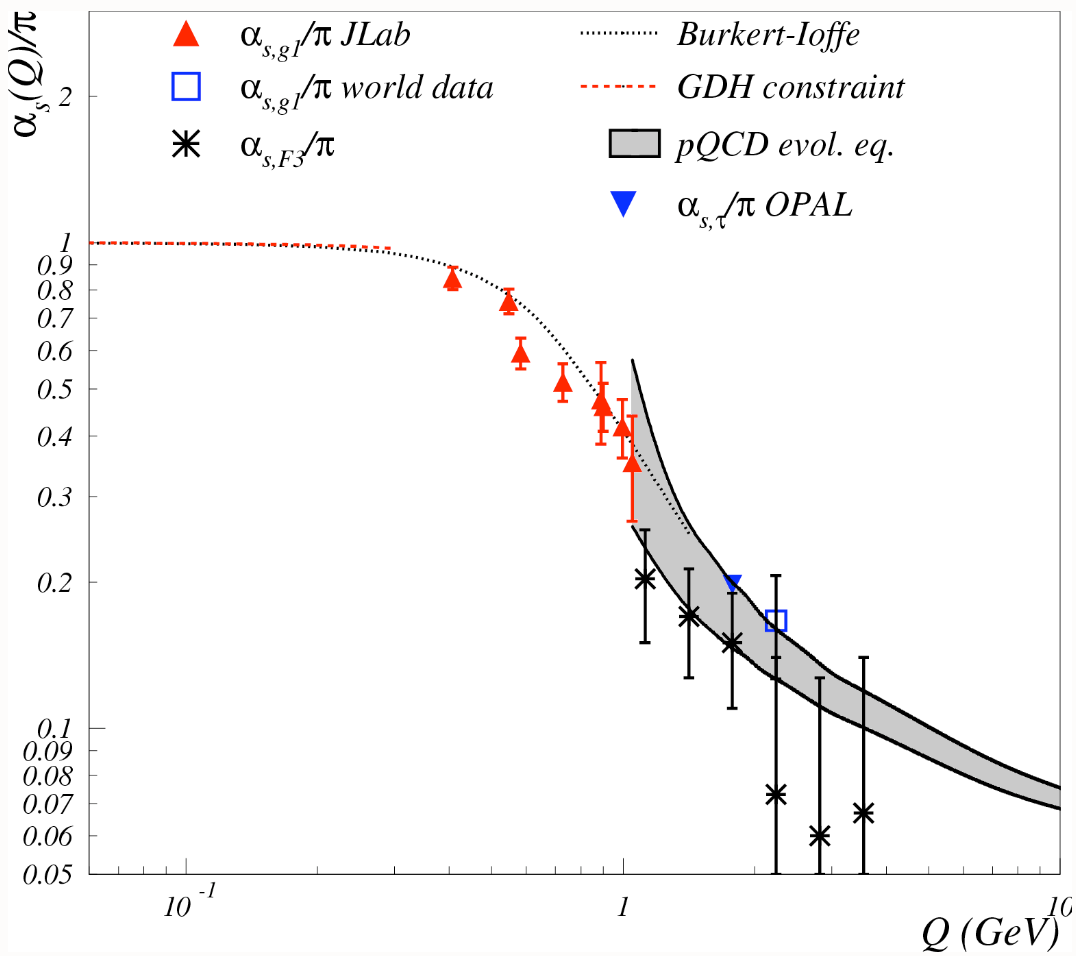
$$\Gamma(\tau \to Xe\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu)\times[1+\frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Effective Charges: analytic at quark mass thresholds, finite at small momenta

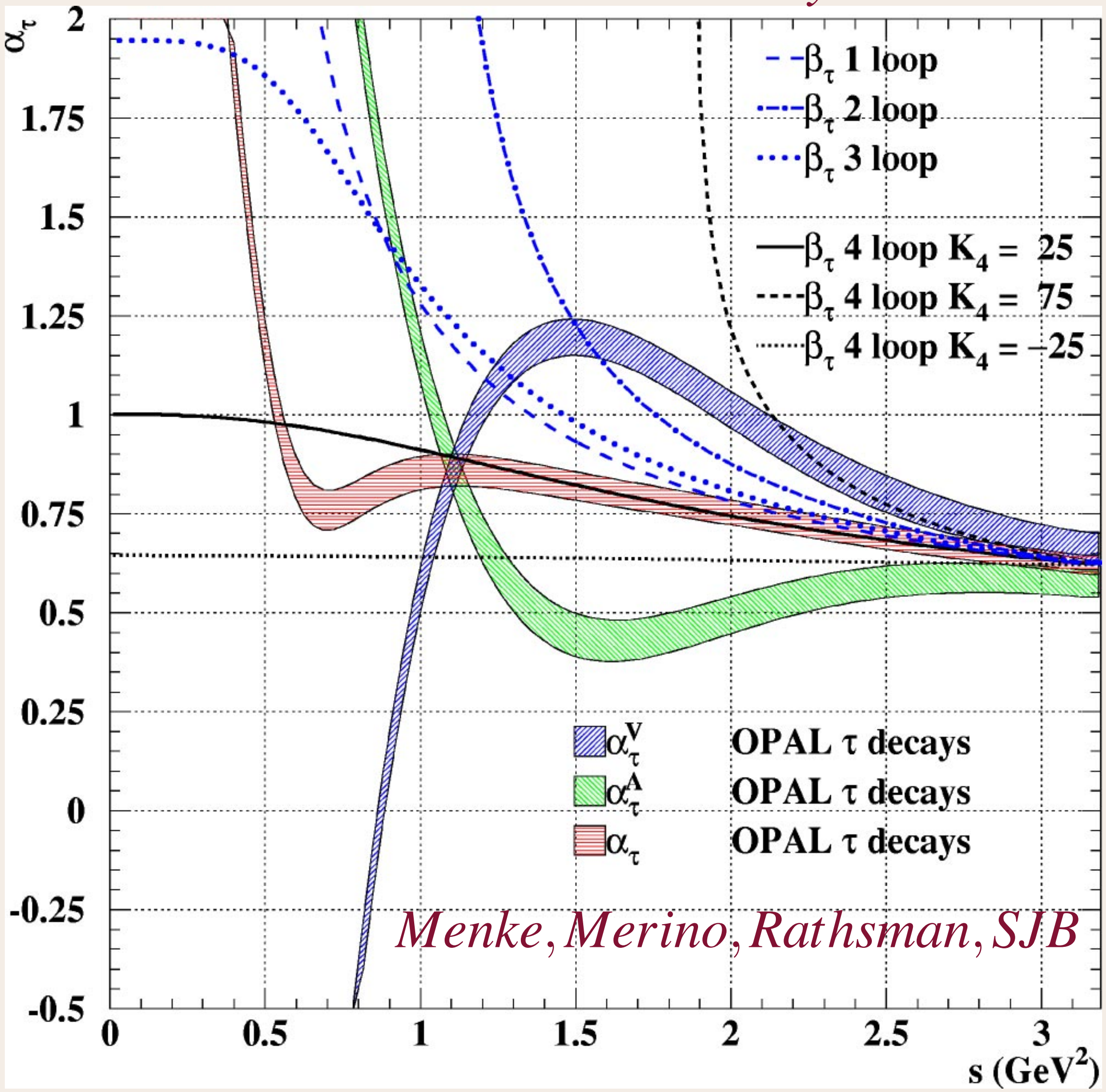
Deur et al: Effective Charge from Bjorken Sum Rule

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

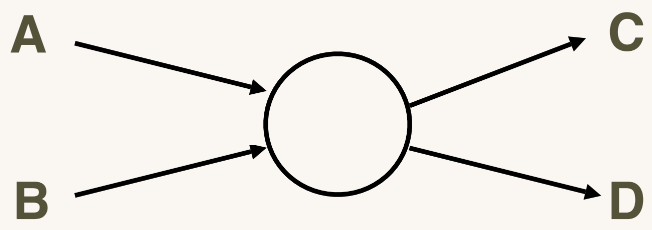
$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



QCD Effective Coupling from
hadronic τ decay



Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

Farrar & sjb; Matveev, Muradyan, Tavkhelidze

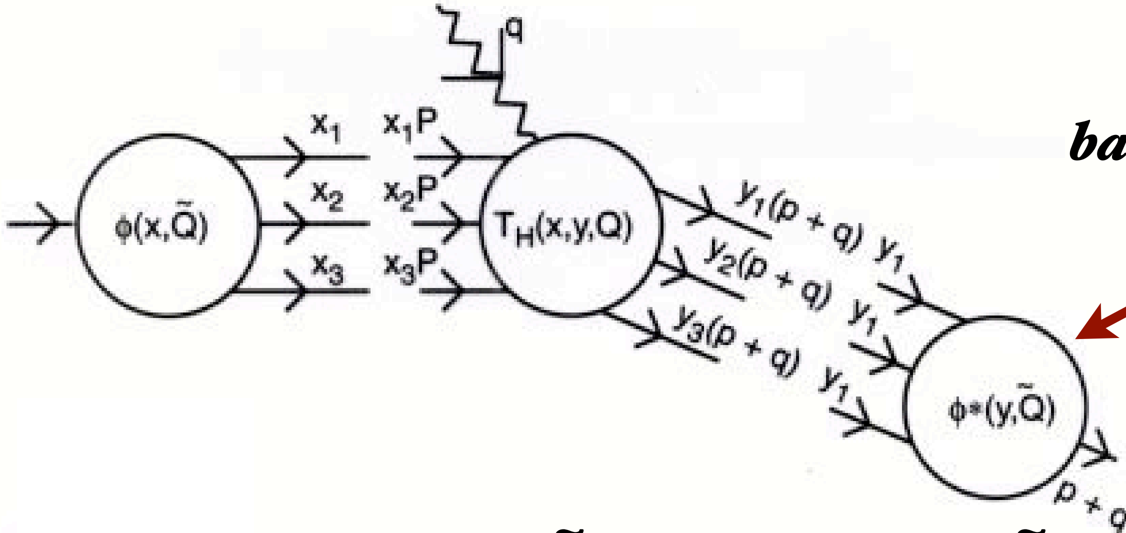
Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests

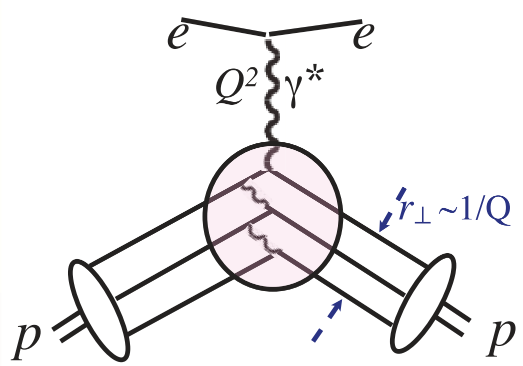
Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb



baryon distribution amplitude

$$M = \int \Pi dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$



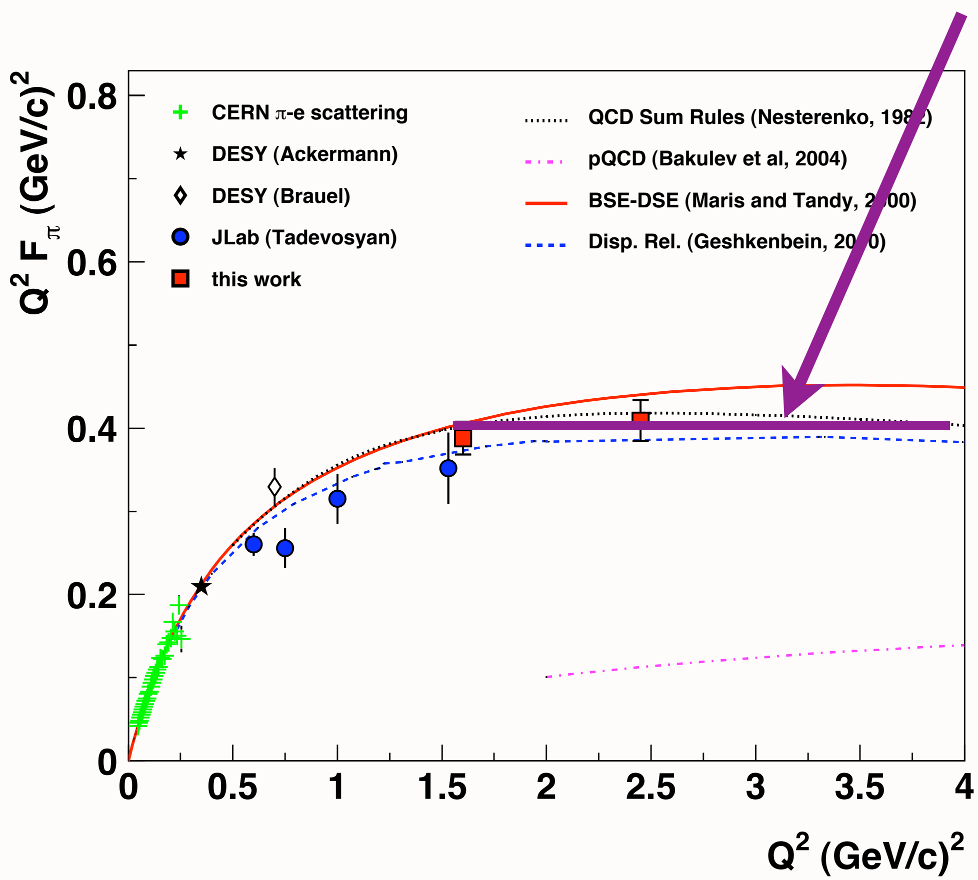
If $\alpha_s(\tilde{Q}^2) \simeq$ constant

$Q^4 F_1(Q^2) \simeq$ constant

Features of Hard Exclusive Processes in PQCD

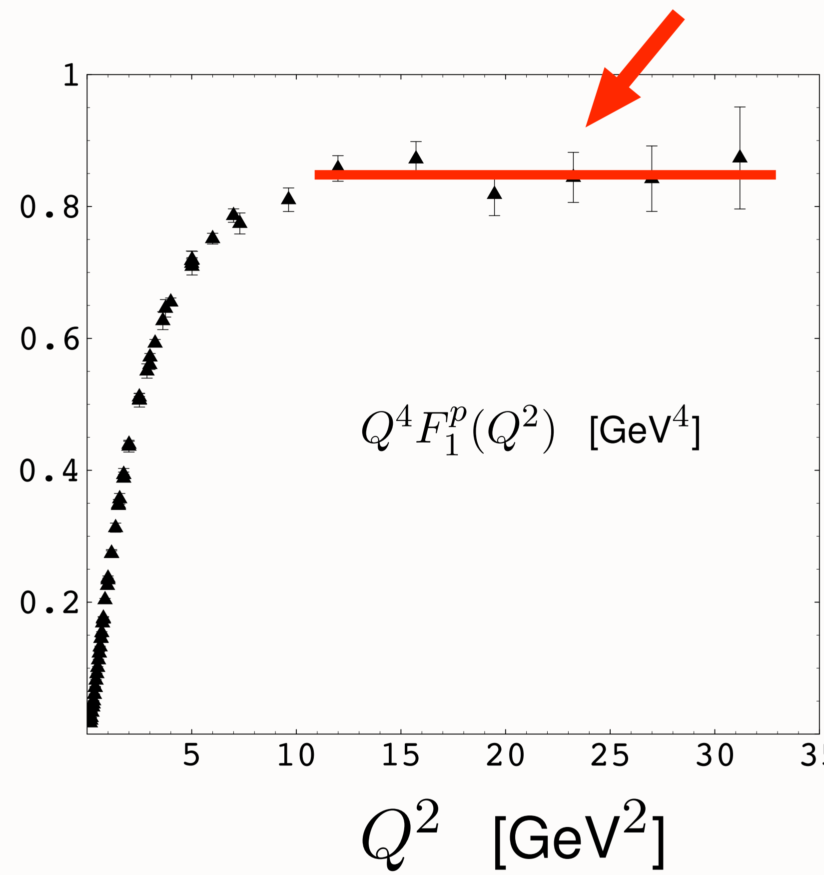
- Factorization of perturbative hard scattering subprocess amplitude and nonperturbative distribution amplitudes $M = \int T_H \times \Pi \phi_i$
- Dimensional counting rules reflect conformal invariance: $M \sim \frac{f(\theta_{CM})}{Q^{N_{tot}-4}}$
- Hadron helicity conservation: $\sum_{initial} \lambda_i^H = \sum_{final} \lambda_j^H$
- Color transparency Mueller, sjb;
- Hidden color Ji, Lepage, sjb;
- Evolution of Distribution Amplitudes Lepage, sjb; Efremov, Radyushkin

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const $\quad Q^4 F_1(Q^2) \to$ const



Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

G. Huber

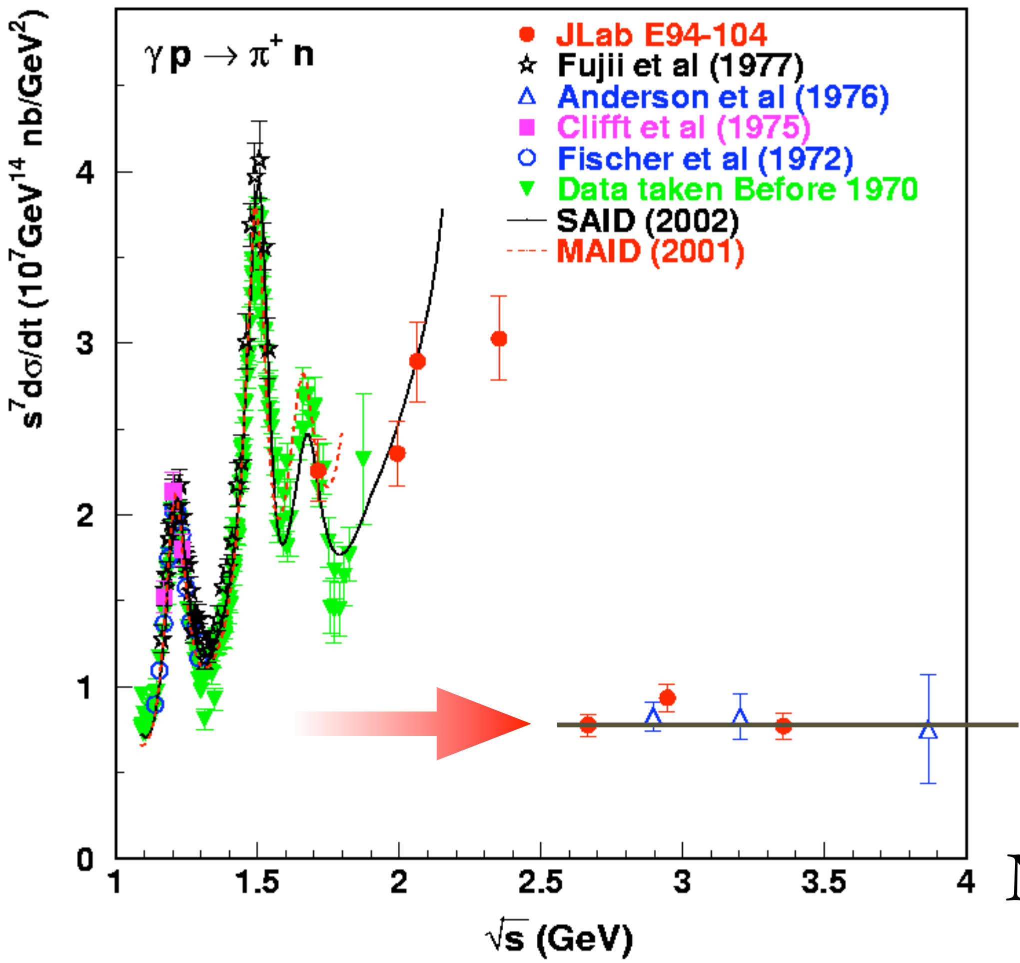


Generalized parton distributions from nucleon form-factor data.
M. Diehl (**DESY**), **Th. Feldmann** (**CERN**),
R. Jakob, **P. Kroll** (**Wuppertal U.**).
DESY-04-146, CERN-PH-04-154, WUB-04-08, Aug 2004. 68pp.
Published in Eur.Phys.J.C39:1-39,2005
e-Print Archive: **hep-ph/0408173**

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

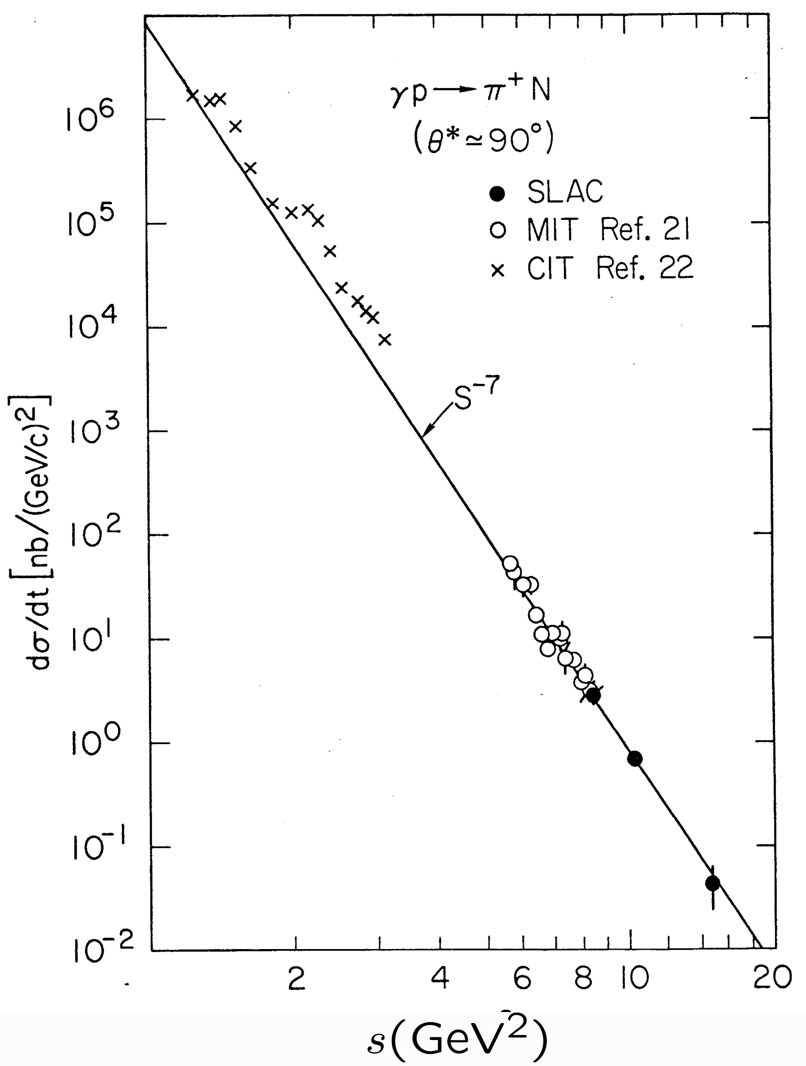
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

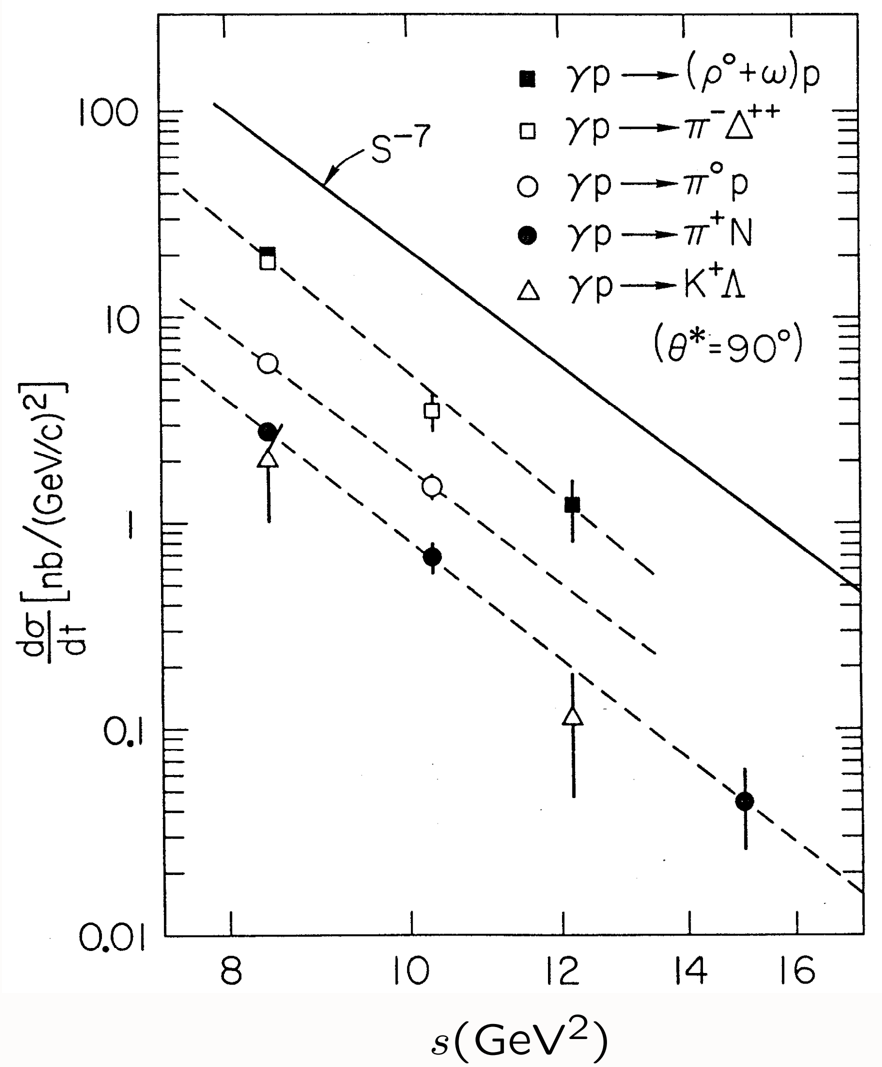
$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

No sign of running coupling

Conformal invariance

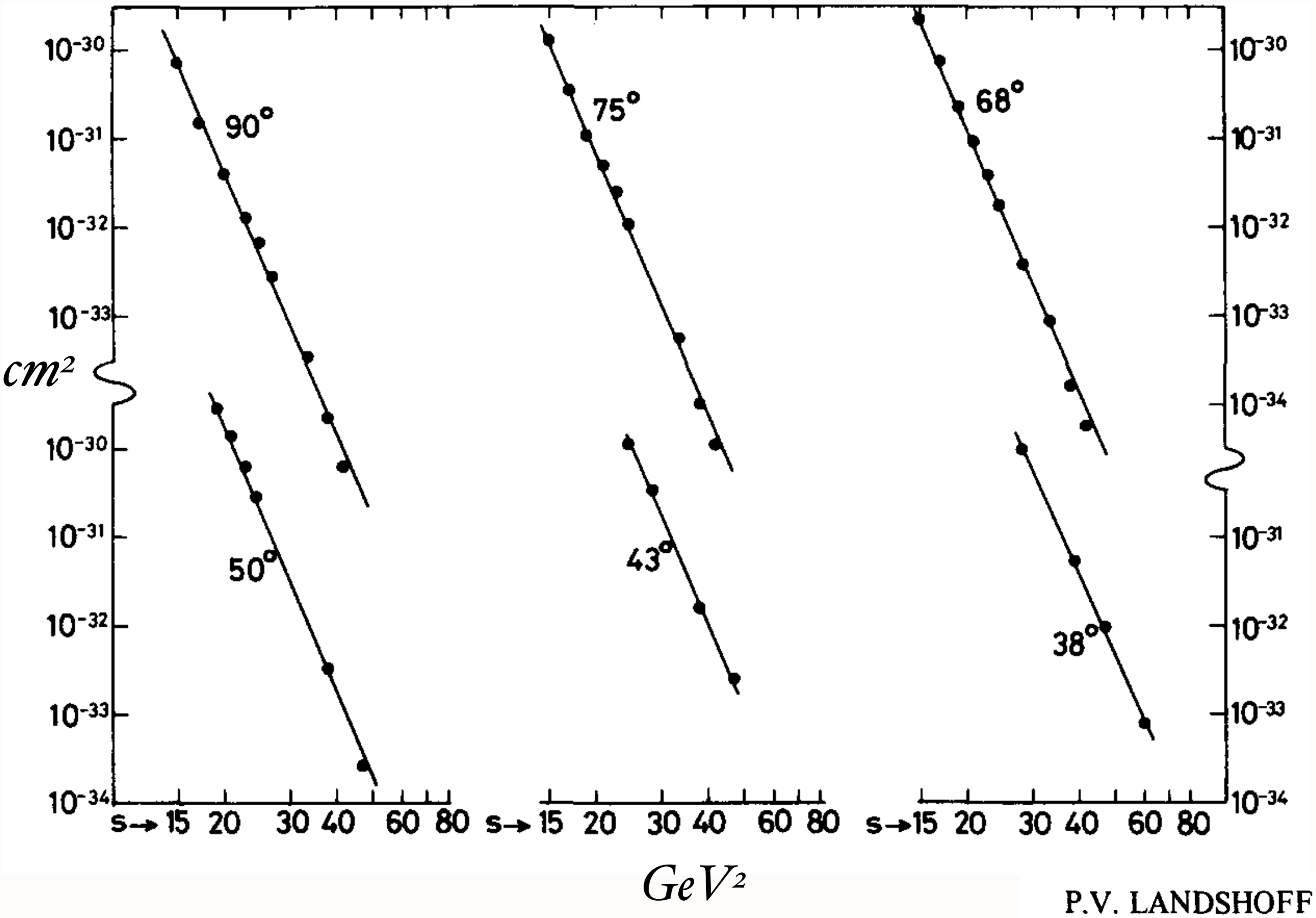




Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$



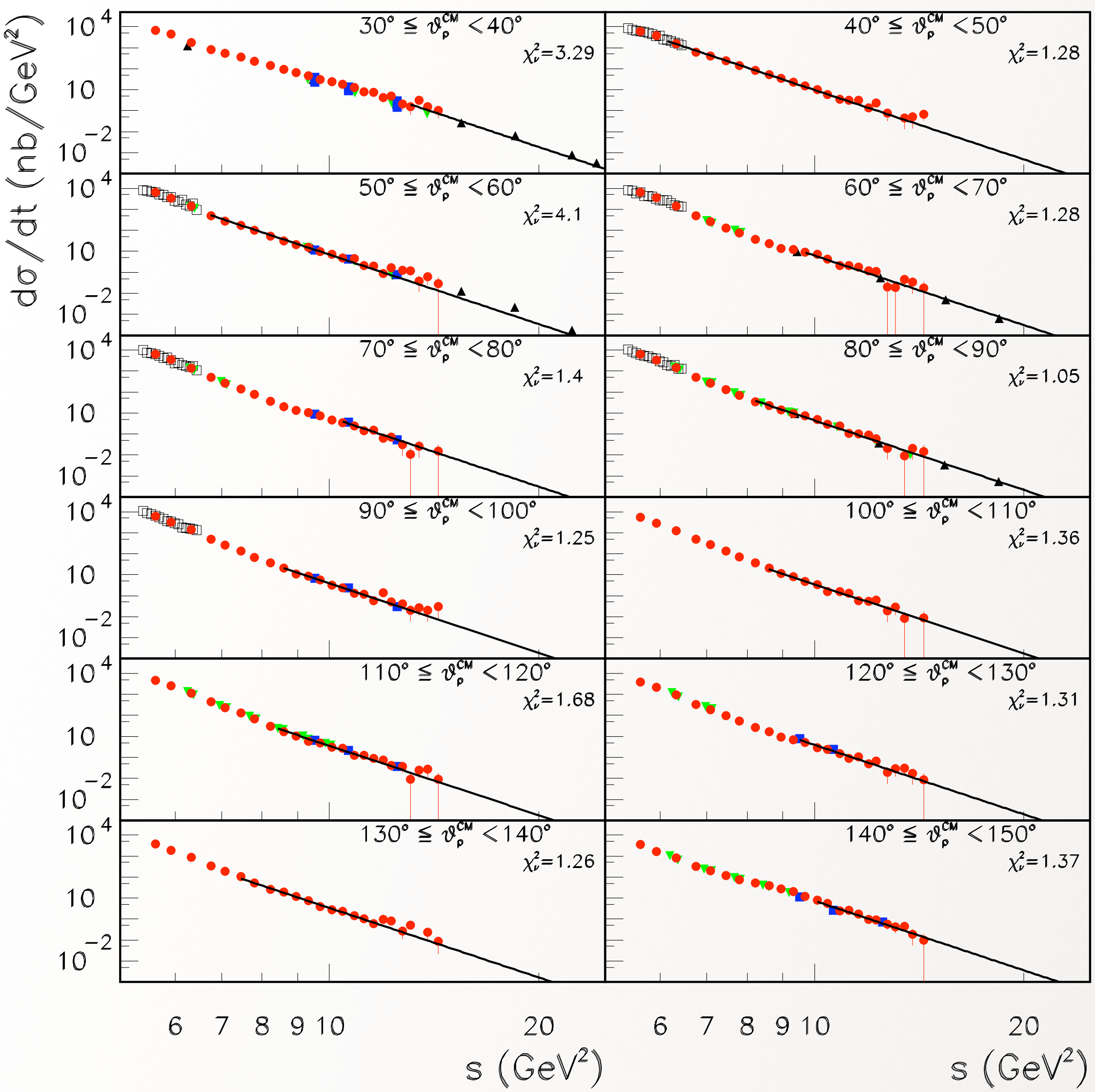
Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

P.V. LANDSHOFF and J.C. POLKINGHORNE

Deuteron Photodisintegration



J-Lab

PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$n_{tot} - 2 =$
$(1 + 6 + 3 + 3) - 2 = 11$

Reflects conformal invariance

Check of CCR

Fit of $d\sigma/dt$ data for the central angles and $P_T \geq 1.1$ GeV/c with

$A\ s^{-11}$

For all but two of the fits

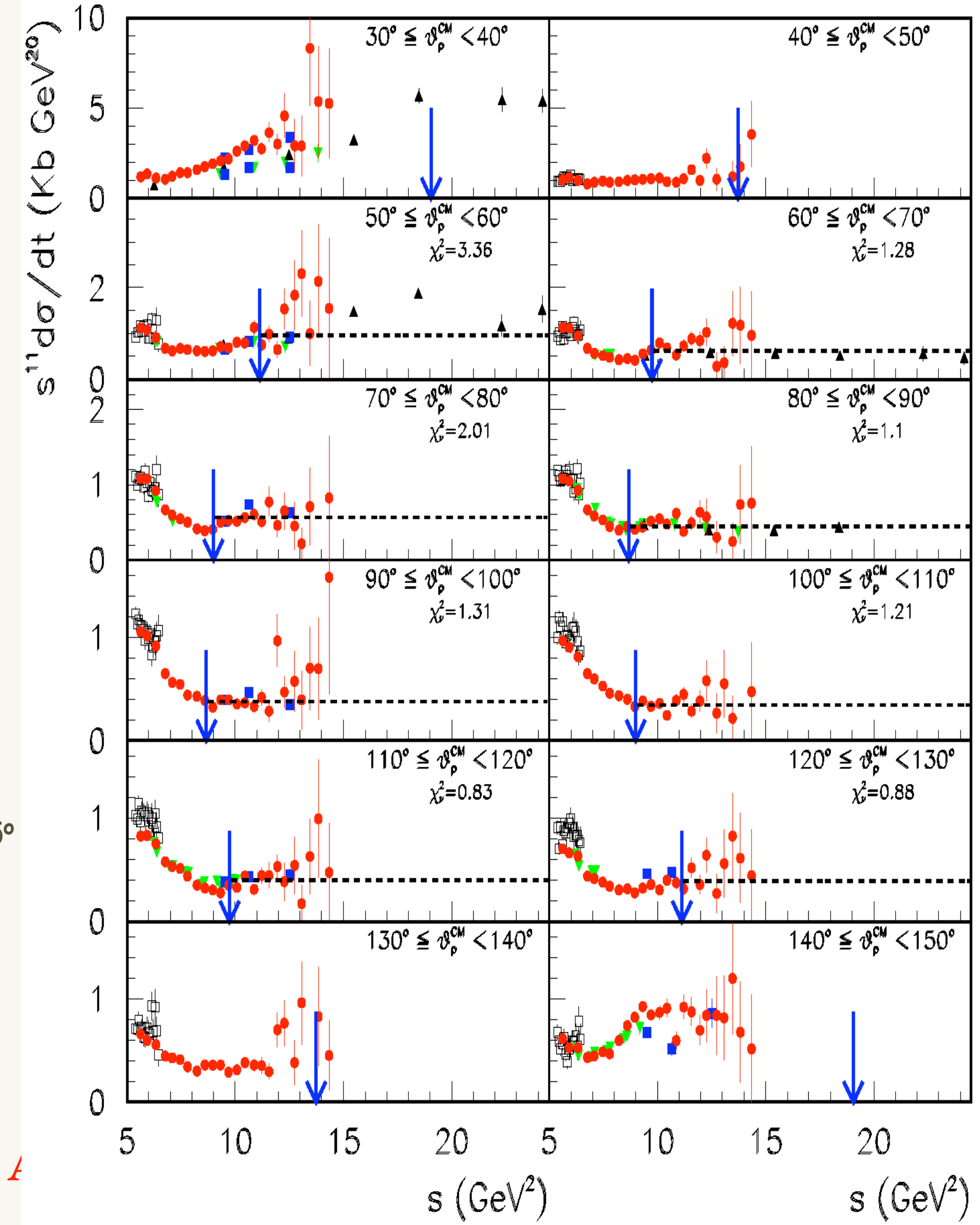
$\chi^2 \leq 1.34$

- Better χ^2 at 55° and 75° if different data sets are renormalized to each other
- No data at $P_T \geq 1.1$ GeV/c at forward and backward angles
- Clear s^{-11} behaviour for last 3 points at 35°

Data consistent with CCR

K. F. Liu Colloquium
University of Kentucky, April 19, 2007

P.Rossi et al, P.R.L. 94, 012301 (2005)



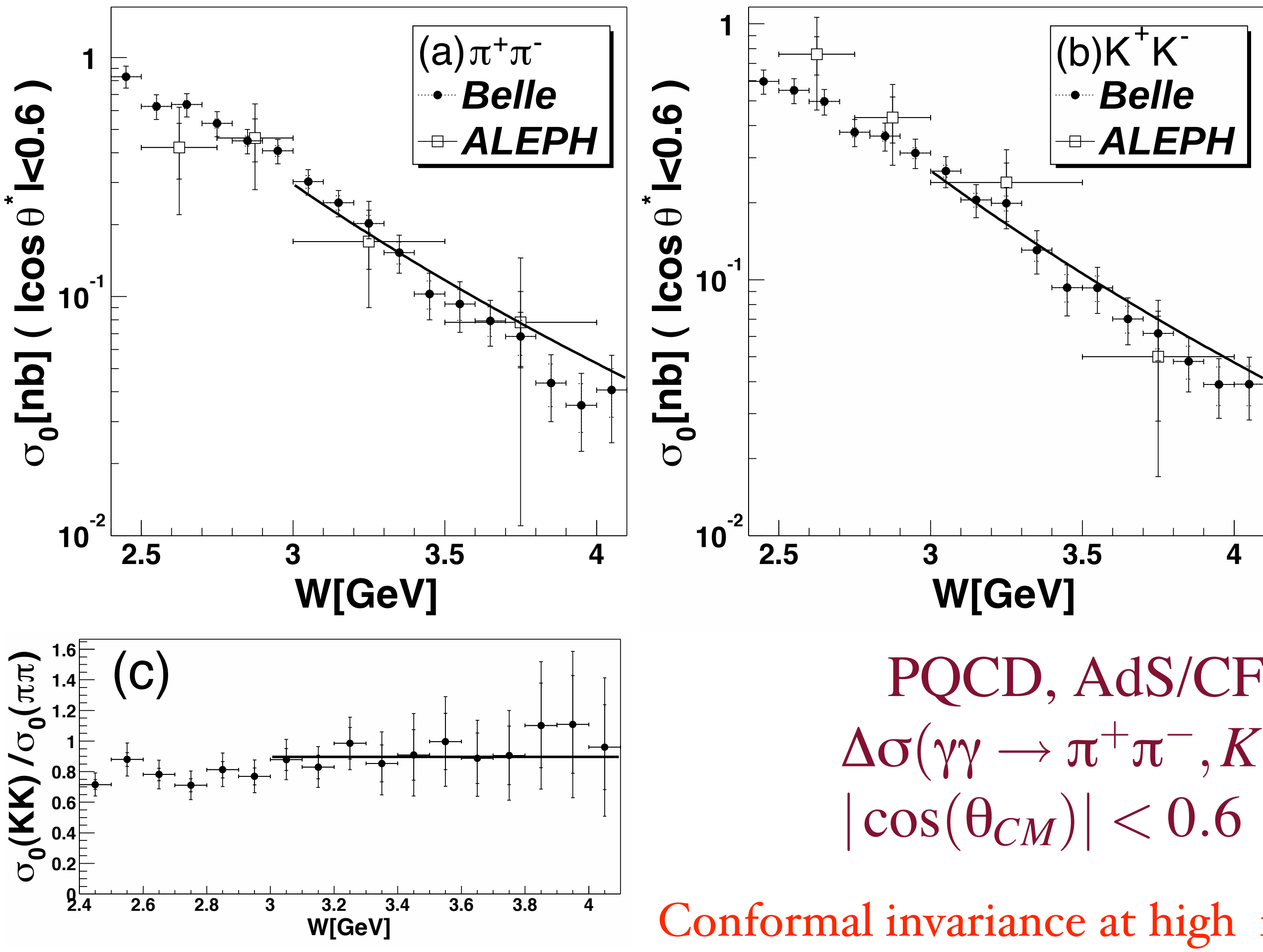
- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \to np$
- $$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$
- $$n_{tot} = 1 + 6 + 3 + 3 = 13$$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$

at high p_T



Two-Photon Reactions

Hard Exclusive Processes: Fixed angle

PQCD, AdS/CFT:

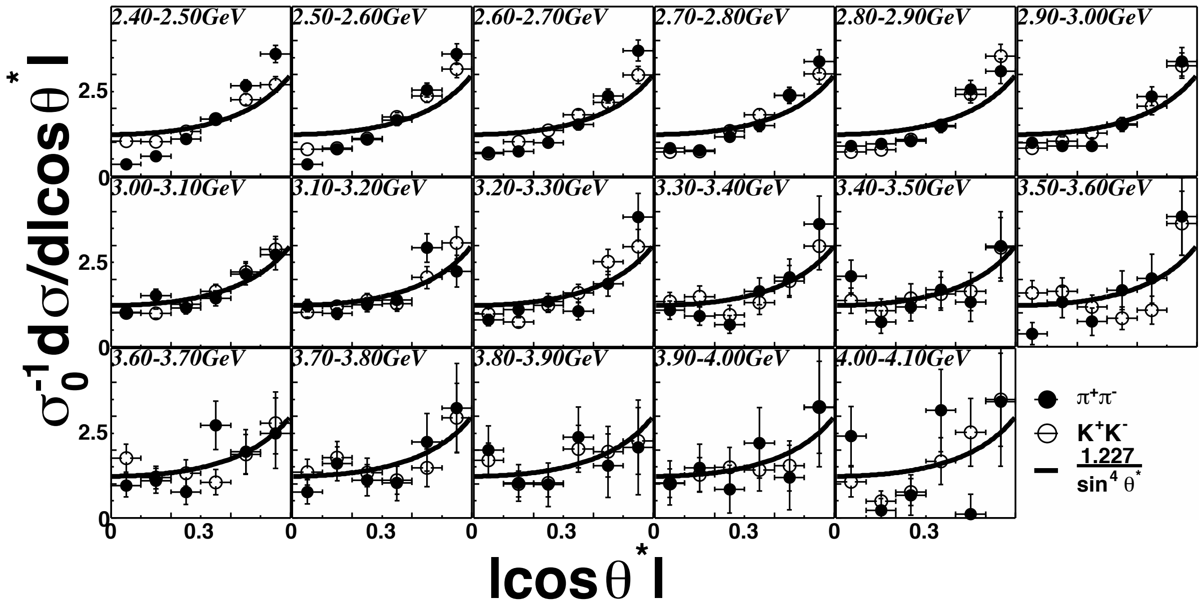
$$\Delta\sigma(\gamma\gamma \to \pi^+\pi^-, K^+, K^-) \sim 1/W^6$$

$$|\cos(\theta_{CM})| < 0.6$$

Conformal invariance at high momentum transfers!

Fig. 5. Cross section for (a) $\gamma\gamma \to \pi^+\pi^-$, (b) $\gamma\gamma \to K^+K^-$ in the c.m. angular region $|\cos\theta^*| < 0.6$ together with a W^{-6} dependence line derived from the fit of $s|R_M|$. (c) shows the cross section ratio. The solid line is the result of the fit for the data above 3 GeV. The errors indicated by short ticks are statistical only.

PQCD: $$\frac{d\sigma}{d|\cos\theta^*|}(\gamma\gamma \to M^+M^-) \approx \frac{16\pi\alpha^2}{s}\frac{|F_M(s)|^2}{\sin^4\theta^*},$$



4. Angular dependence of the cross section, $\sigma_0^{-1} d\sigma/d|\cos\theta^*|$, for the $\pi^+\pi^-$(closed circles) and K^+K^-(open circles) processes. The curves are $1.227 \times \sin^{-4}\theta^*$. The errors are statistical only.

Measurement of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ processes at energies of 2.4–4.1 GeV

Belle Collaboration

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Define "Reduced" Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^{\,2}(Q^2/4)}.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

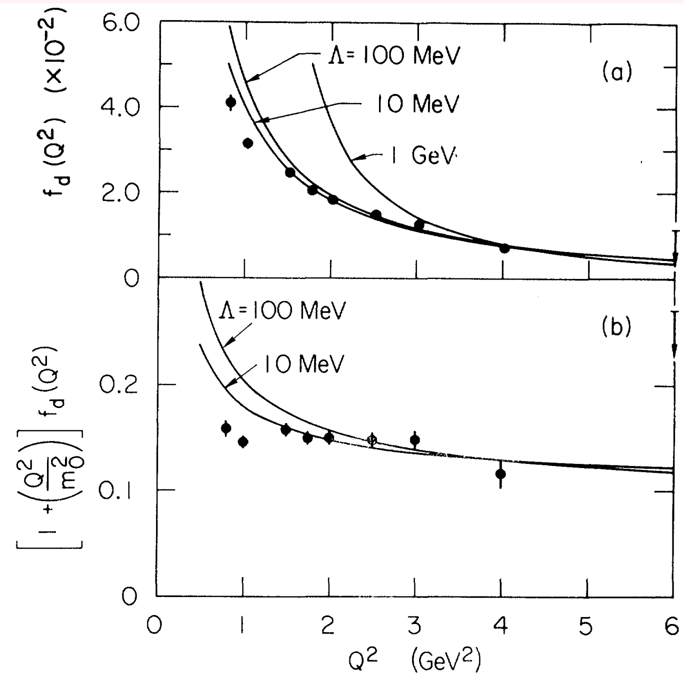


FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28\ \mathrm{GeV}^2$ is used (Ref. 8).

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions x_i ($i = 1, 2, \ldots, 6$) can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, C_d, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum Q, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy] = \delta(1 - \sum_{i=1}^{6} y_i) \prod_{i=1}^{6} dy_i$, $C_F = (n_c^2 - 1)/2n_c = \frac{4}{3}$, $\beta = 11 - \frac{2}{3} n_f$, and n_f is the effective number of flavors$\}$

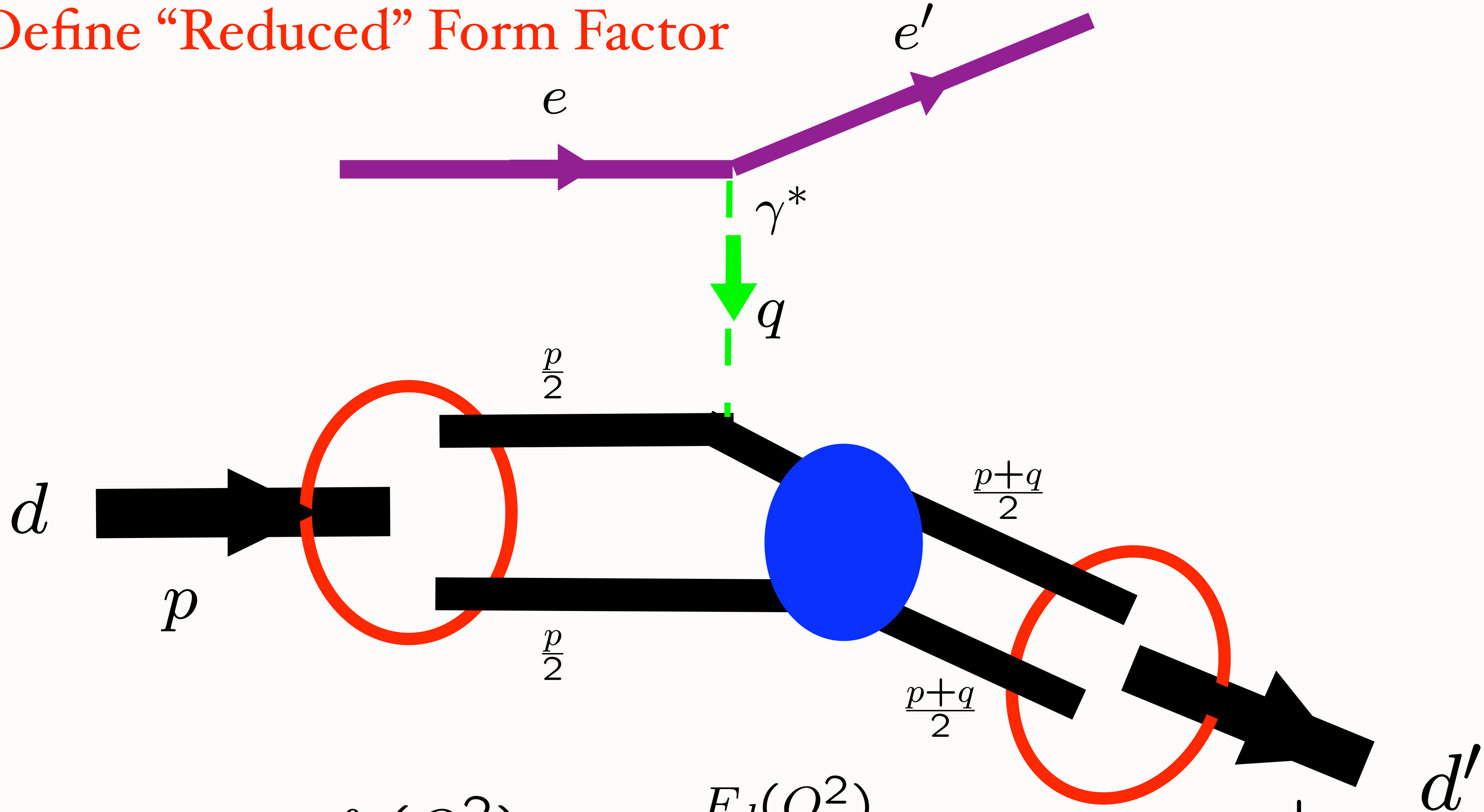
$$\prod_{k=1}^{6} x_k \left[\frac{\partial}{\partial \xi} + \frac{3C_F}{\beta}\right] \tilde{\Phi}(x_i, Q) = -\frac{C_d}{\beta} \int_0^1 [dy] V(x_i, y_i) \tilde{\Phi}(y_i, Q),$$

$$\xi(Q^2) = \frac{\beta}{4\pi} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \alpha_s(k^2) \sim \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i, y_i) = 2 \prod_{k=1}^{6} x_k \sum_{i \neq j}^{6} \theta(y_i - x_i) \prod_{l \neq i,j}^{6} \delta(x_l - y_l) \frac{y_j}{x_j} \left(\frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i}\right)$$

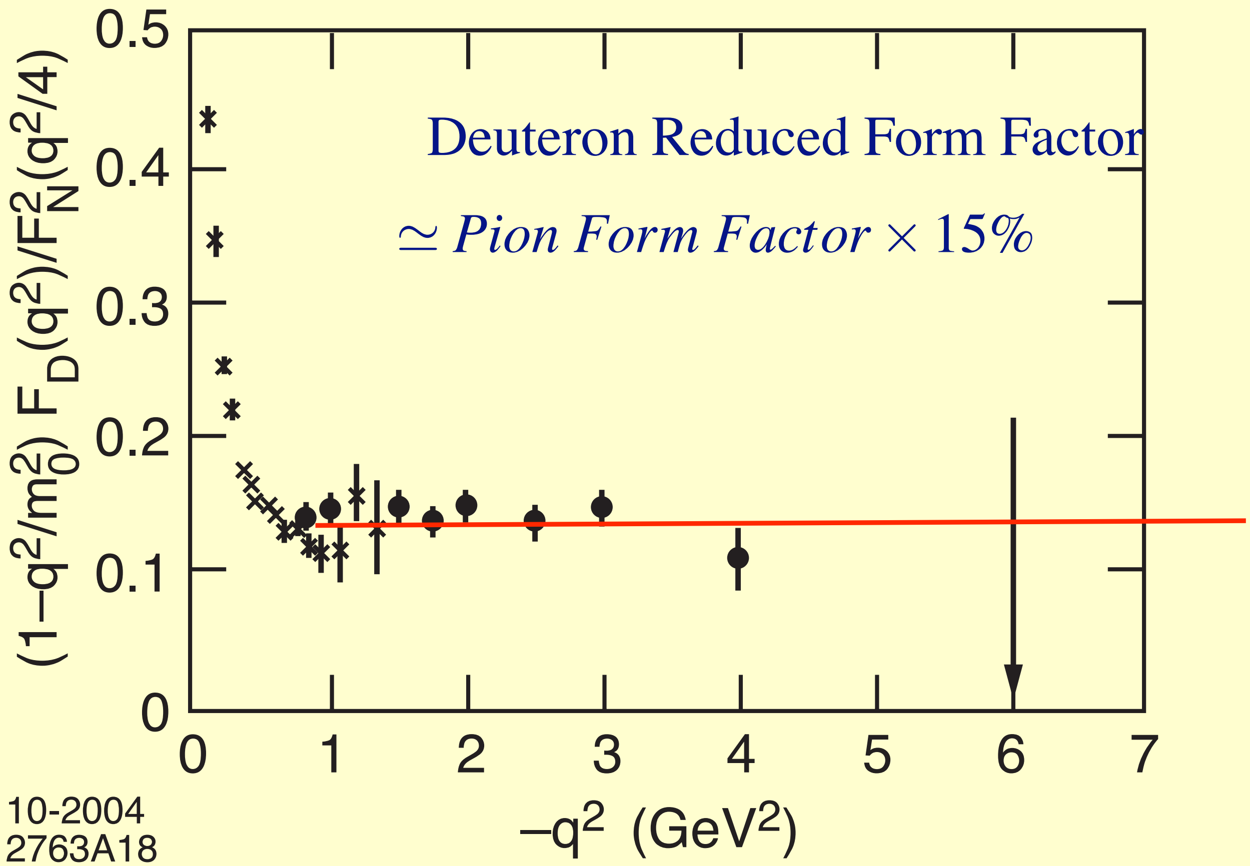
where $\delta_{h_i \bar{h}_j} = 1$ (0) when the helicities of the constituents $\{i, j\}$ are antiparallel (parallel). The infrared singularity at $x_i = y_i$ is cancelled by the factor $\Delta \tilde{\Phi}(y_i, Q) = \tilde{\Phi}(y_i, Q) - \tilde{\Phi}(x_i, Q)$ since the deuteron is a color singlet.

Define "Reduced" Form Factor



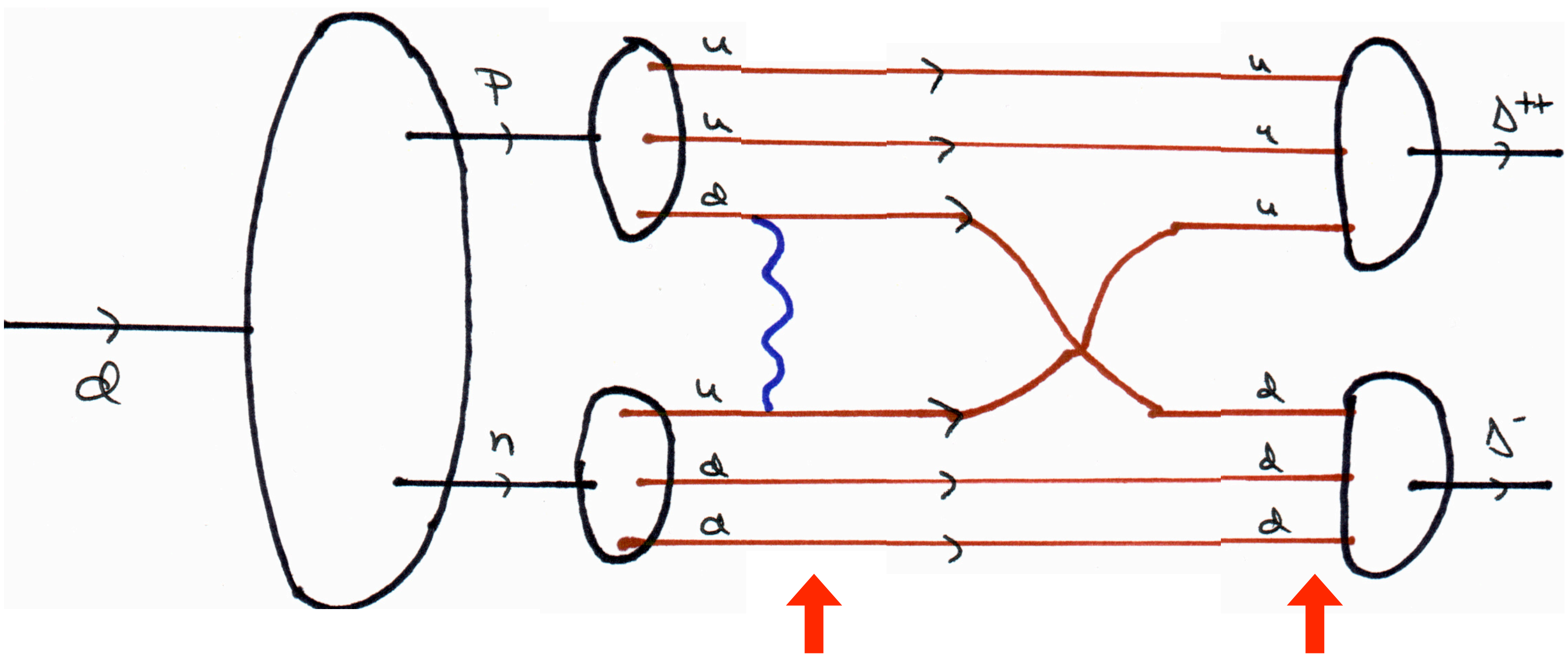
$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering



- Evidence for Hidden Color in the Deuteron

Structure of Deuteron in QCD



Hidden Color Fock State

Delta-Delta Fock State

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six-quark wavefunction
- 5 color-singlet combinations of 6 color-triplets -- only one state is | n p
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

Key Test of Hidden Color

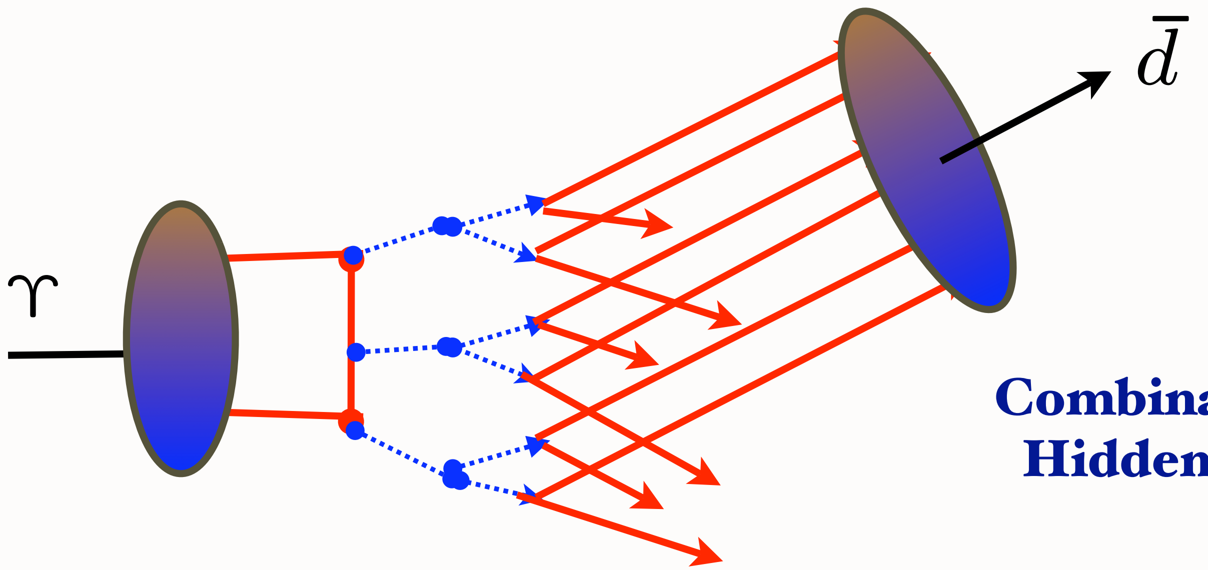
- CLEO measurement: Upsilon decay to anti-deuteron $\Upsilon \to ggg \to \bar{d}X$
- Is ratio of deuteron production to production of anti-nucleon pairs determined by Nuclear Physics?

$$R = \frac{\Gamma(\Upsilon \to \bar{d}X)}{\Gamma(\Upsilon \to \bar{p}\bar{n}X)}$$

$$\frac{E}{\sigma_{\text{tot}}} \frac{\mathrm{d}^3\sigma(\mathrm{d})}{\mathrm{d}^3 p} = C \left(\frac{E}{\sigma_{\text{tot}}} \frac{\mathrm{d}^3\sigma(\mathrm{p})}{\mathrm{d}^3 p} \right)^2 \quad C = \frac{4\pi}{3} p_0^3 / m_{\mathrm{p}} \quad p_0 \approx 130 \mathrm{MeV}$$

Gustafson, Hakkinen

Hadronization at the Amplitude Level



Combinatoric Advantage for Hidden-Color Fock States

$$\Upsilon \to ggg \to q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q} \to \bar{d}\ X$$

Anti-Deuteron vs. double antibaryon production

$$\Upsilon \to ggg \to q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q}\ q\bar{q} \to \bar{p}\ \bar{n}\ X$$

Why do dimensional counting rules work so well?

- **PQCD predicts log corrections from powers of α_s, logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin; Landshoff; Mueller, Duncan

- **DSE: QCD coupling (mom scheme) has IR Fixed point** Alkofer, Fischer, von Smekal et al.

- **Lattice results show similar flat behavior** Furui, Nakajima

- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**

- **Analogous to the Schrodinger Equation for Atomic Physics**

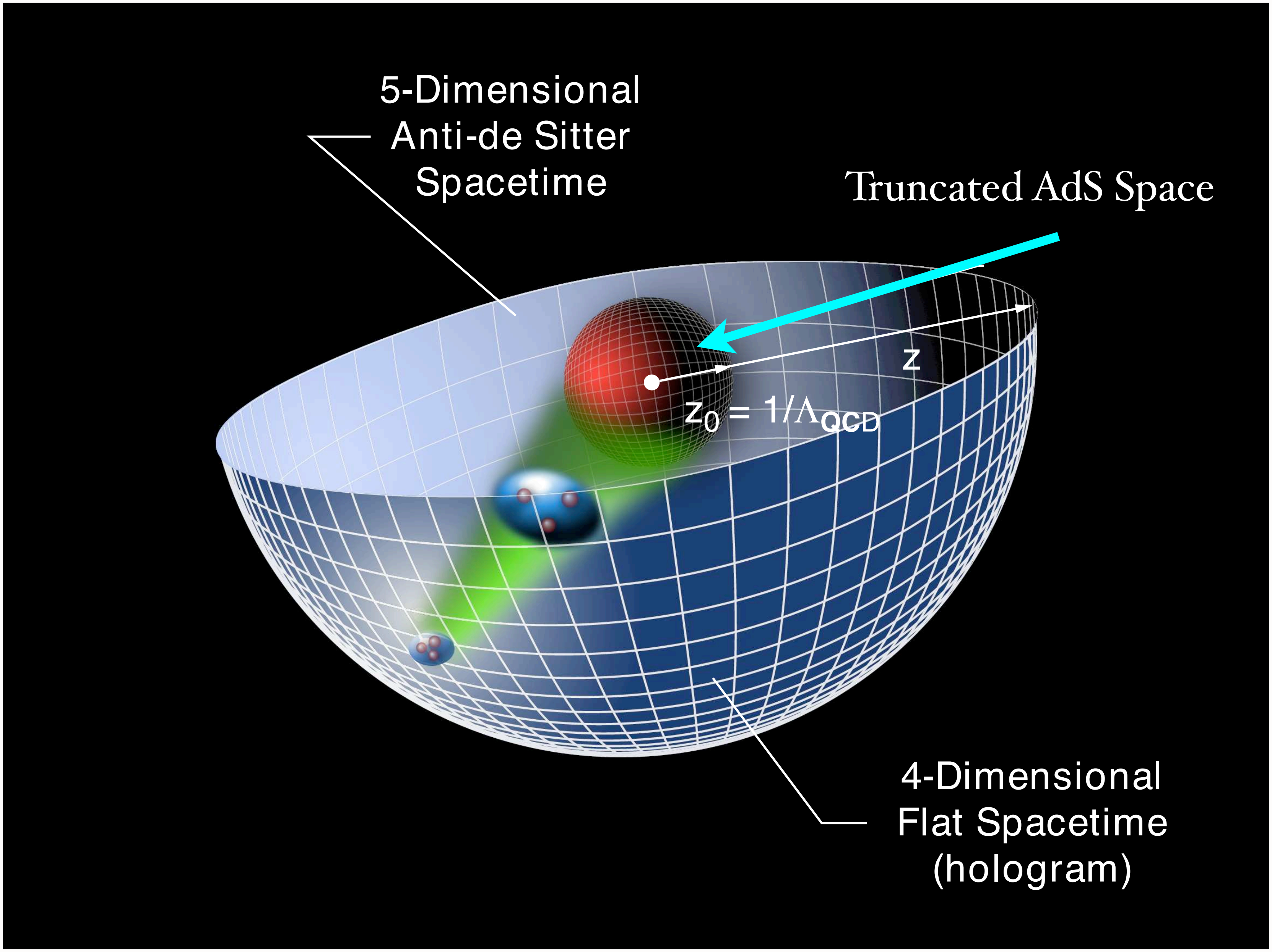
- *AdS/QCD Holographic Model*

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5


5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD}; variational methods

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$ ← *invariant measure*

$x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

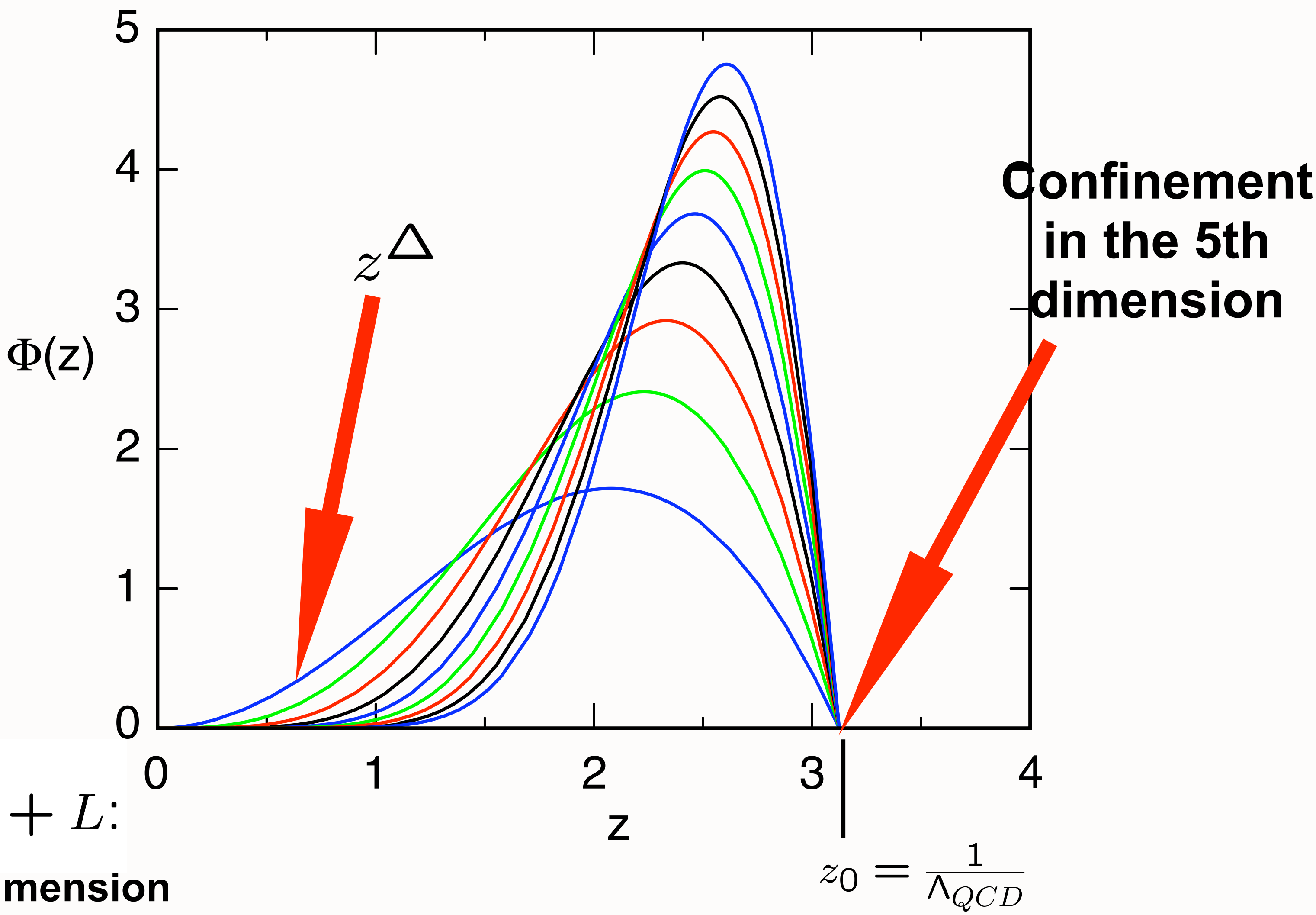
Guy de Teramond
SJB

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Identify hadron by its interpolating operator at z --> 0



$\Delta = 3 + L$:

Twist dimension of baryon

de Teramond, sjb

$$\Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

Truncated space

$$V(z) = -\frac{1-4L^2}{4z^2}$$

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Alternative: Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

Karch, et al.

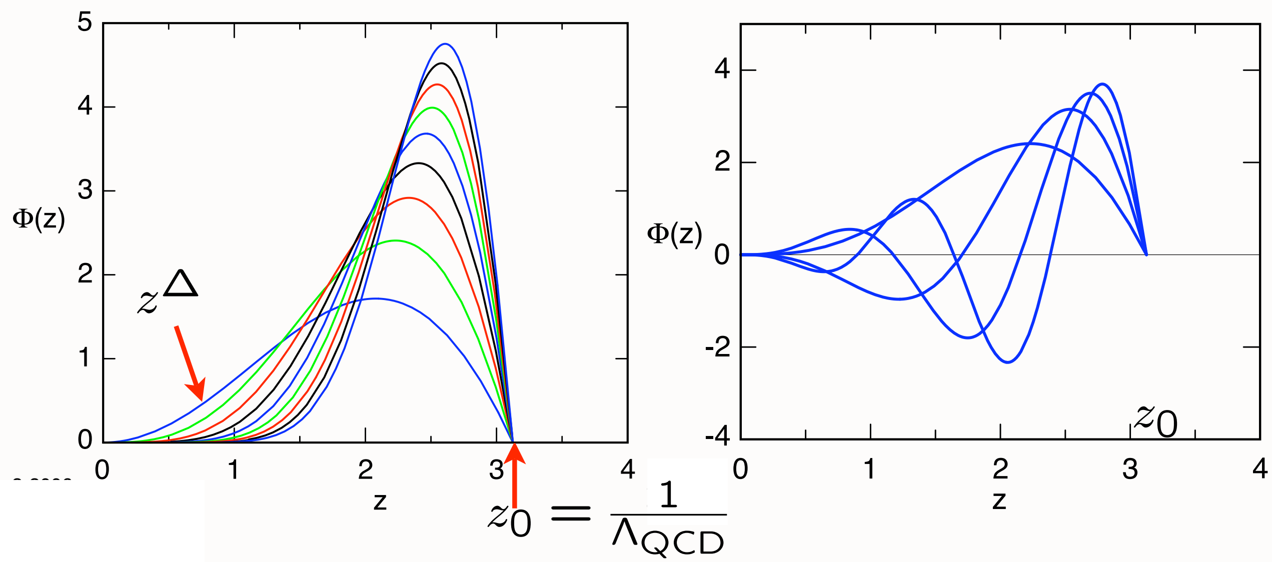
Derived from variation of Action in AdS5

Match fall-off at small z to conformal twist dimension at short distances

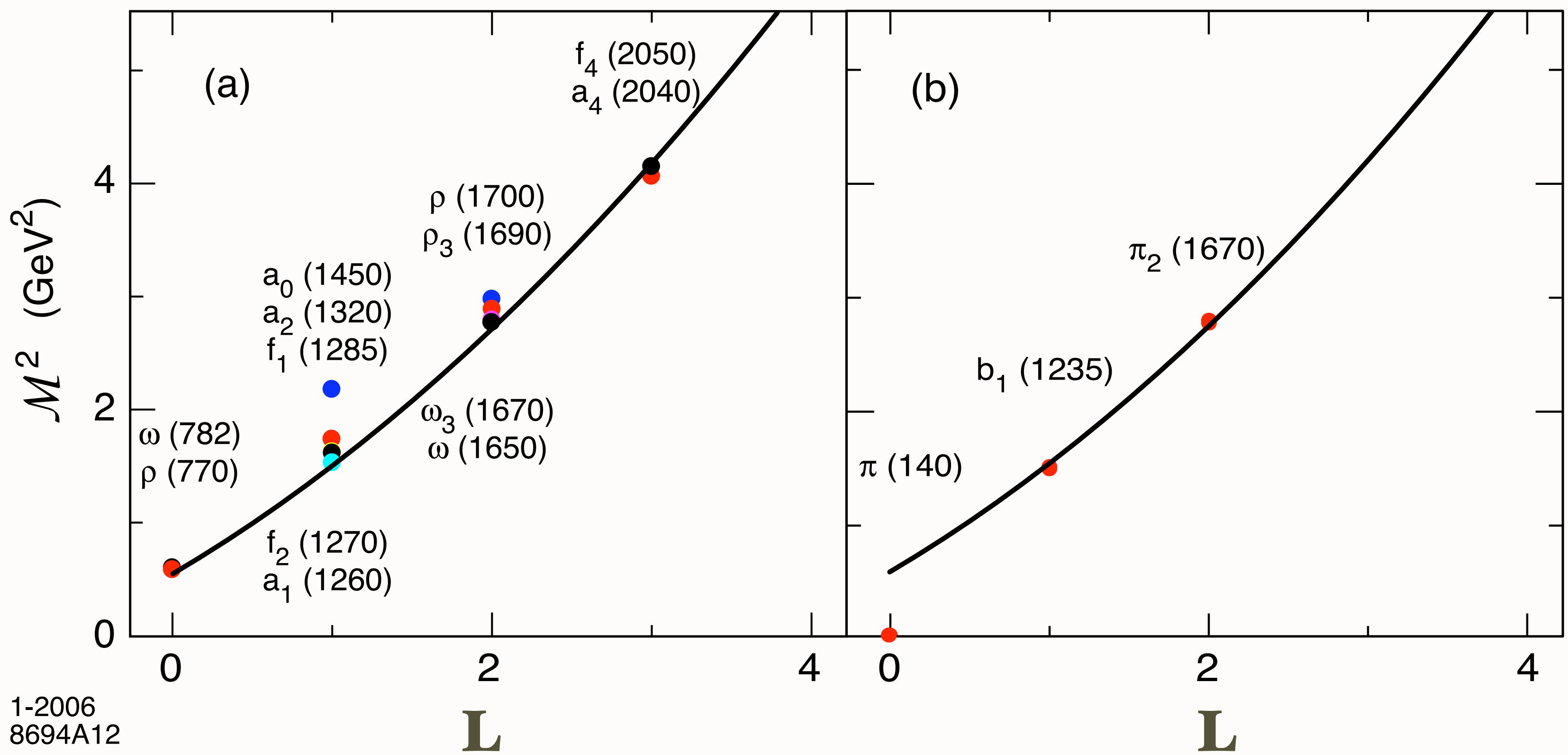
- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.



Light meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

Guy de Teramond
SJB

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i.$$

Wave Equation: $\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \left[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P) \right].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

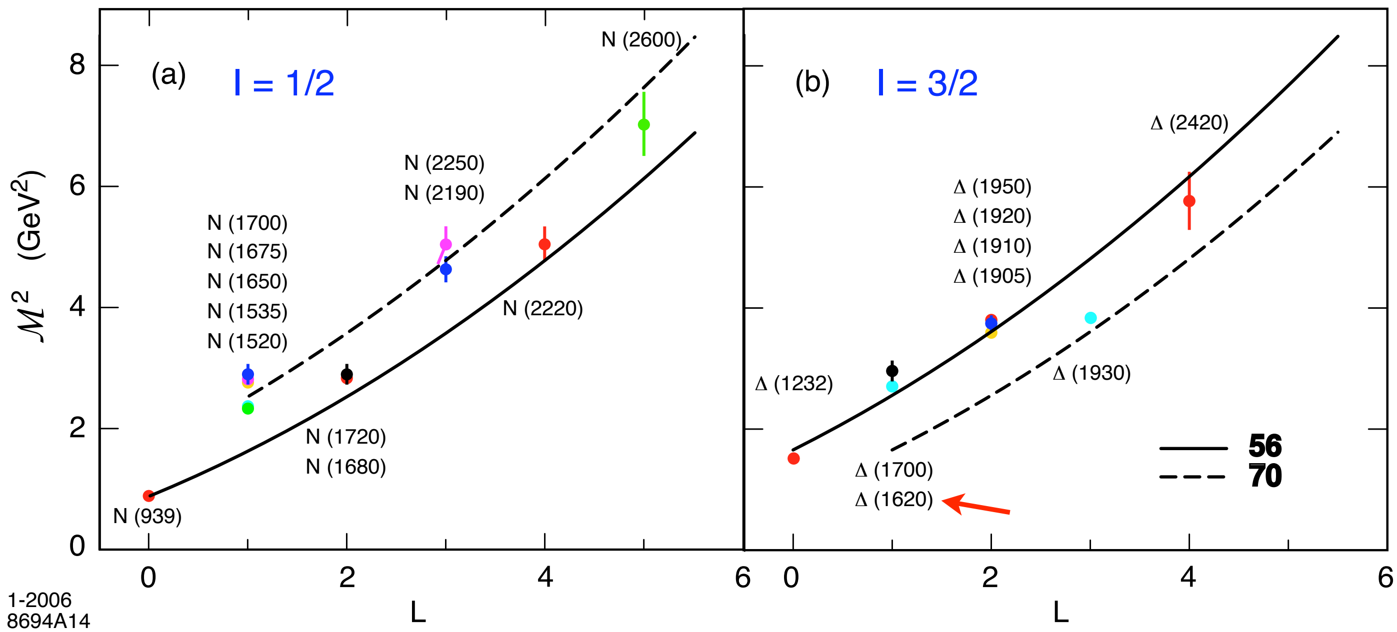


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P=+$ states, and the $\mathbf{70}$ to L odd $P=-$ states.

Guy de Teramond
SJB

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances

+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

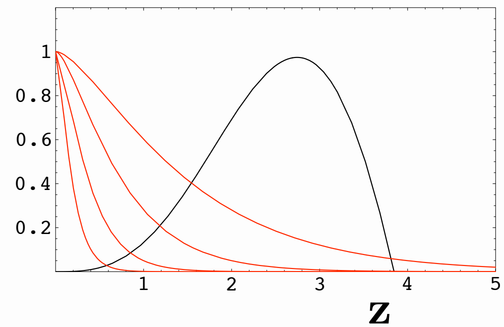
Hadron Spectra, Wavefunctions, Dynamics

Hadron Form Factors from AdS/CFT

- Propagation of external perturbation suppressed inside AdS. $J(Q,z) = zQK_1(zQ)$
- At large Q^2 the important integration region is $z \sim 1/Q$.

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z) J(Q,z)\Phi_I(z)$$

$\mathbf{J(Q,z)}, \ \mathbf{\Phi(z)}$



Polchinski, Strassler
de Teramond, sjb

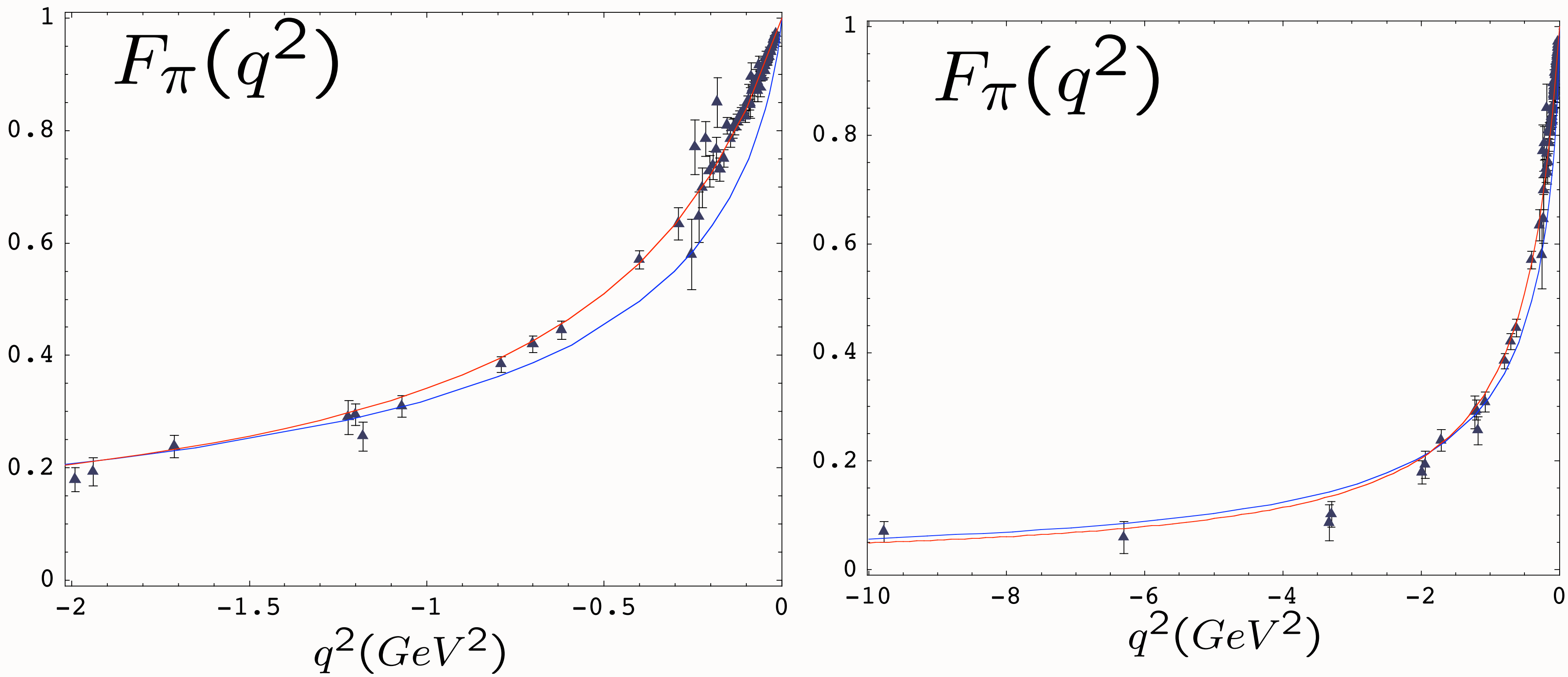
- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

Truncated Space Confinement

One parameter - set by pion decay constant

G. de Teramond, sjb

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

$$\Phi(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2 e^{-\kappa^2 z^2/2}. \qquad J(Q,z) = zQK_1(zQ).$$

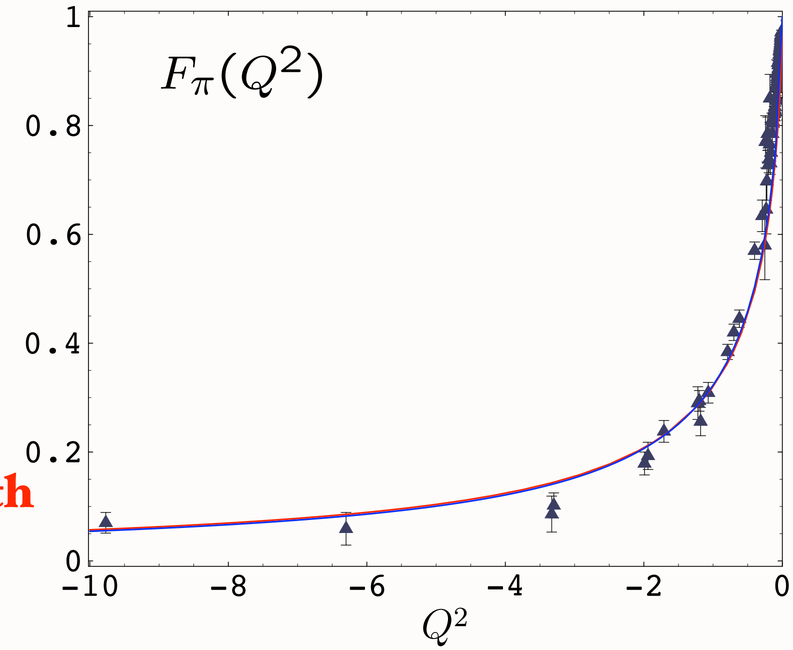
$$F(Q^2) = 1 + \frac{Q^2}{4\kappa^2} \exp\left(\frac{Q^2}{4\kappa^2}\right) Ei\left(-\frac{Q^2}{4\kappa^2}\right) \qquad Ei(-x) = \int_\infty^x e^{-t}\frac{dt}{t}.$$

Space-like Pion Form Factor

$\kappa = 0.4$ GeV

$\Lambda_{QCD} = 0.2$ GeV.

Identical Results for both confinement models

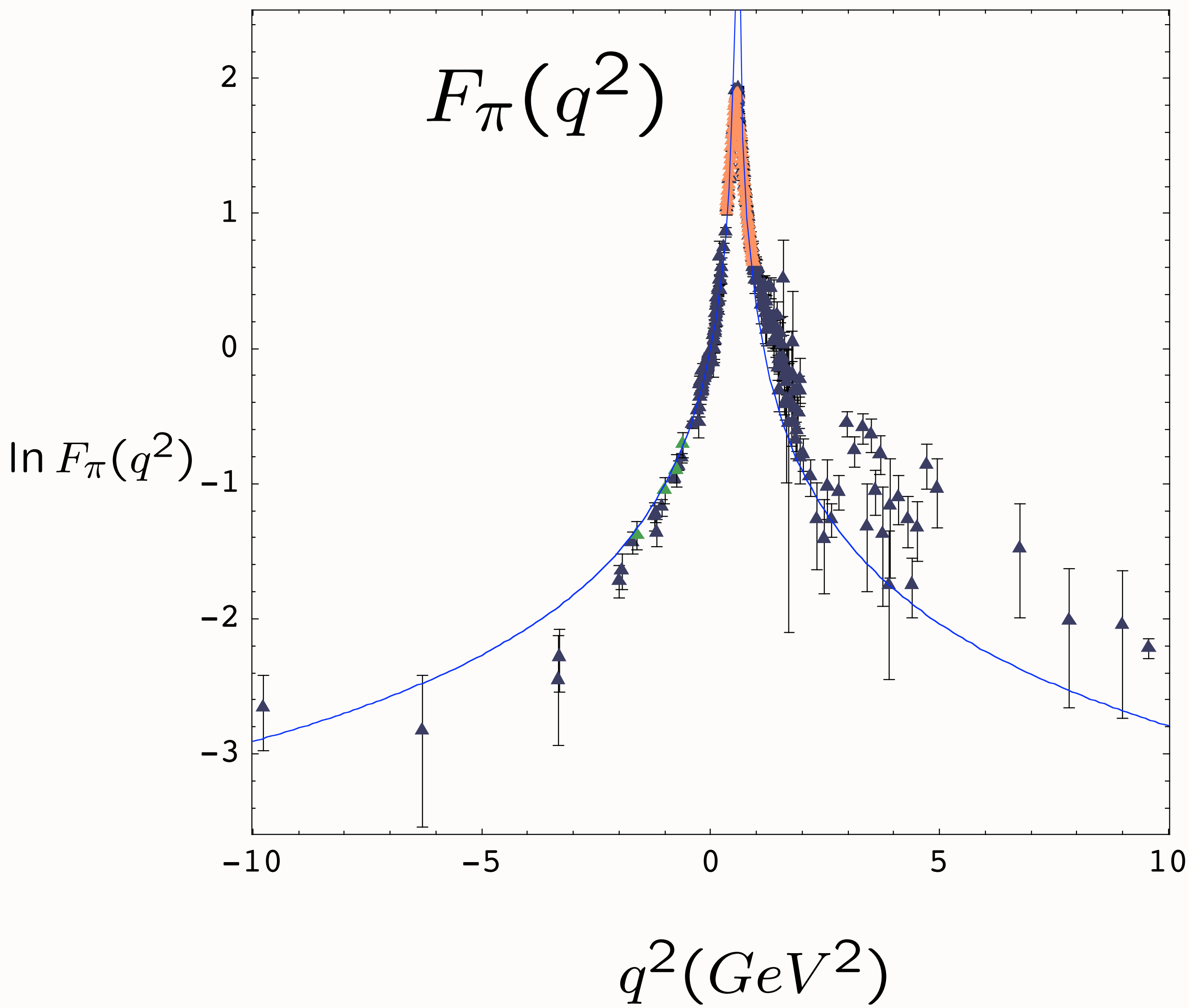


$$F(Q^2) \to \frac{4\kappa^2}{Q^2}$$

$\kappa = 2\Lambda_{QCD}$

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb

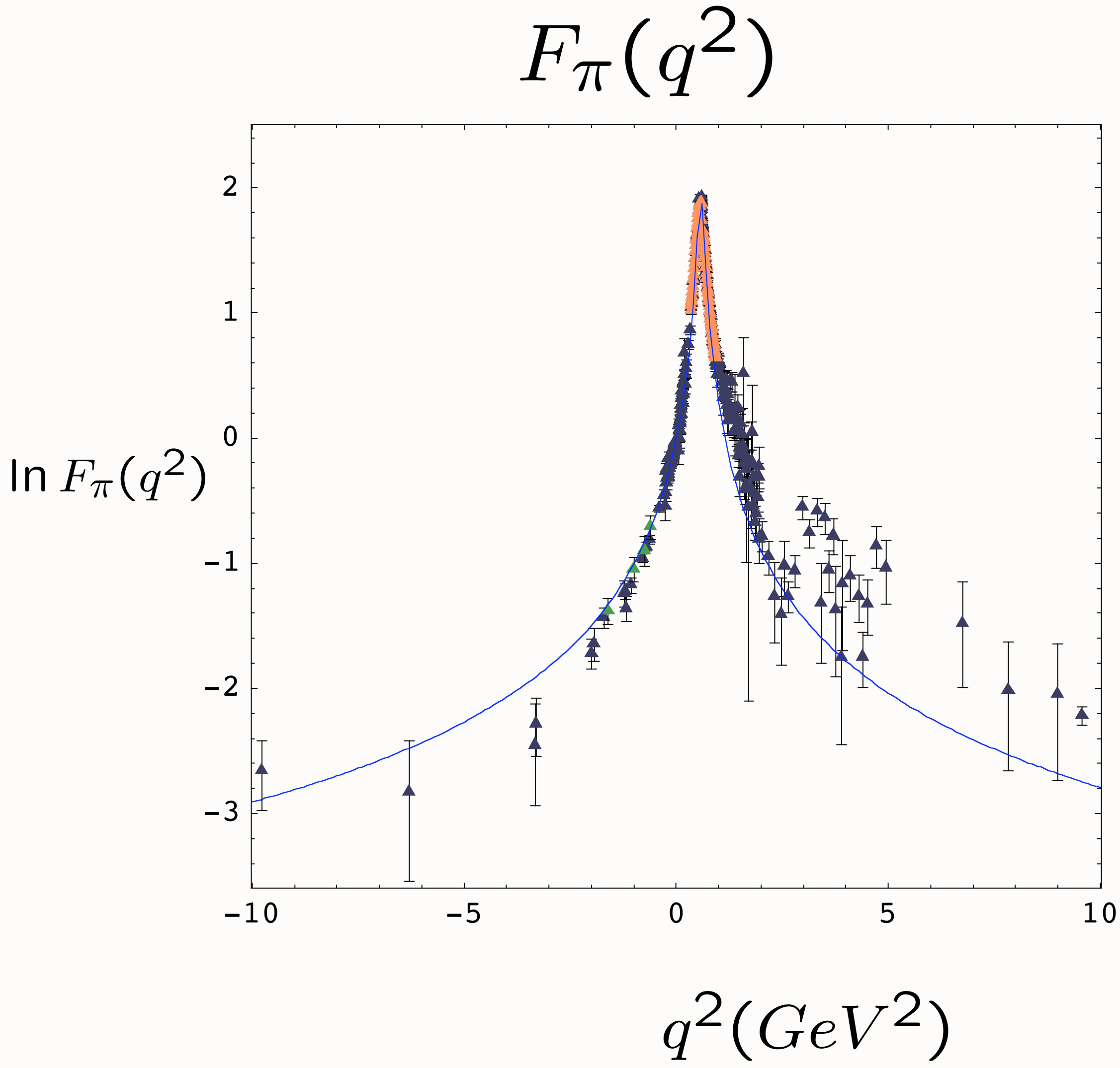


Harmonic Oscillator Confinement scale set by pion decay constant

$\kappa = 0.38$ GeV

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



Harmonic Oscillator Confinement

$\kappa = 0.38$ GeV

Analytic continue to timelike momenta and introduce width

$$q^2 \to q^2 + i\epsilon \to q^2 + iM\Gamma$$

Fit to height, predict width

$\Gamma_\rho = 111$ MeV

$\Gamma_\rho^{exp} = 150.3 \pm 1.6$ MeV

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu\Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \qquad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2}\, u(P).$$

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}.$$

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} \, J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

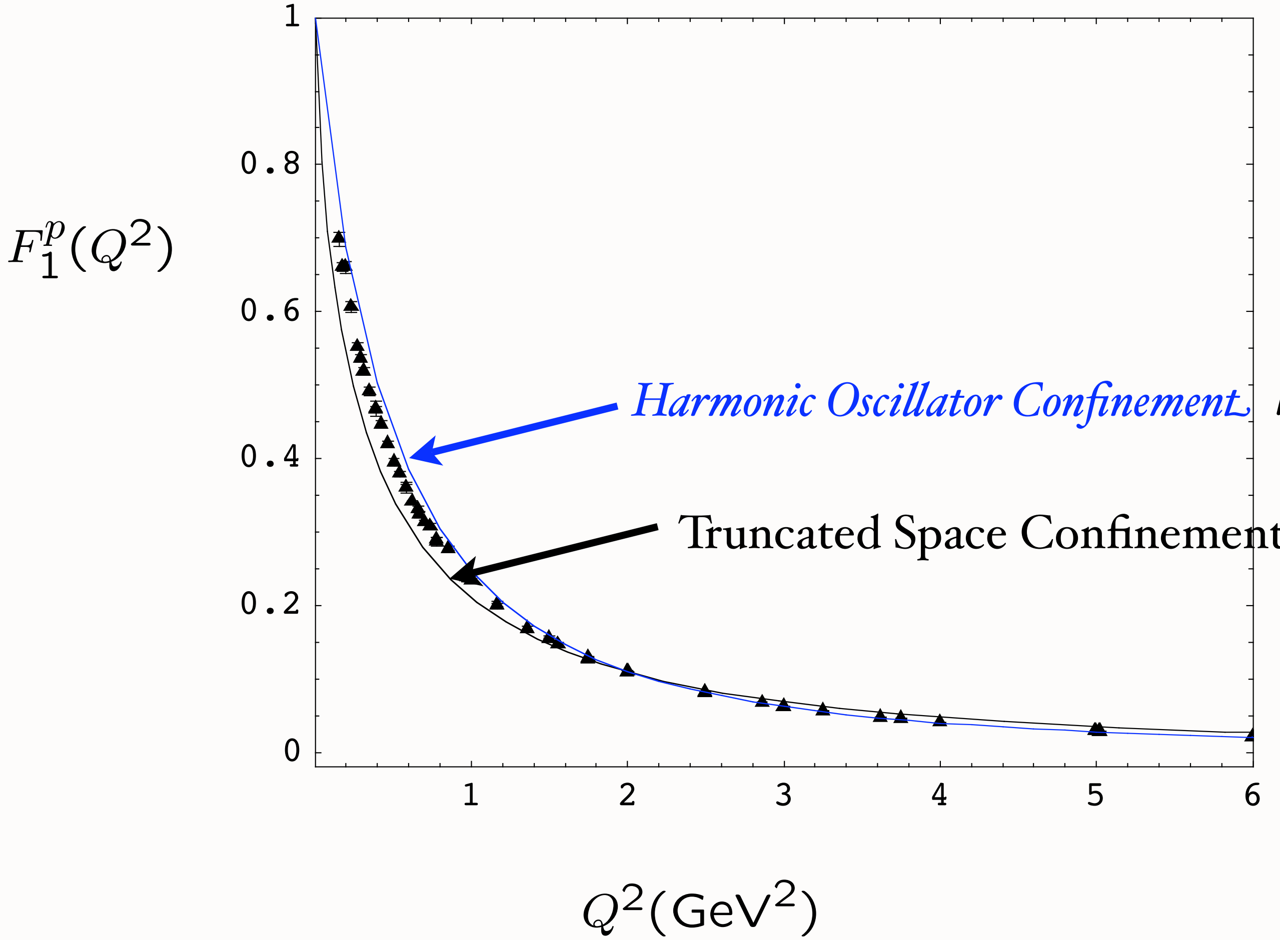
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb

G. de Teramond, sjb

Preliminary



$\kappa = 0.424$ GeV

$\Lambda = 0.2$ GeV

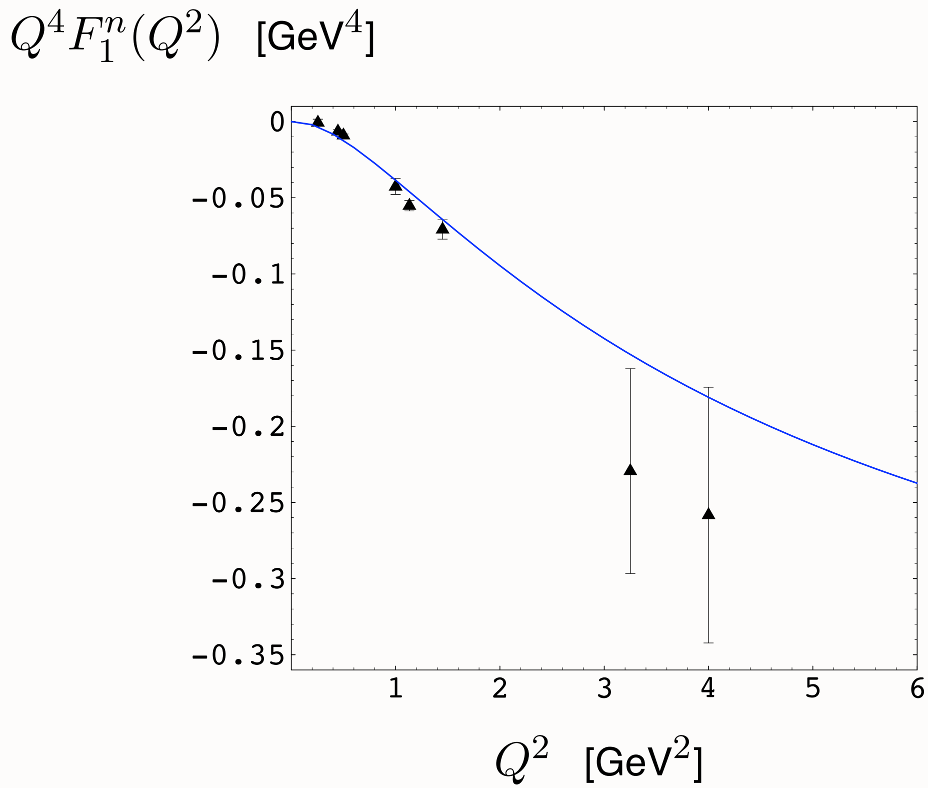
Current modified by metric

$$F_1(Q^2)_{I\to F} = \int \frac{dz}{z^3} \Phi^{\uparrow}_F(z) J(Q,z) \Phi^{\uparrow}_I(z)$$

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement



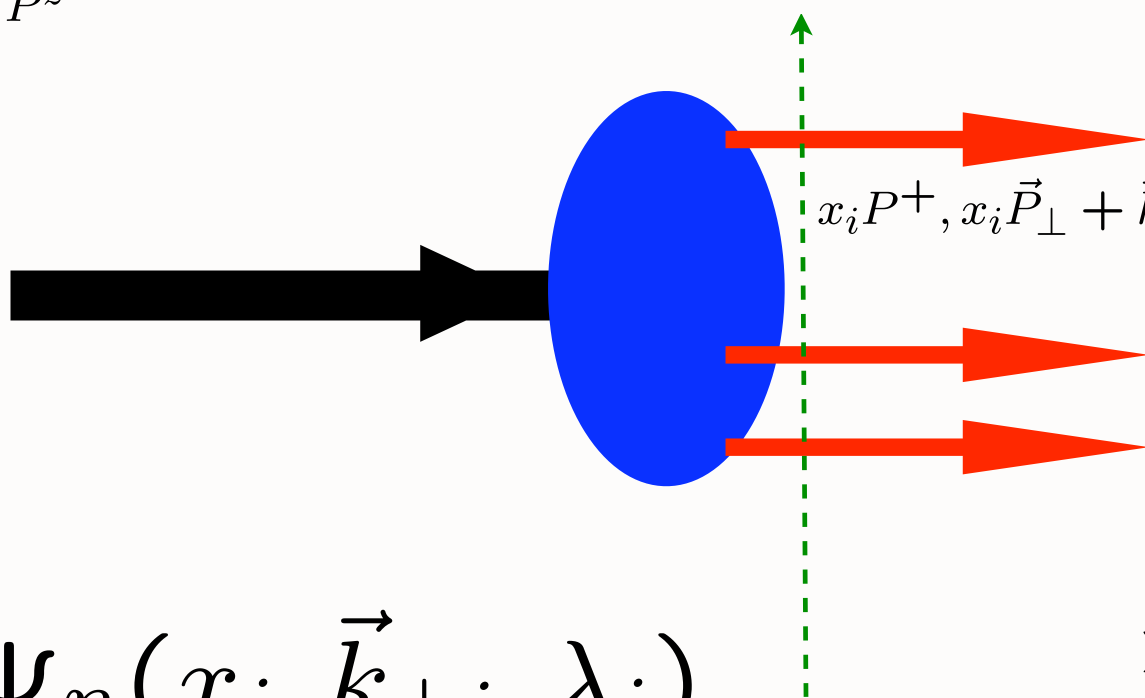
Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Light-Front Wavefunctions

$P^+ = P^0 + P^z$

Fixed $\tau = t + z/c$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

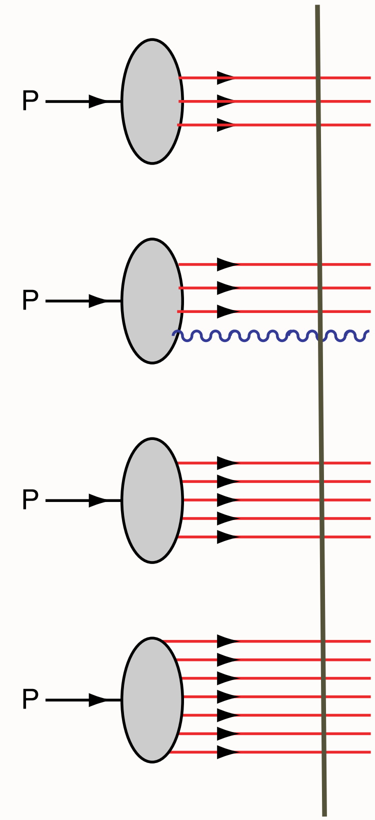
$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp) \qquad x_i = \frac{k_i^+}{P^+}$$



$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Intrinsic gluons, sea quarks, asymmetries

Angular Momentum on the Light-Front

A^+=0 gauge: No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment requires
Nonzero orbital angular momentum

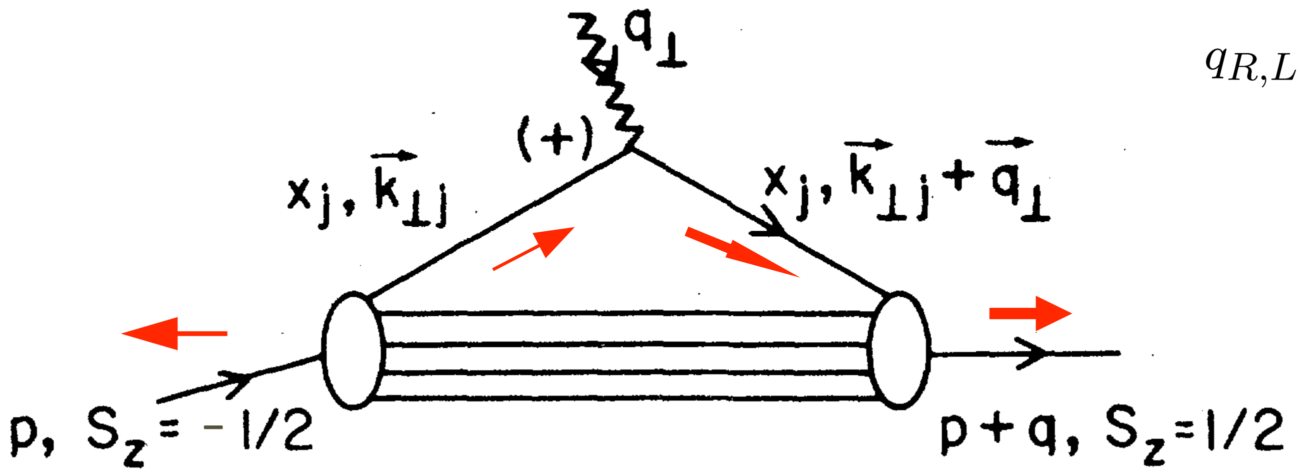
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

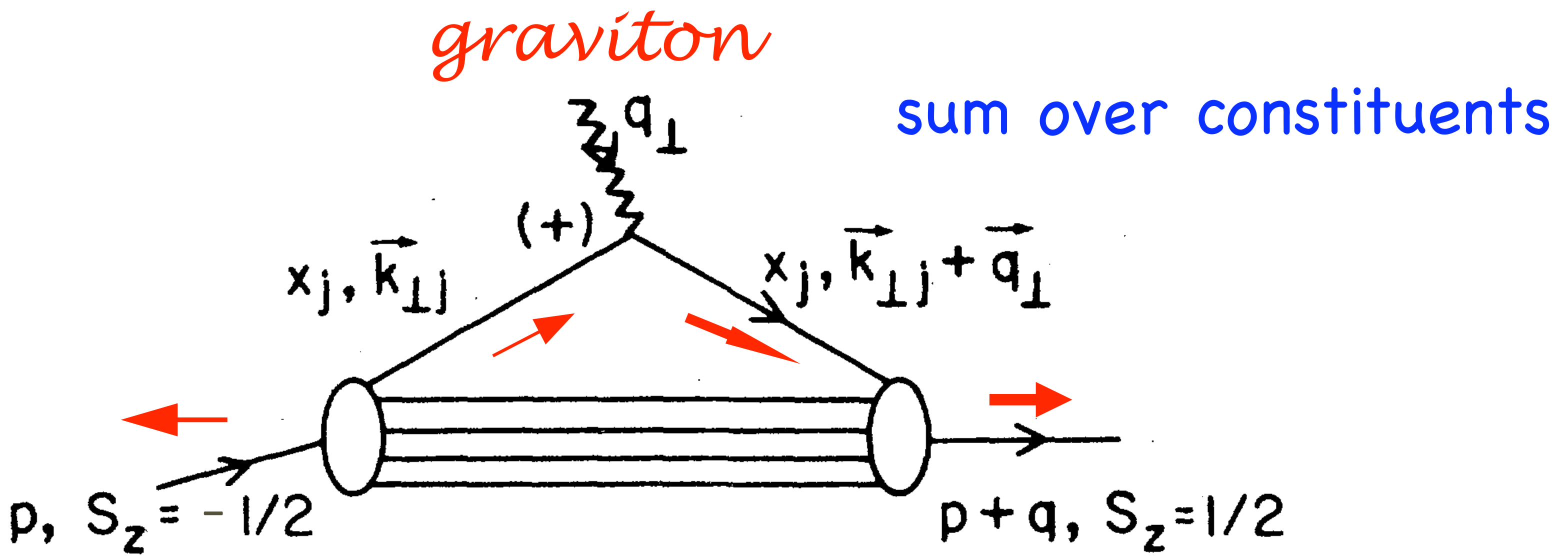
$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Anomalous gravitomagnetic moment B(0)

Okun et al: *B(0) Must vanish because of Equivalence Theorem*



Hwang, Schmidt, sjb;
Holstein et al

B(0) = 0

Each Fock State

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle =$$

$$\bar{U}(P', \lambda') \left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha + F_3(q^2)\frac{-1}{2M}\sigma^{\mu\alpha}\gamma_5 q_\alpha \right] U(P, \lambda)$$

$$\kappa = \frac{e}{2M}\left[F_2(0)\right] \ , \qquad d = \frac{e}{M}\left[F_3(0)\right]$$

We will find a close connection between κ and d, as long anticipated. [Bigi, Uralstev, NPB 1991]

Gardner, Hwang, sjb,

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame,
imply ($q^{R/L} \equiv q^1 \pm iq^2$):

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big] ,$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{i}{2} \times$$

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big] ,$$

$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$ for the struck constituent j and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \Longrightarrow$ only $n' = n$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Gardner, Hwang, sjb,

CP-violating phase

$$F_3(q^2) = F_2(q^2) \times \tan\phi$$

Fock state by Fock state

Gardner, Hwang, sjb,

Light-Front Representation of Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Identical DYW and AdS5 Formulae: Two-parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\mathrm{QCD}}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

Same result for LF and AdS5

$\zeta \leftrightarrow \mathbf{z}$

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for the electromagnetic potential in AdS space, and

$$\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\mathrm{QCD}}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\mathrm{QCD}}}{x(1-x)}\right)$$

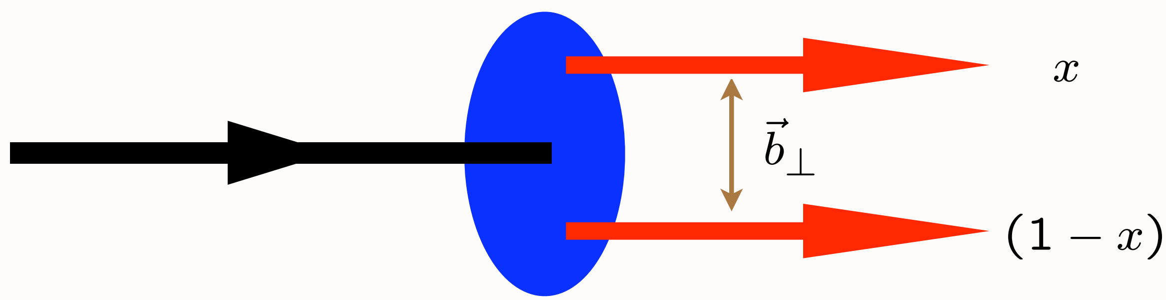
the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

LF(3+1) AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

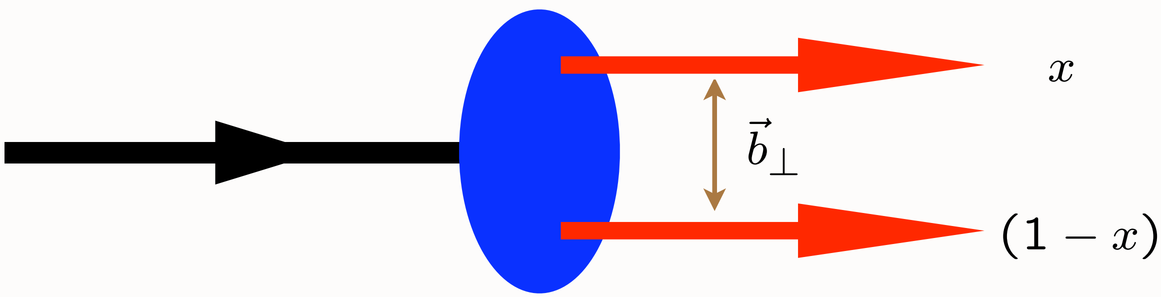
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

G. de Teramond and sjb

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

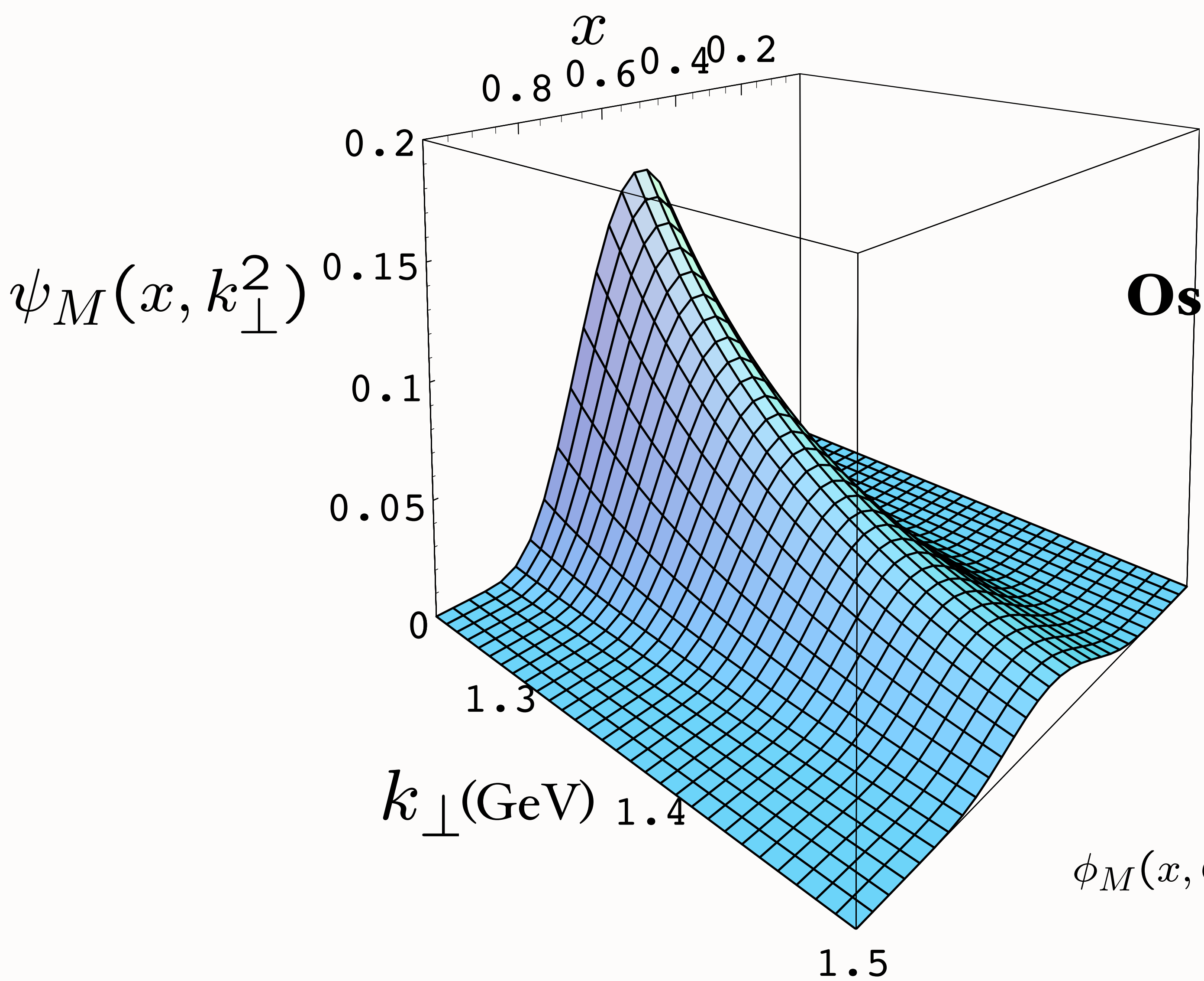
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

Prediction from AdS/CFT: Meson LFWF

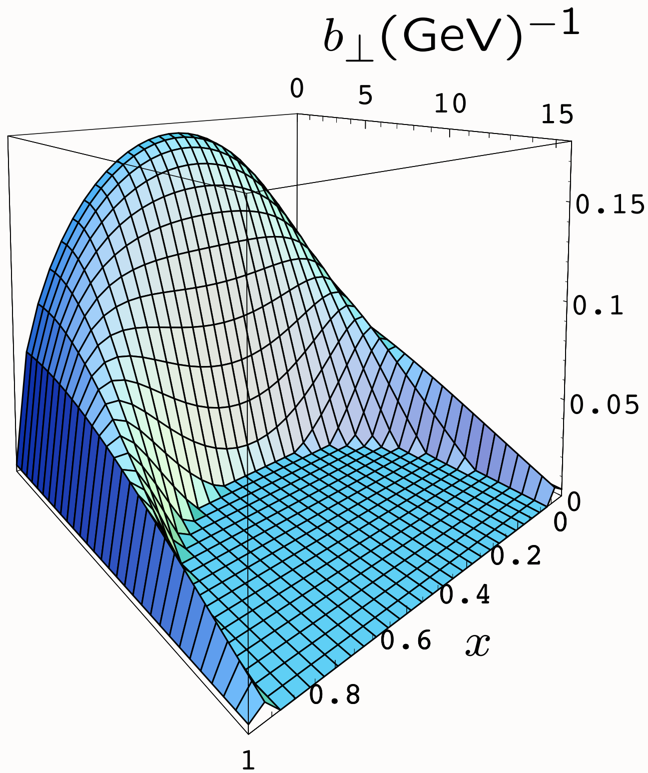


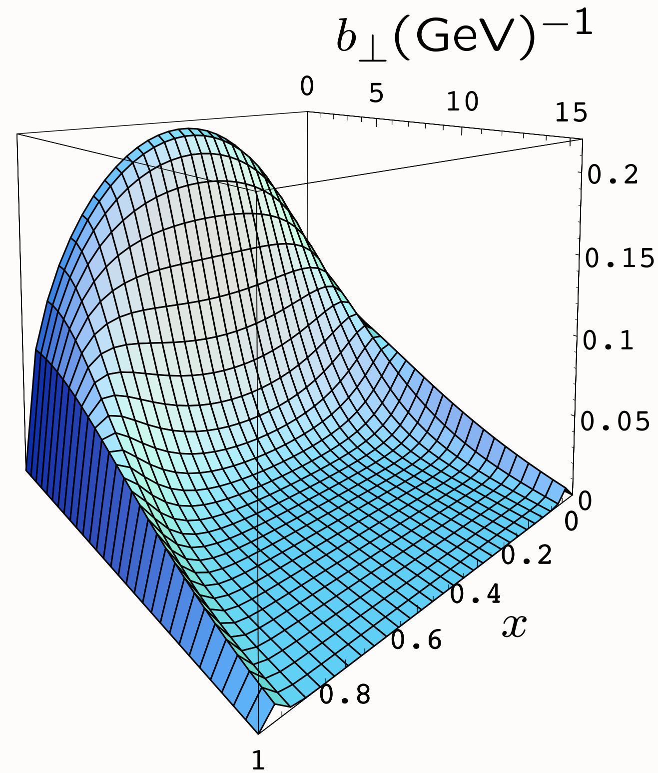
Harmonic Oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



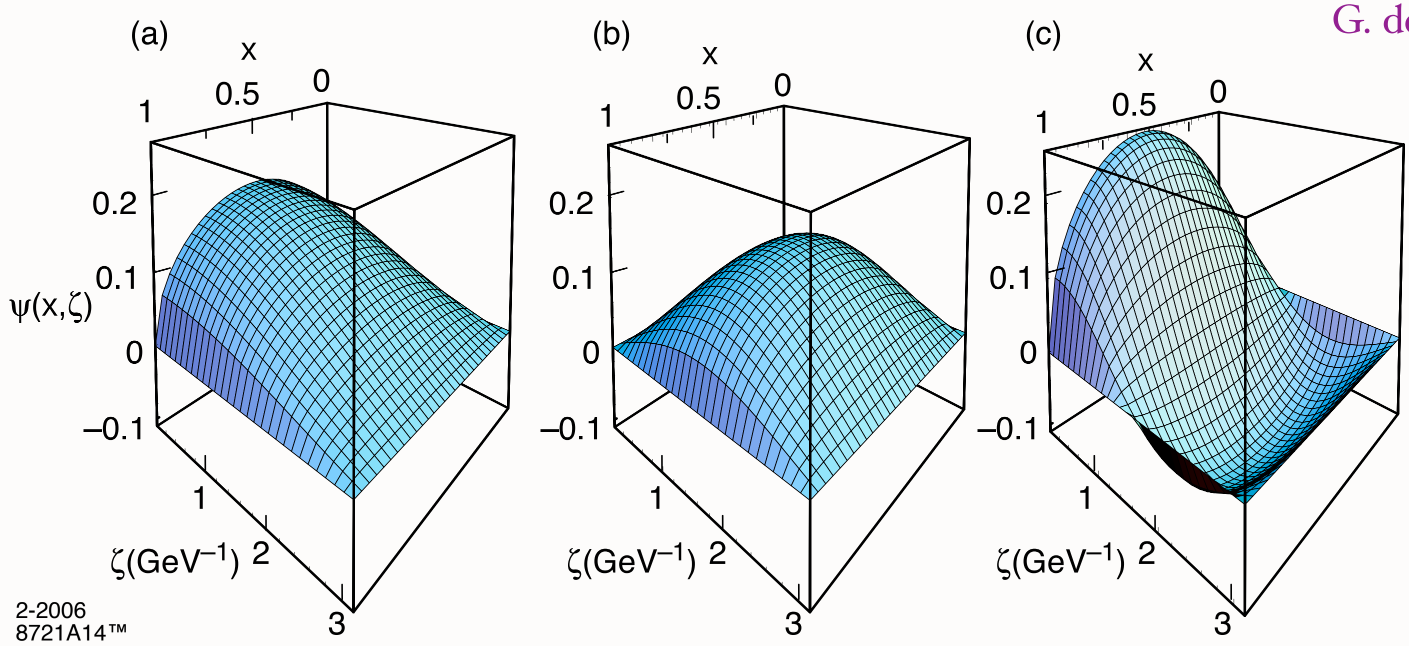


$\Lambda_{QCD} = 0.32$ GeV $\qquad \kappa = 0.76$ GeV

Truncated Space $\qquad$ ***Harmonic Oscillator***

AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L=0,\ k=1$; (b) first orbital exited state $L=1,\ k=1$; (c) first radial exited state $L=0,\ k=2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)$$

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

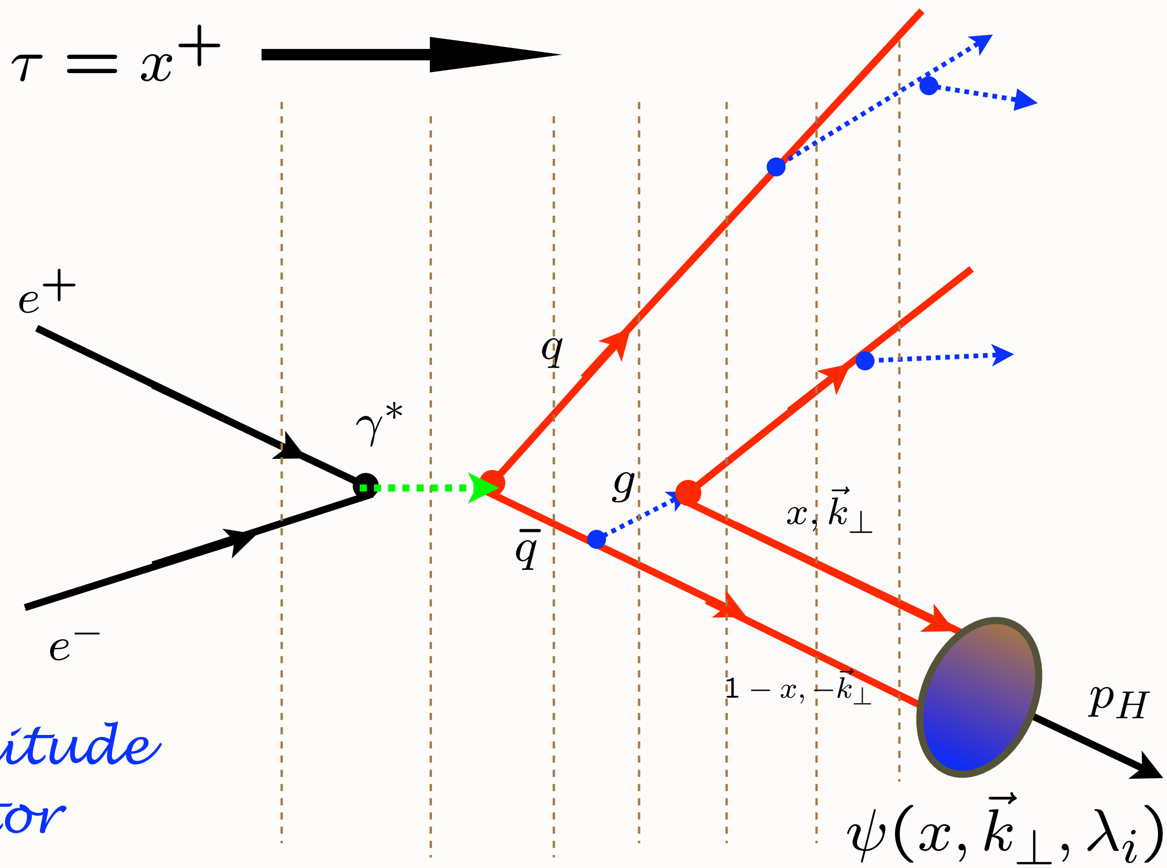
- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

Hadronization at the Amplitude Level



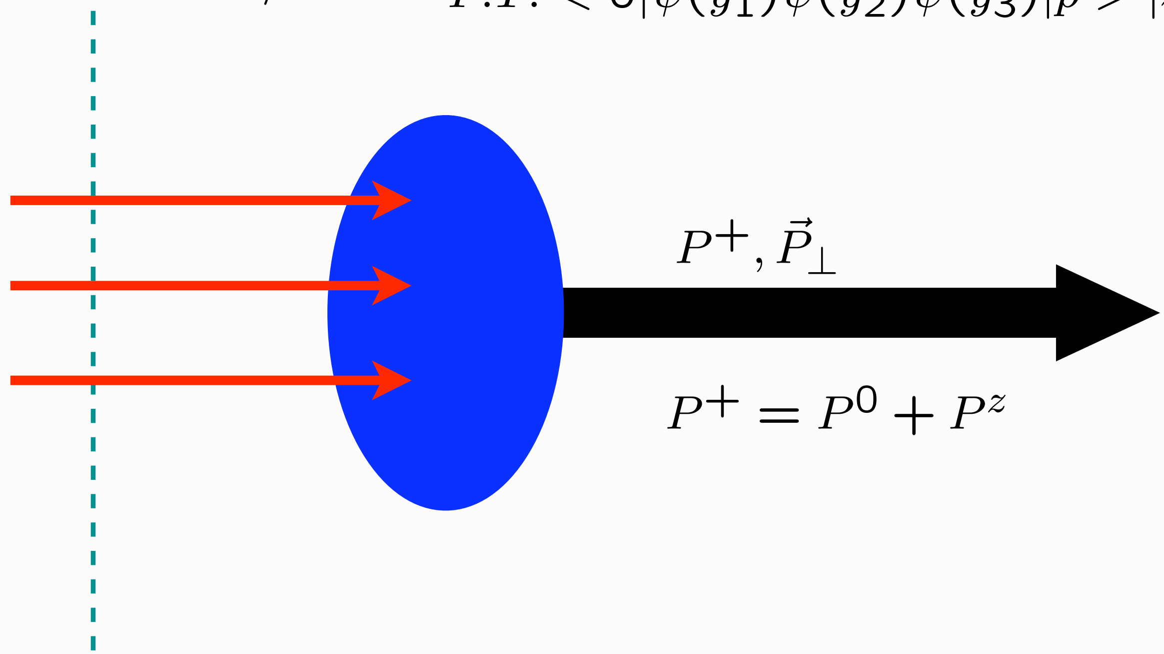
Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$F.T. < 0|\psi(y_1)\psi(y_2)\psi(y_3)|p > |_{\tau_i=0}$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

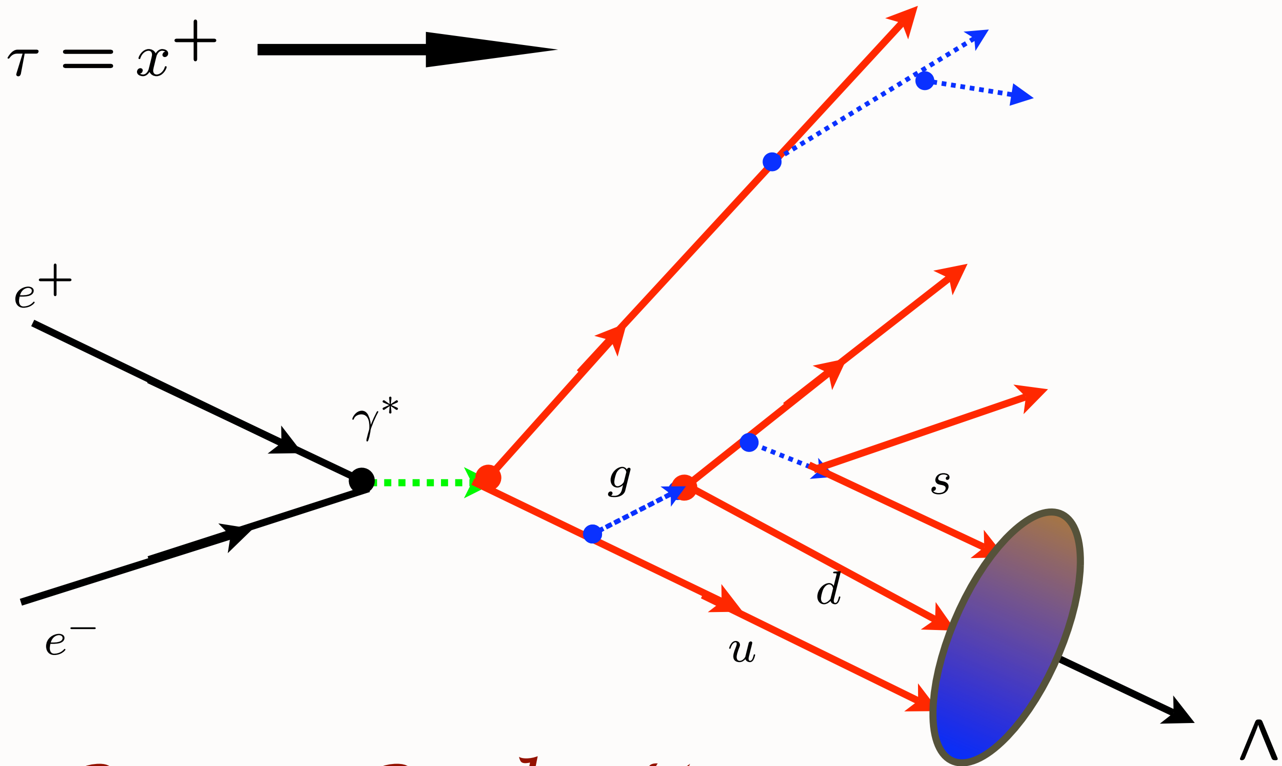
$P^+, \vec{P}_\perp$

$P^+ = P^0 + P^z$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

Hadronization at the Amplitude Level



Baryon Production

$\psi(x, \vec{k}_\perp, \lambda_i)$

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

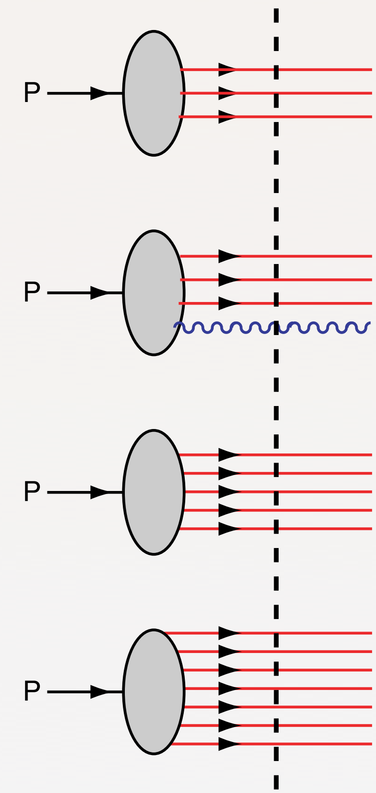
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

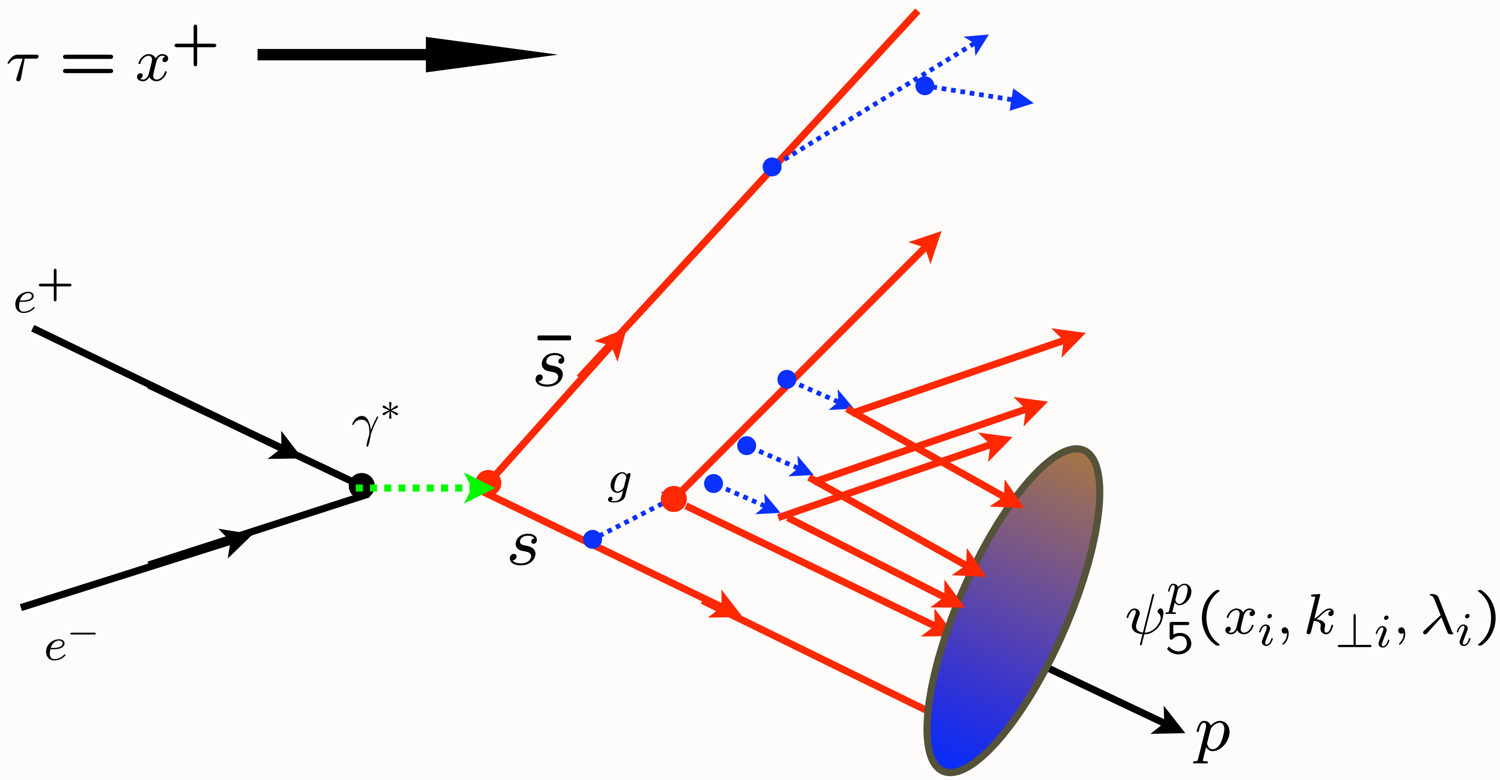


Fixed LF time

Intrinsic heavy quarks, $\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

Hadronization at the Amplitude Level



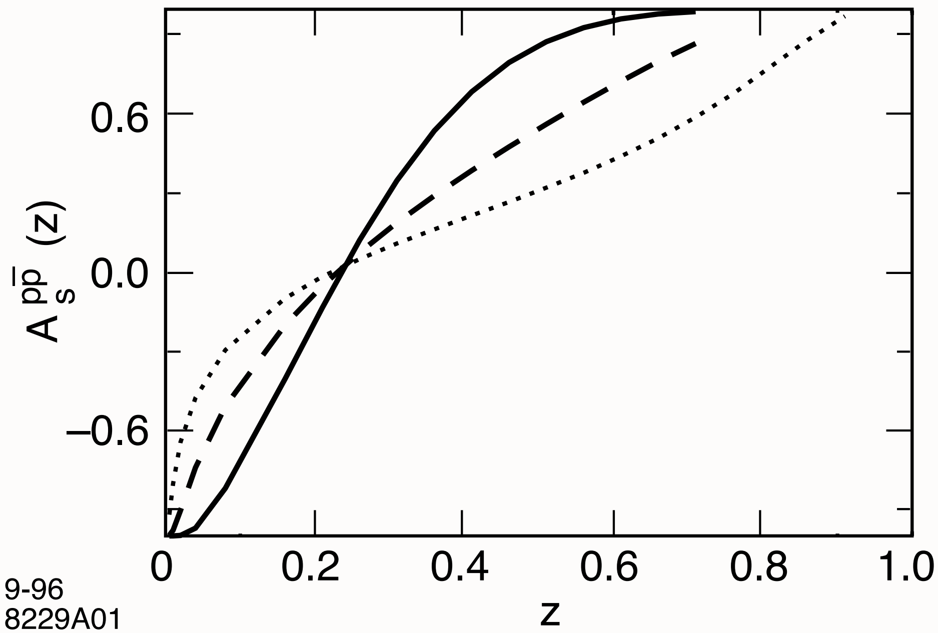
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$

B-Q Ma, sjb

B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to \bar{p}}(z)}{D_{s\to p}(z) + D_{s\to \bar{p}}(z)}$$

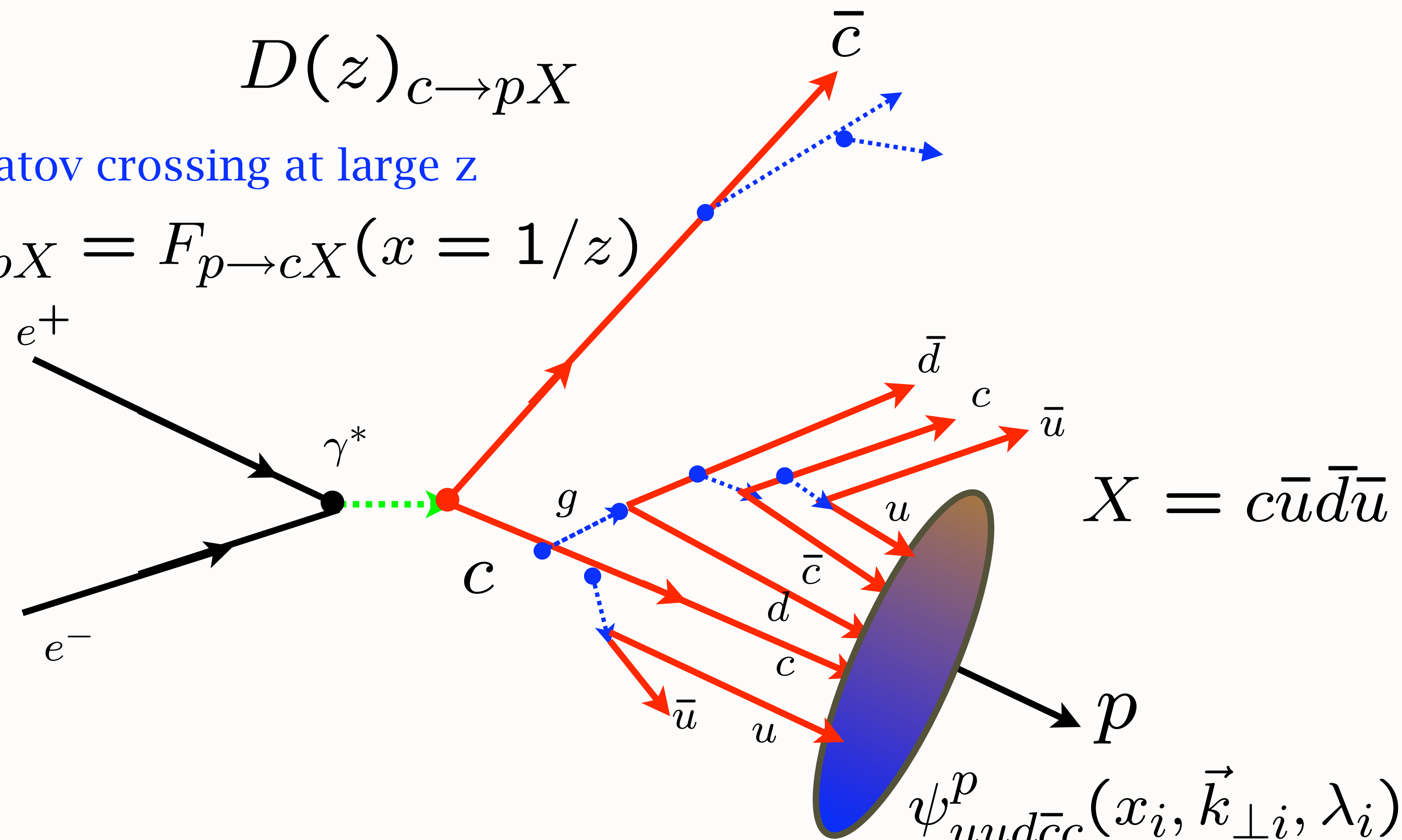
Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s}> \simeq |K^+\Lambda>$

Timelike Test of Charm Distribution in Proton

$$D(z)_{c \to pX}$$

Gribov-Lipatov crossing at large z

$$zD(z)_{c \to pX} = F_{p \to cX}(x = 1/z)$$

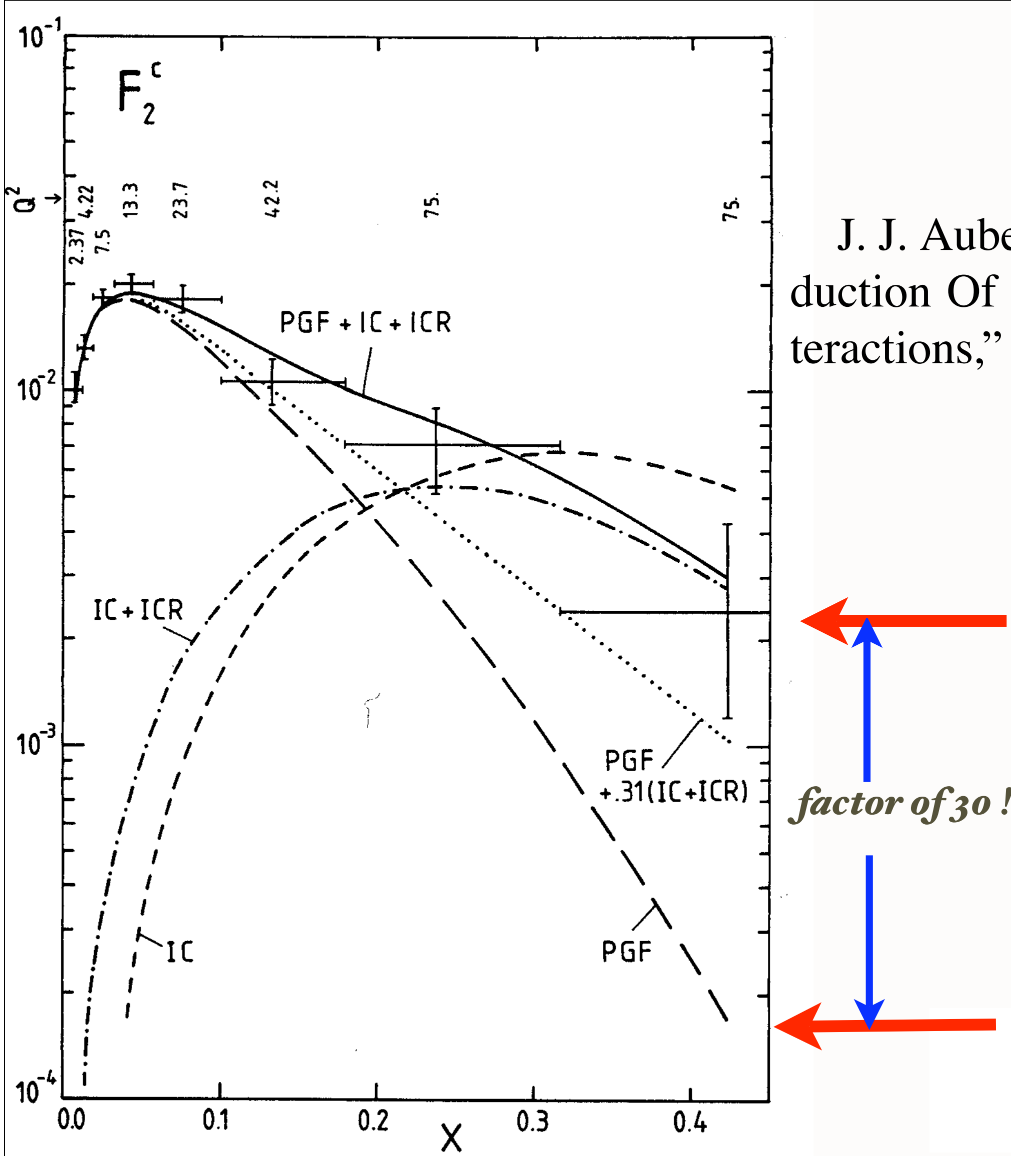


Intrinsic charm model:
predict proton at same rapidity as charm quark: high z

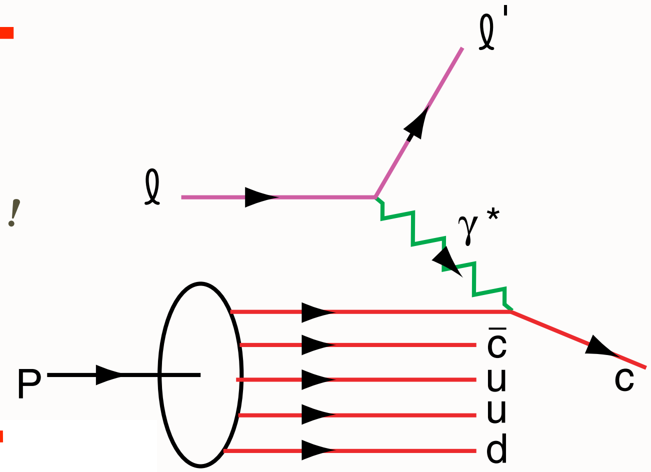
$$z_i \propto m_{\perp i} = \sqrt{m_i^2 + k_\perp^2}$$

Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



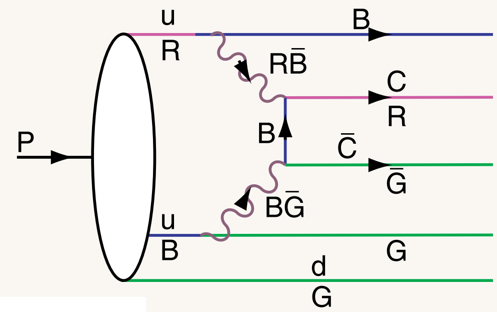
First Evidence for Intrinsic Charm



DGLAP / Photon-Gluon Fusion: factor of 30 too small

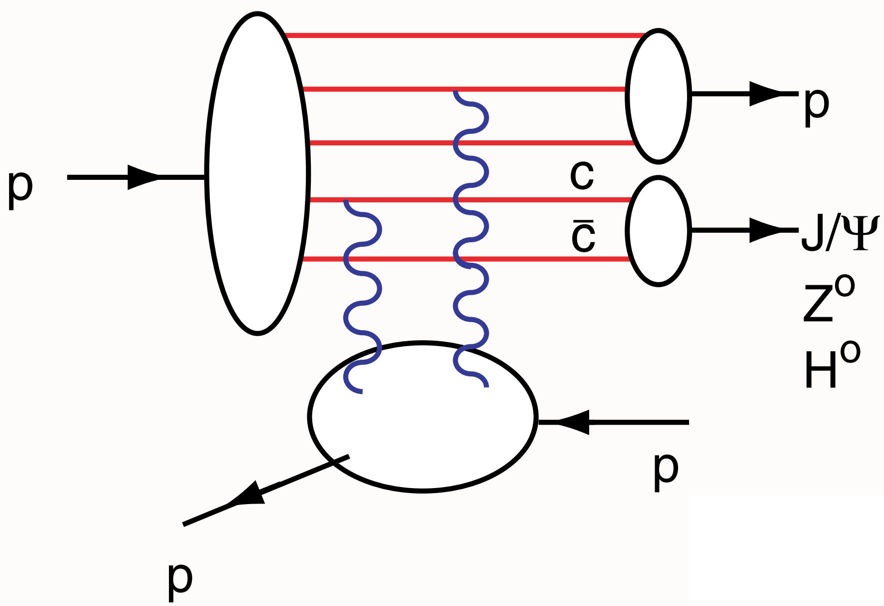
Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



- Rigorous prediction of QCD, OPE
- Color-Octet Fock State
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin)

Intrinsic Charm Mechanism for Exclusive Diffraction Production



$$\mathrm{p\,p} \to J/\psi\ p\ p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

Exclusive Diffractive High-X_F Higgs Production

Kopeliovitch, Schmidt, Soffer, sjb

Intrinsic $c\bar{c}$ pair formed in color octet 8_C in proton wavefunction Large Color Dipole

Collision produces color-singlet J/ψ through color exchange

RHIC Experiment

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

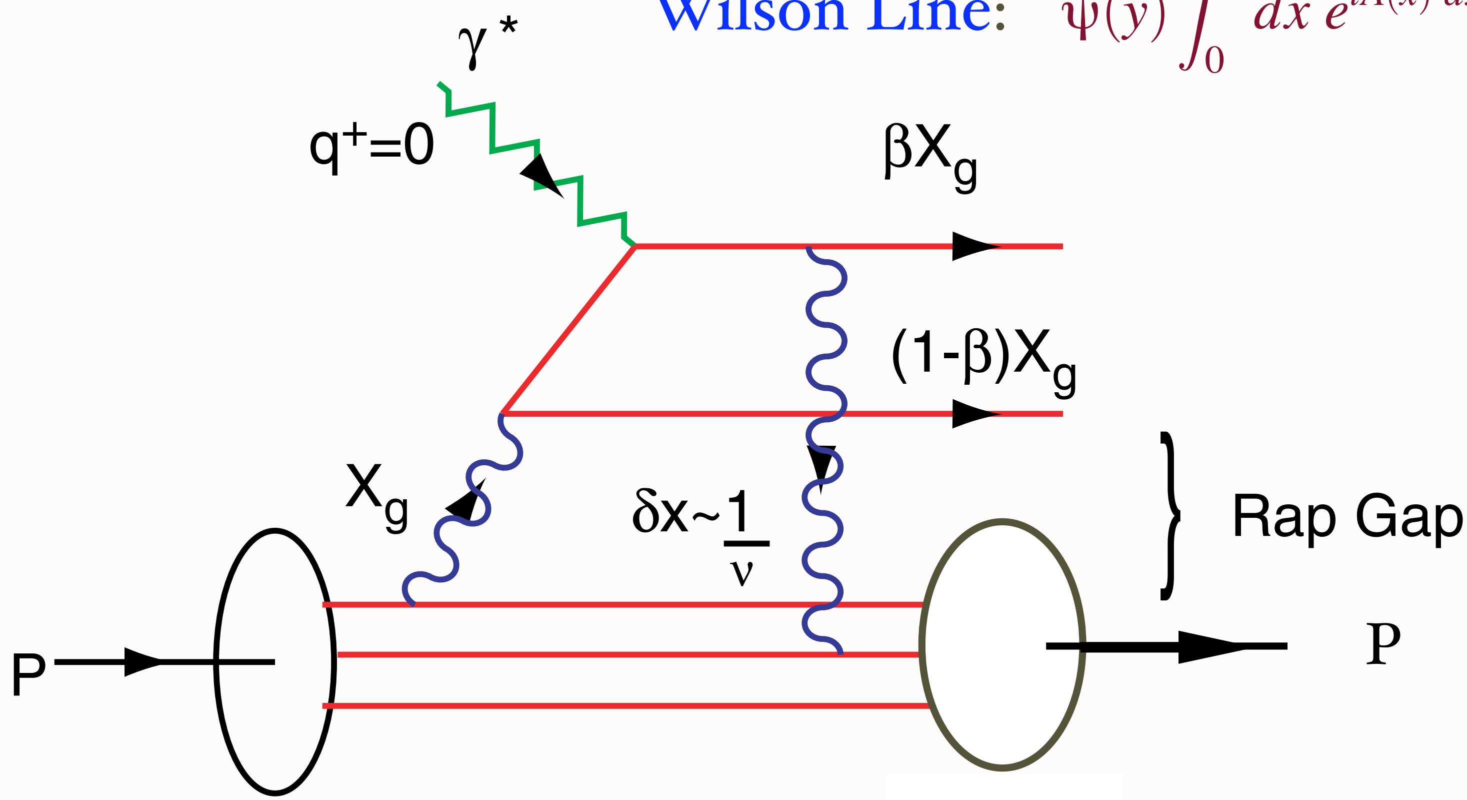
- High x_F $pp \to \Lambda_b X$

- High x_F $pp \to \Xi(ccd)X$ (SELEX)

Hoyer, Marchal, Peigne, Sannino, sjb

QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$

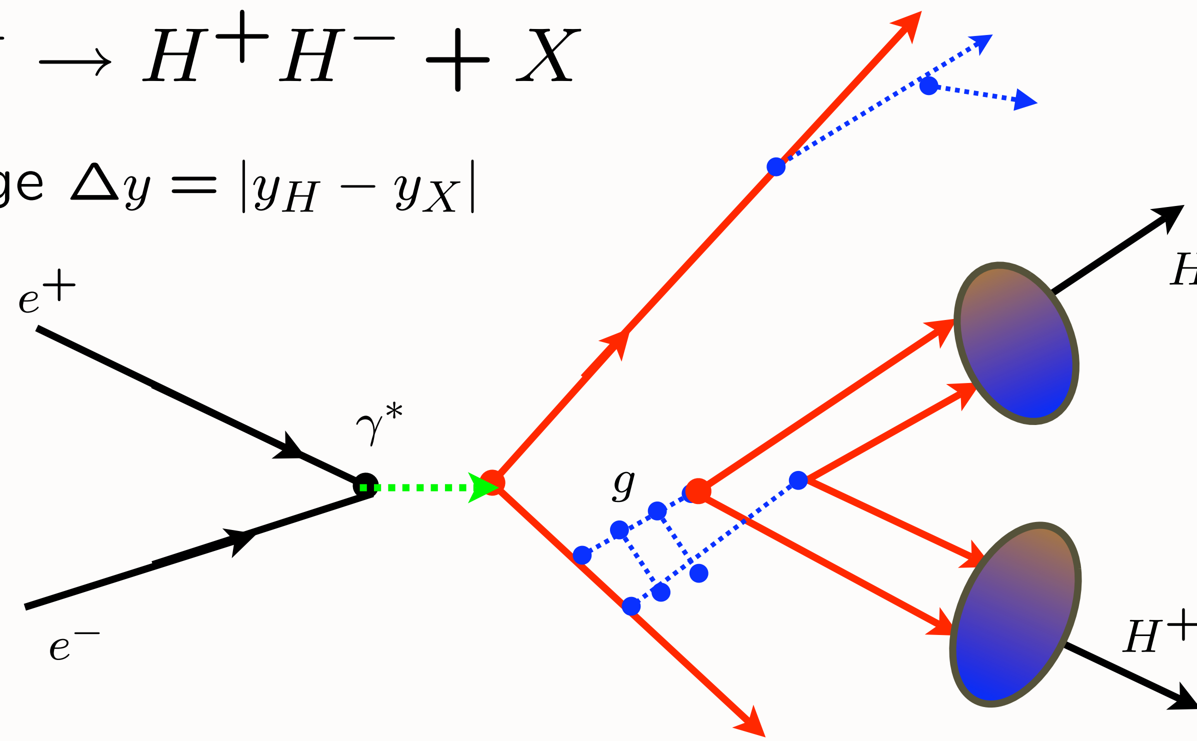


Reproduces lab-frame color dipole approach

Hadronization at the Amplitude Level

$$e^+e^- \to H^+H^- + X$$

Large $\Delta y = |y_H - y_X|$



Bjorken, Lu, sjb
Kopeliovich, Schmidt, sjb

Timelike Pomeron

C= + Gluonium Trajectory

Large Rapidity Gap Events

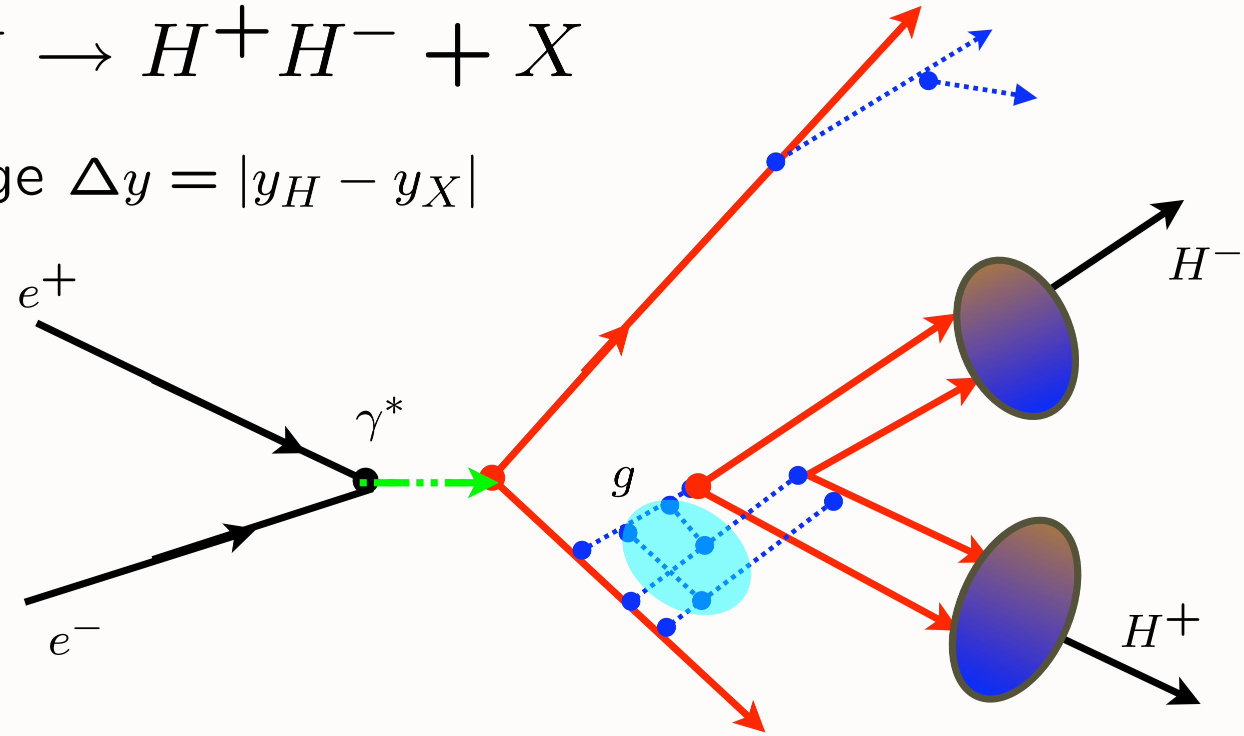
Crossing analog of Diffractive DIS $eH \to eH + X$

Hadronization at the Amplitude Level

$$e^+e^- \to H^+H^- + X$$

Kopeliovich, Schmidt, sjb

Large $\Delta y = |y_H - y_X|$



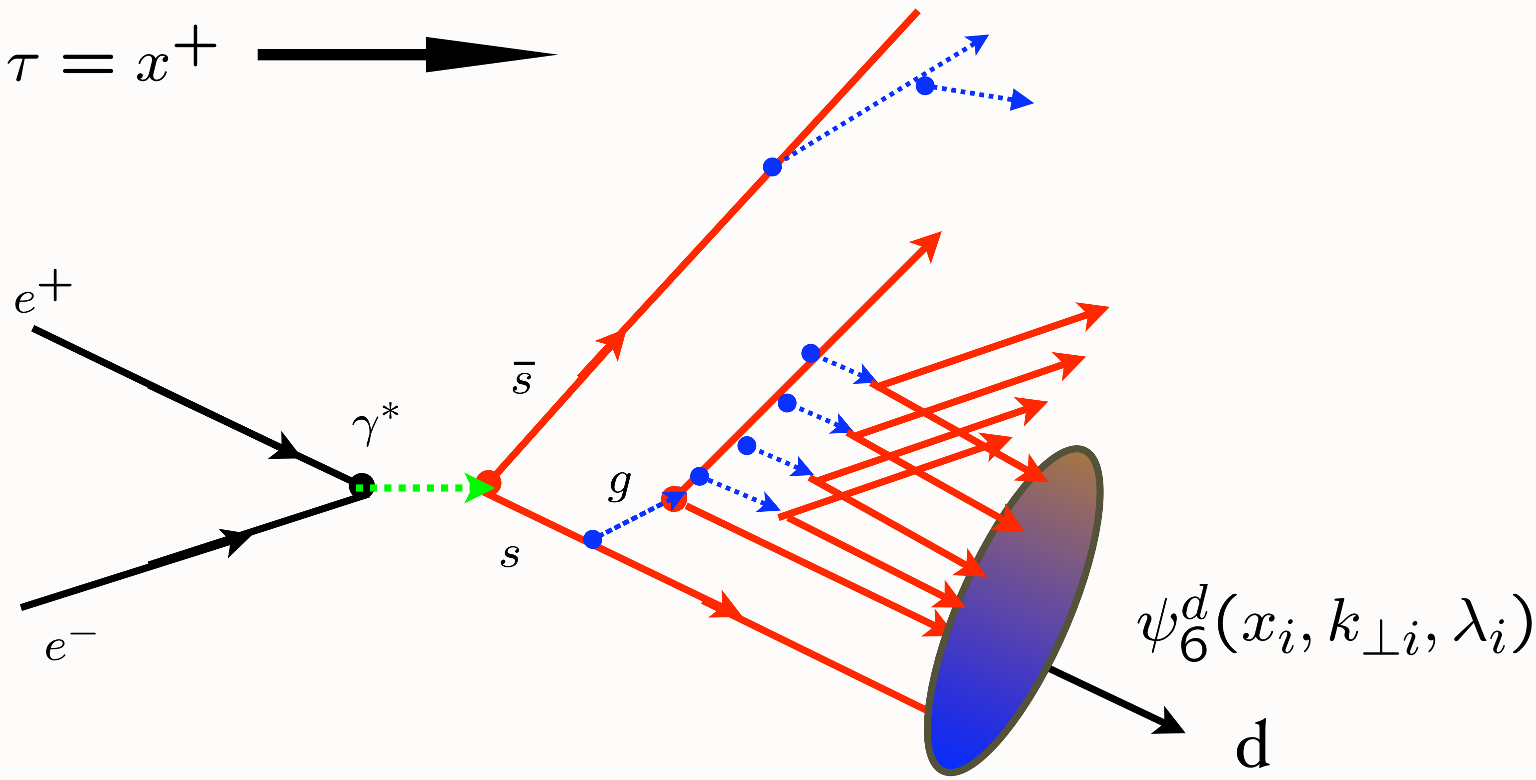
Timelike Odderon

Large Rapidity Gap Events

C= - Gluonium Trajectory

H^+H^- asymmetry from Odderon-Pomeron interference

Hadronization at the Amplitude Level

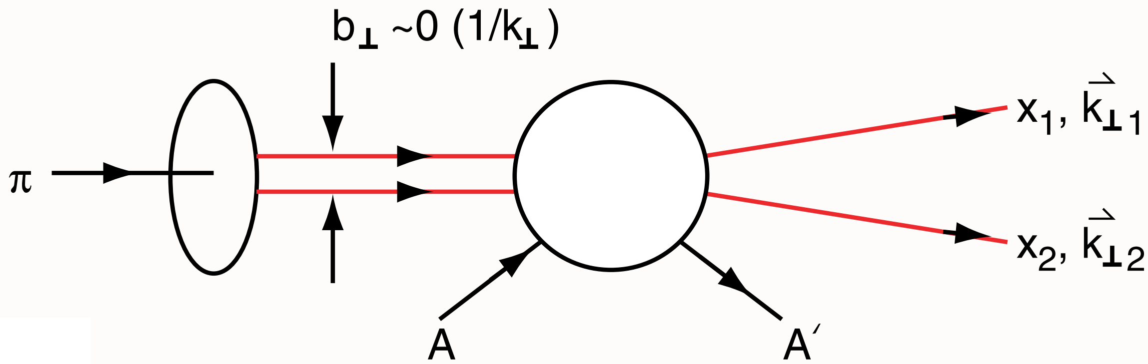


"Hidden-Color" Components $|(uud)_{8C}(ddu)_{8C}>$

New Hadronization Mechanism

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



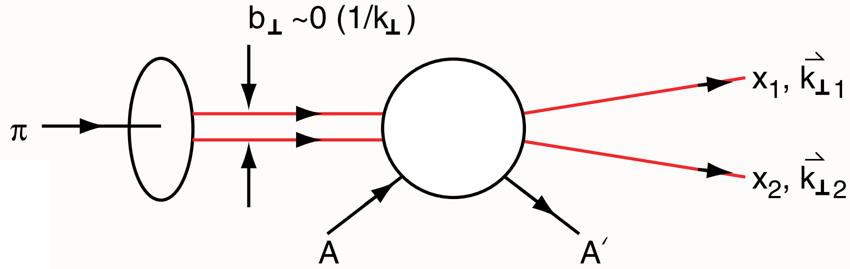
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

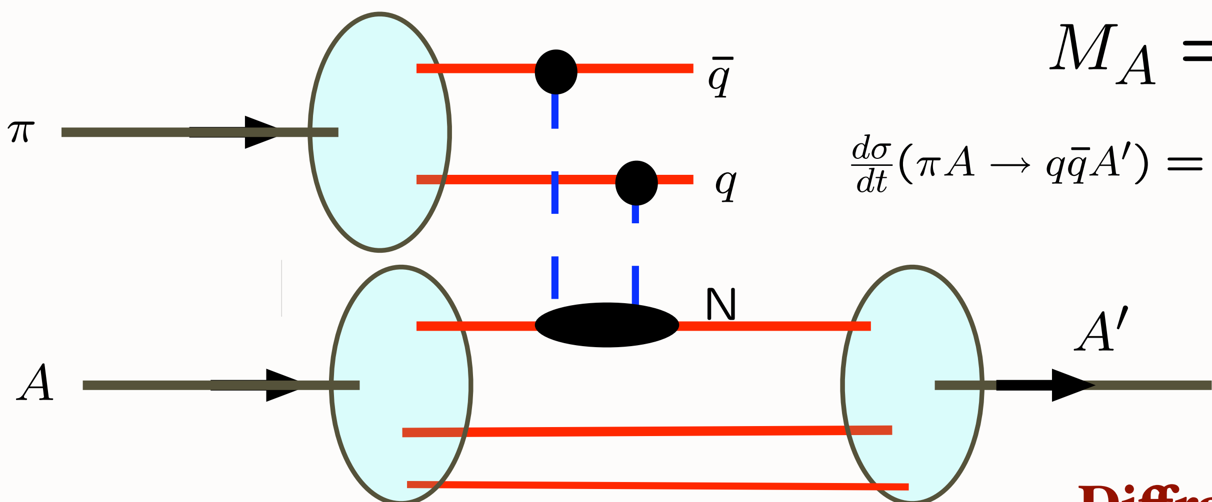
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \rightarrow q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \rightarrow q\bar{q}N')\ F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

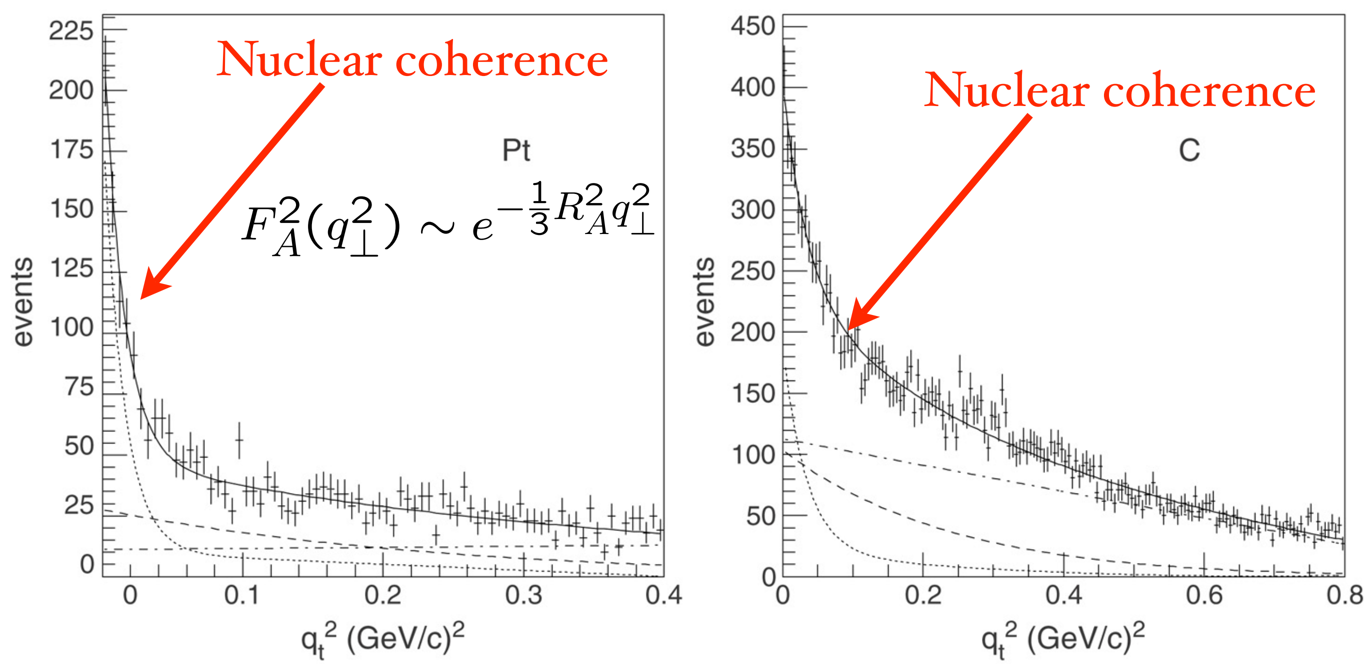
- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

- **Fully coherent interactions between pion and nucleons.**
- **Emerging Di-Jets do not interact with nucleus.**

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

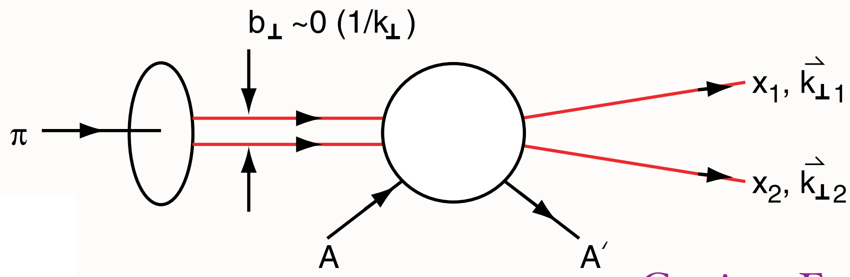
Ashery E791

α (Incoh.) $= 0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

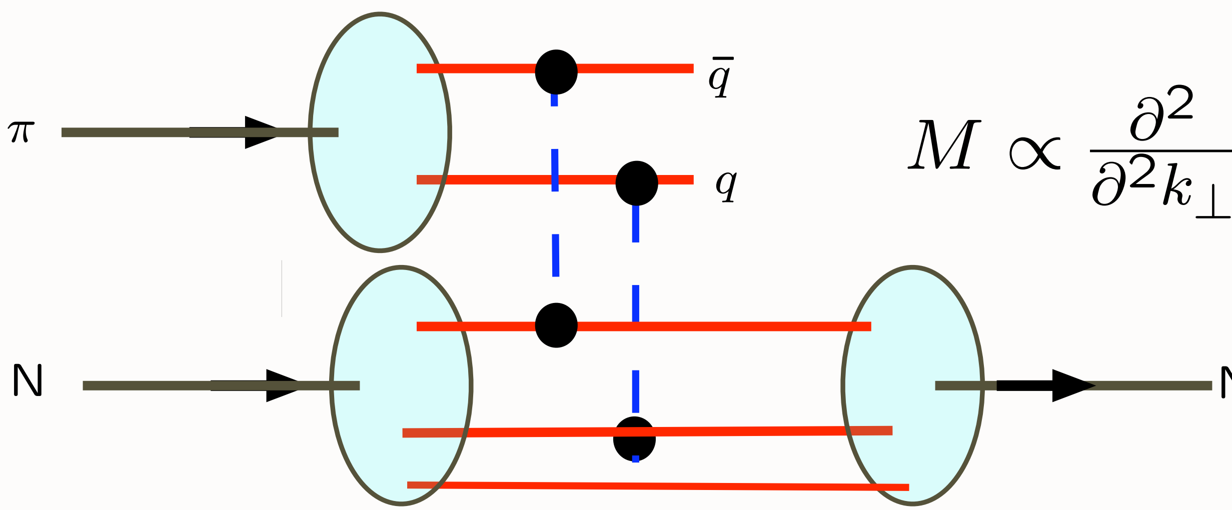
Factor of 7

Key Ingredients in Ashery Experiment



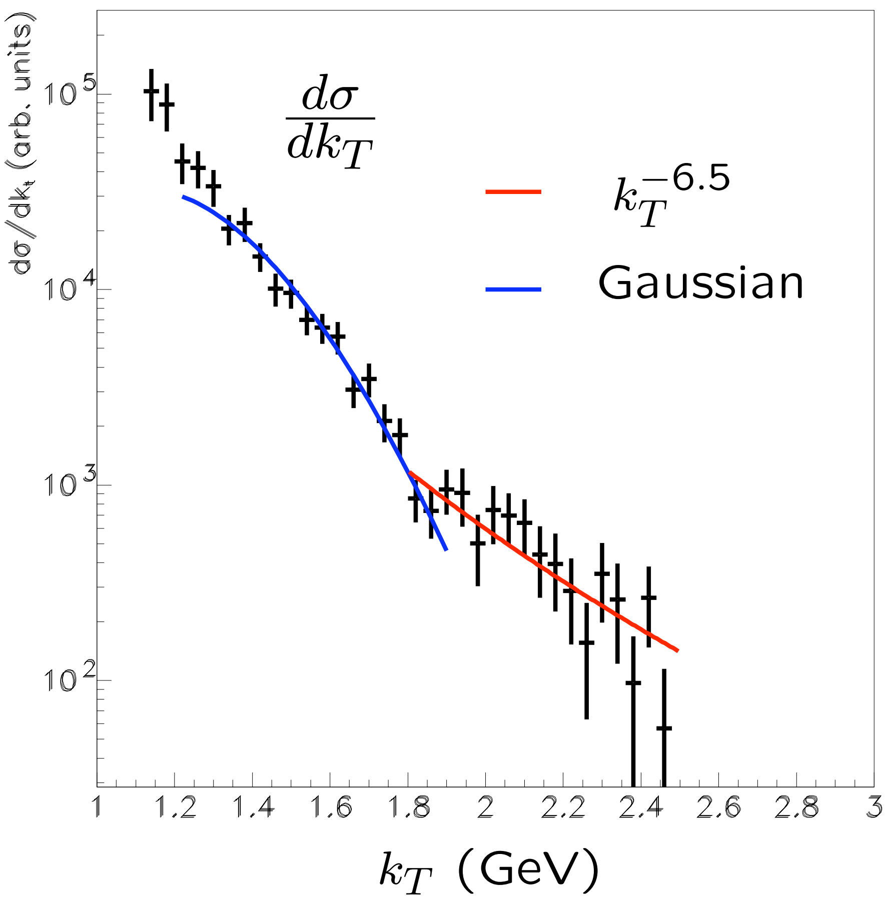
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution

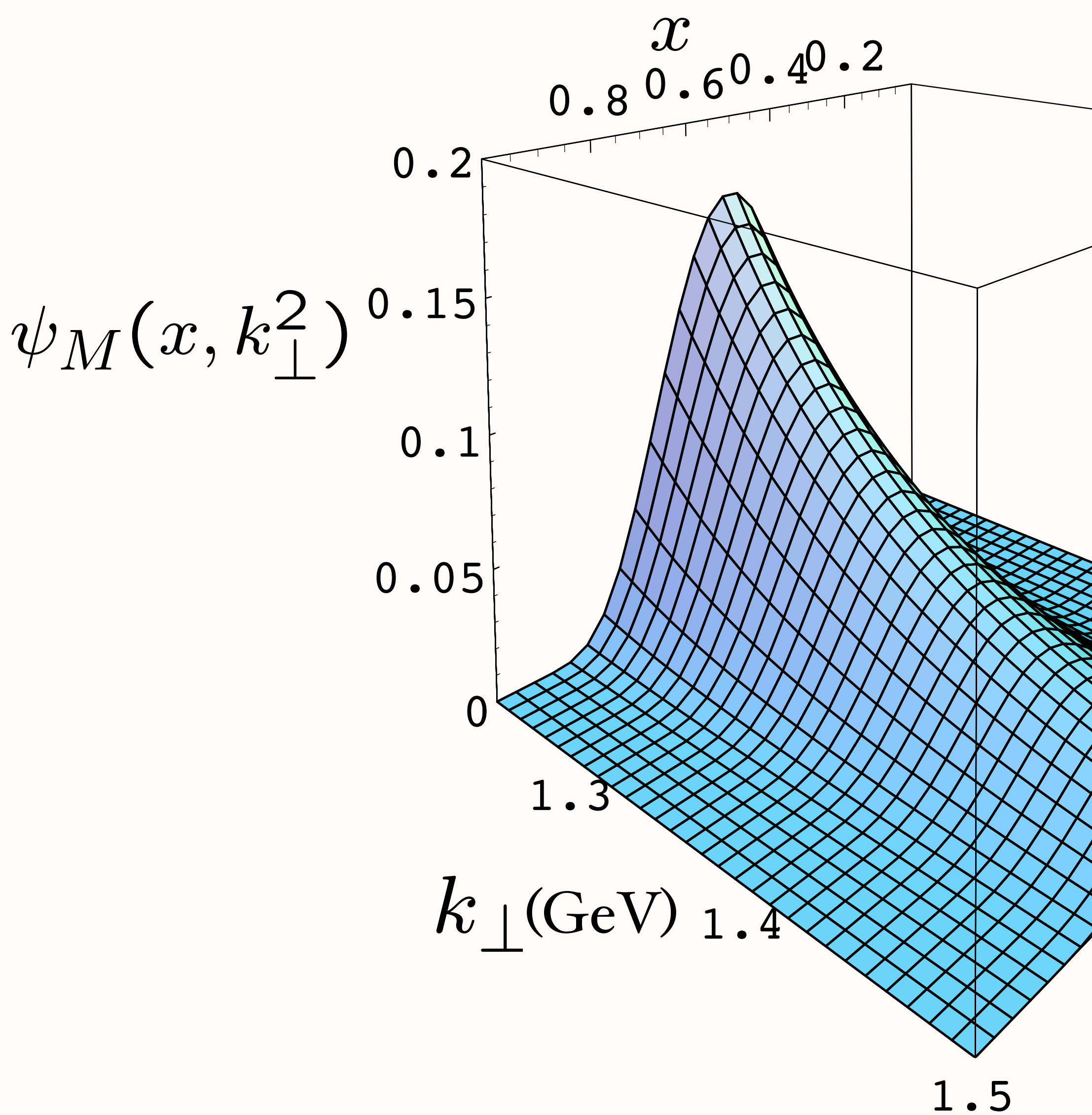


Two Components

High Transverse momentum dependence $k_T^{-6.5}$ *consistent with PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF

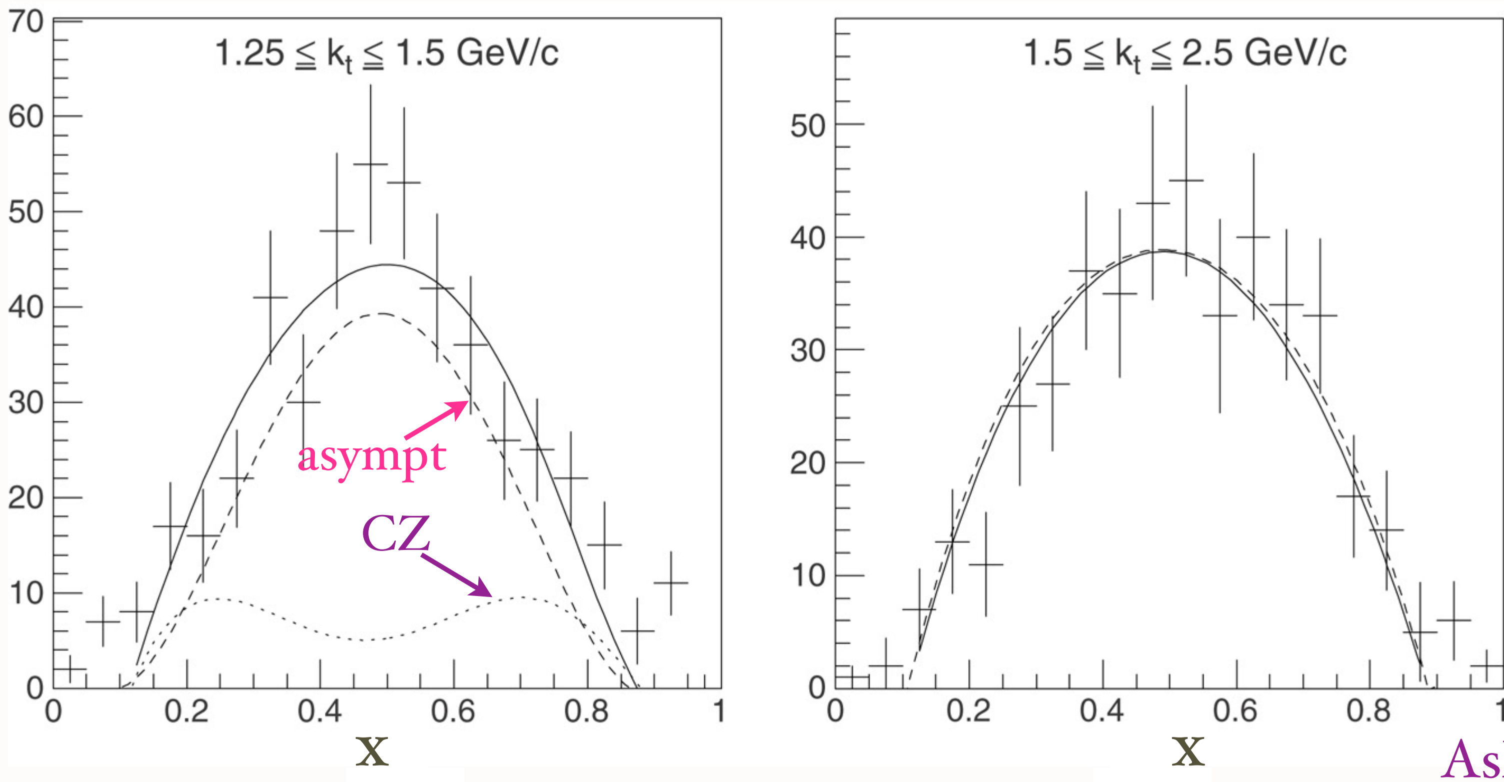
Prediction from AdS/CFT: Meson LFWF



Harmonic oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$



Ashery E791

Narrowing of x distribution at higher jet transverse momentum

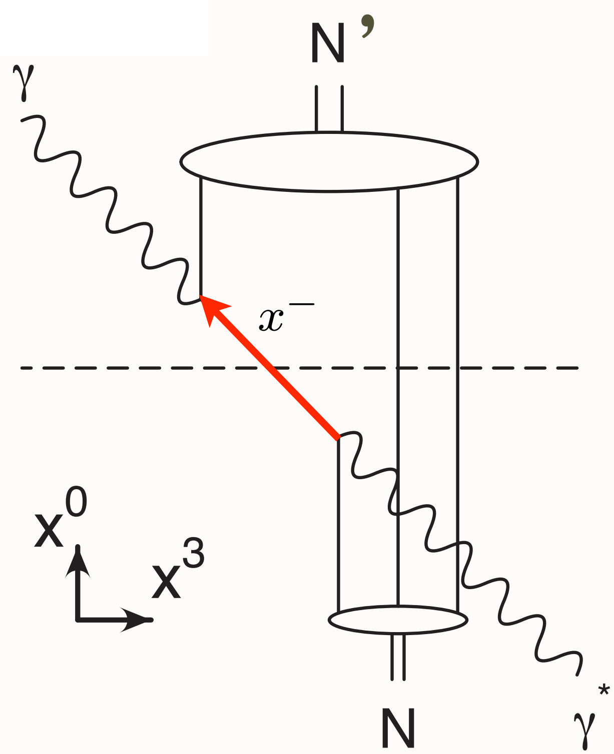
x distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

Space-time picture of DVCS

P. Hoyer

$\sigma = \frac{1}{2} x^- P^+$



$x^+ = \mathbf{x}_\perp = 0)$

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Take Fourier transform of skewness,
the longitudinal momentum transfer

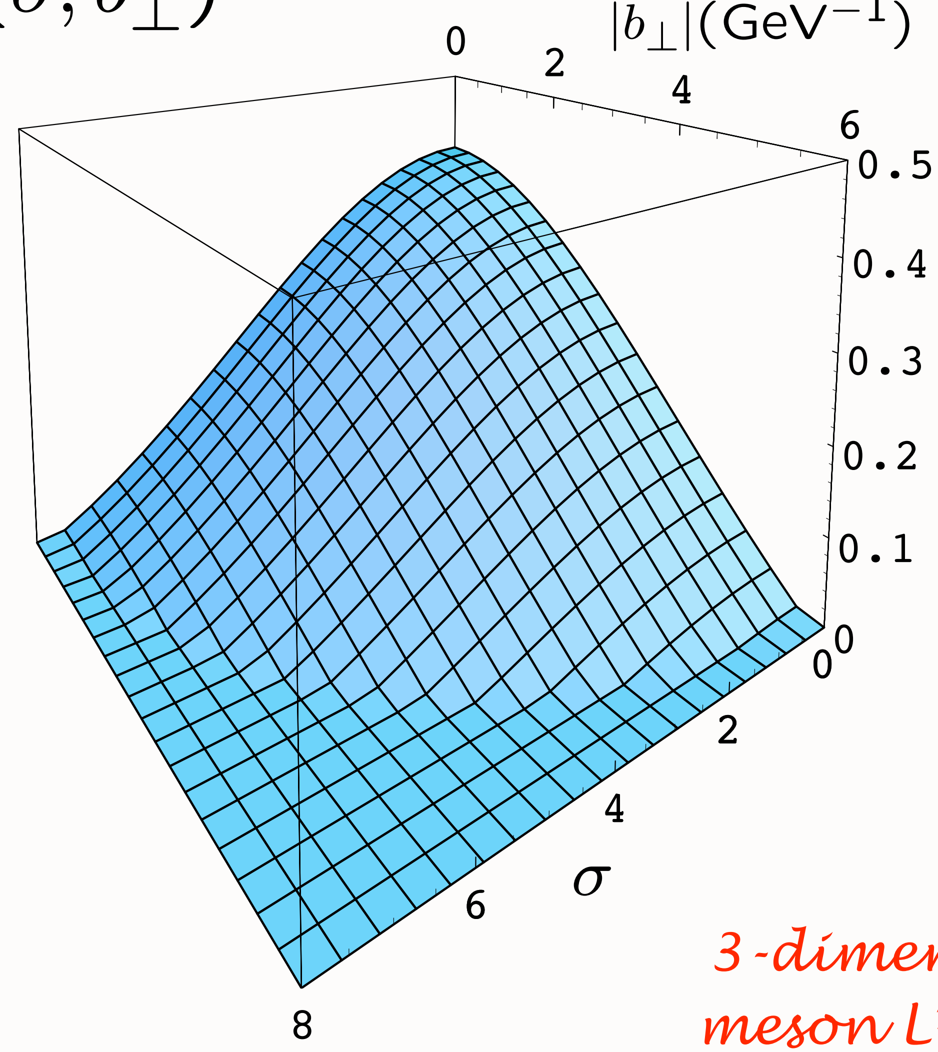
$\zeta = \frac{Q^2}{2p\cdot q}$

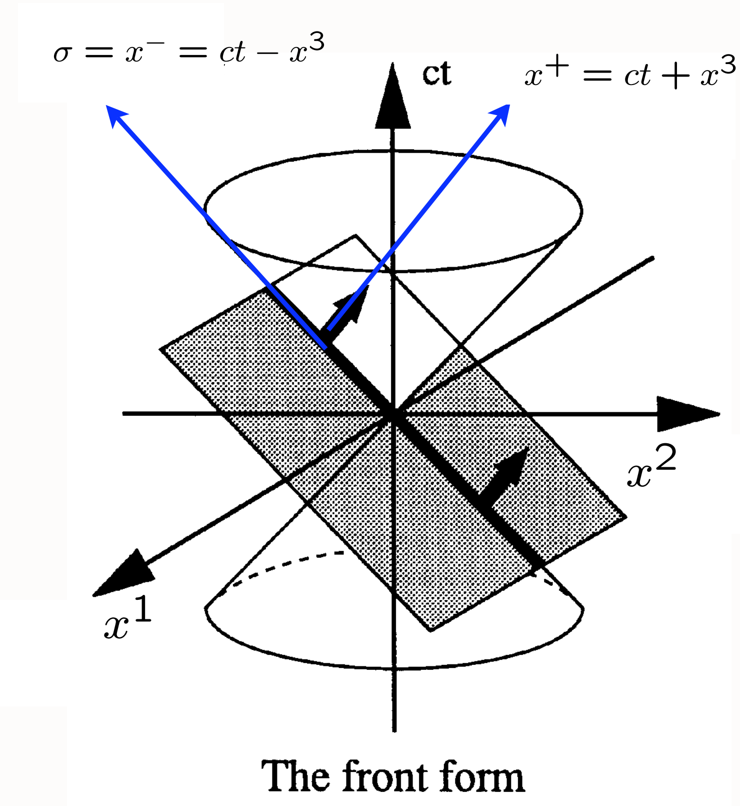
S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

AdS/CFT Holographic Model

G. de Teramond
SJB

$\psi(\sigma, b_\perp)$



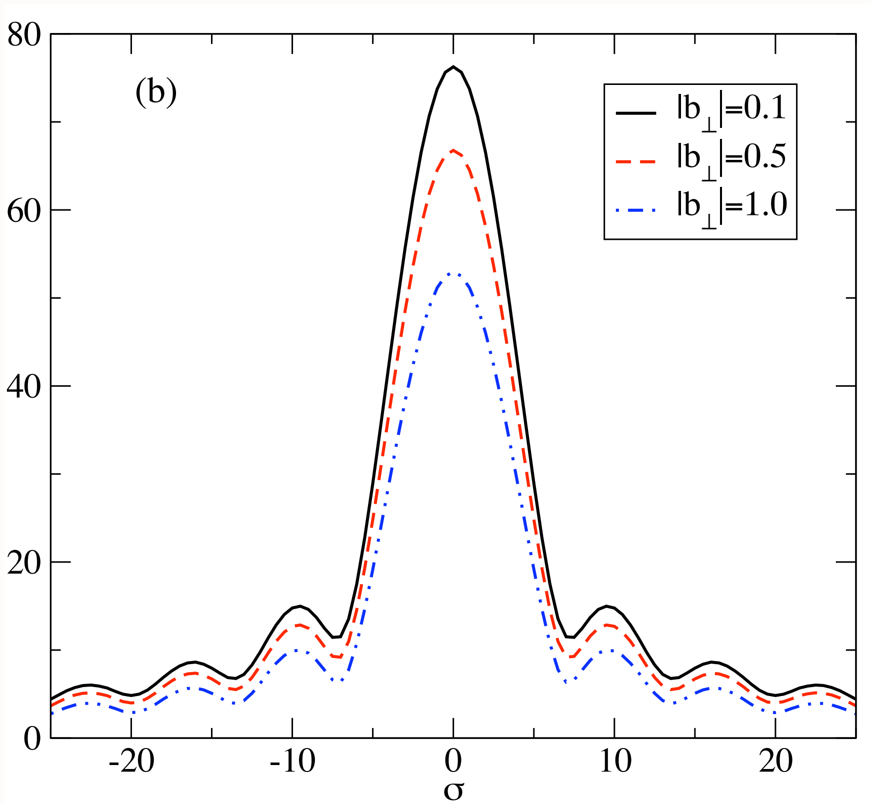


3-dimensional photograph: meson LFWF at fixed LF Time

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi}\int d\zeta e^{i\sigma\zeta}\tilde{A}(b_\perp, \zeta) \qquad \sigma = \tfrac{1}{2}x^- P^+ \qquad \zeta = \frac{Q^2}{2p\cdot q}$$

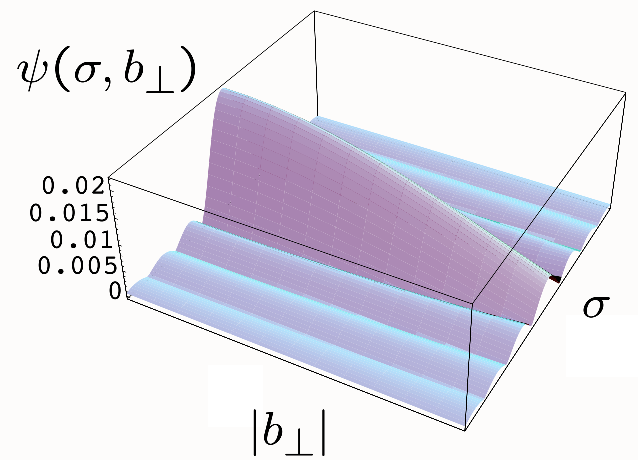


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$

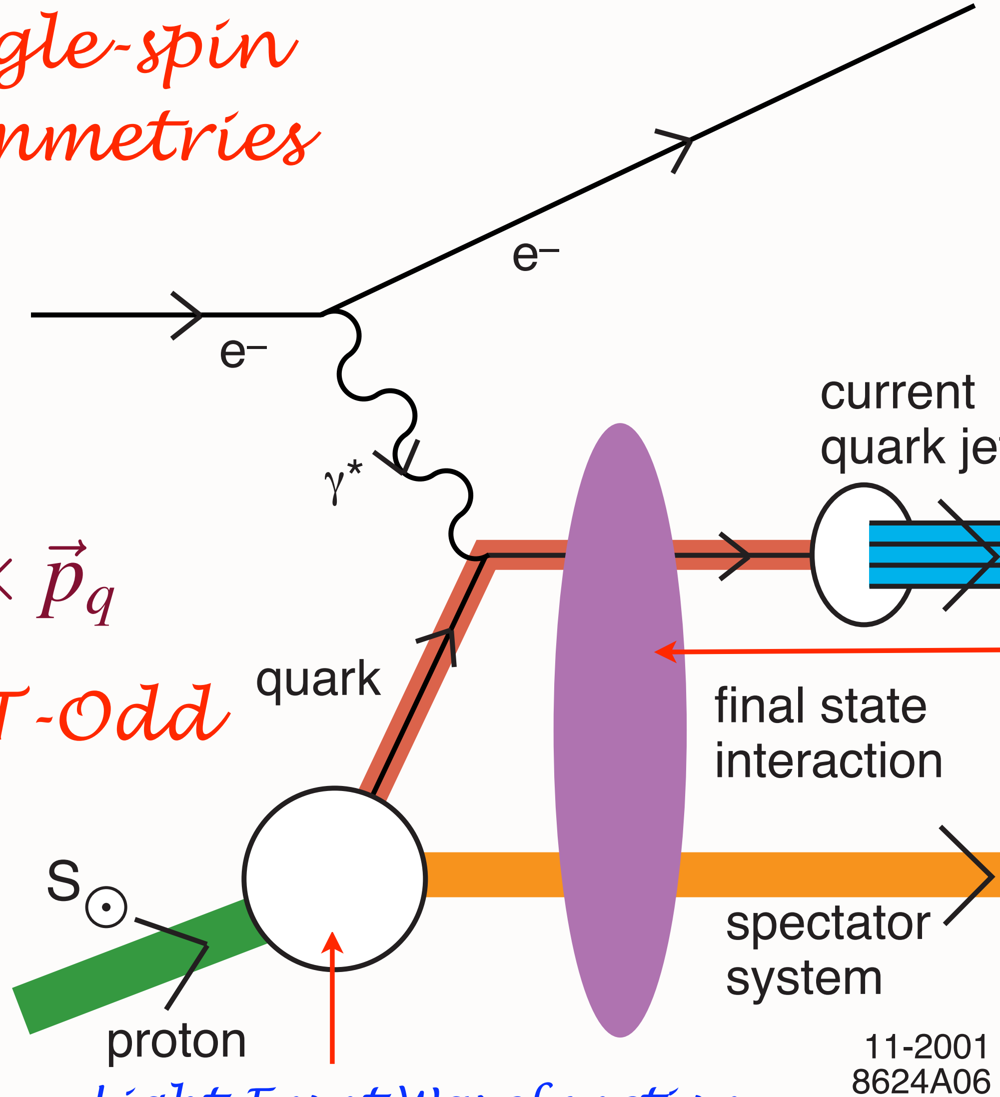


Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Single-spin asymmetries

Leading Twist Sivers Effect



$$i \, \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

QCD S- and P-Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

Final-State Interactions Produce T-Odd (Sivers Effect) $i\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$

- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment

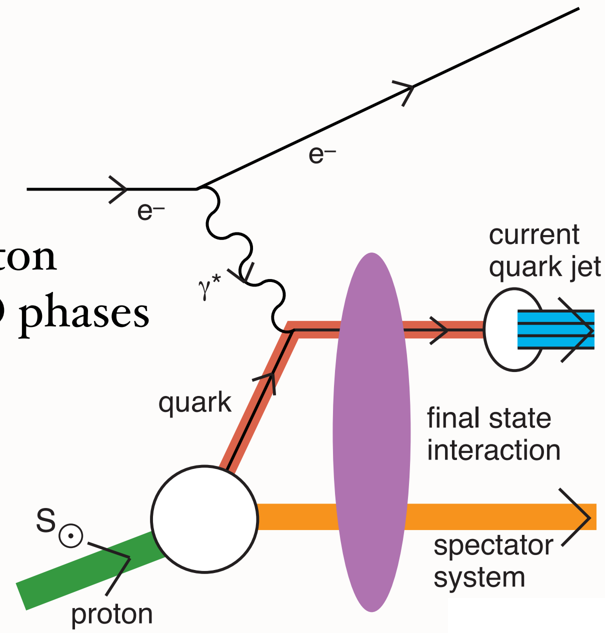
Hwang, Schmidt. sjb;
Burkardt

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark!

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Unexpected QCD Effect -- thought to be zero!
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD Coulomb phase at soft scale
- Measure in jet trigger or leading hadron
- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero gravito-anomalous magnetic moment: B(0)= 0)



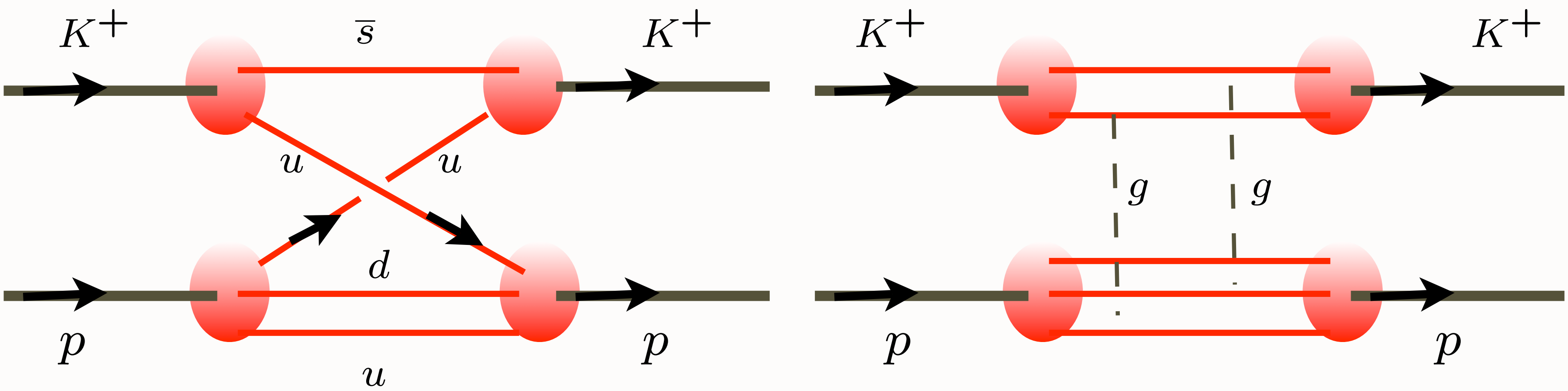
Features of Light-Front Formalism

- *Hidden Color* Nuclear Wavefunction
- *Color Transparency, Opaqueness*
- *Intrinsic glue, sea quarks, intrinsic charm*
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- *Direct mapping to AdS/CFT* (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)

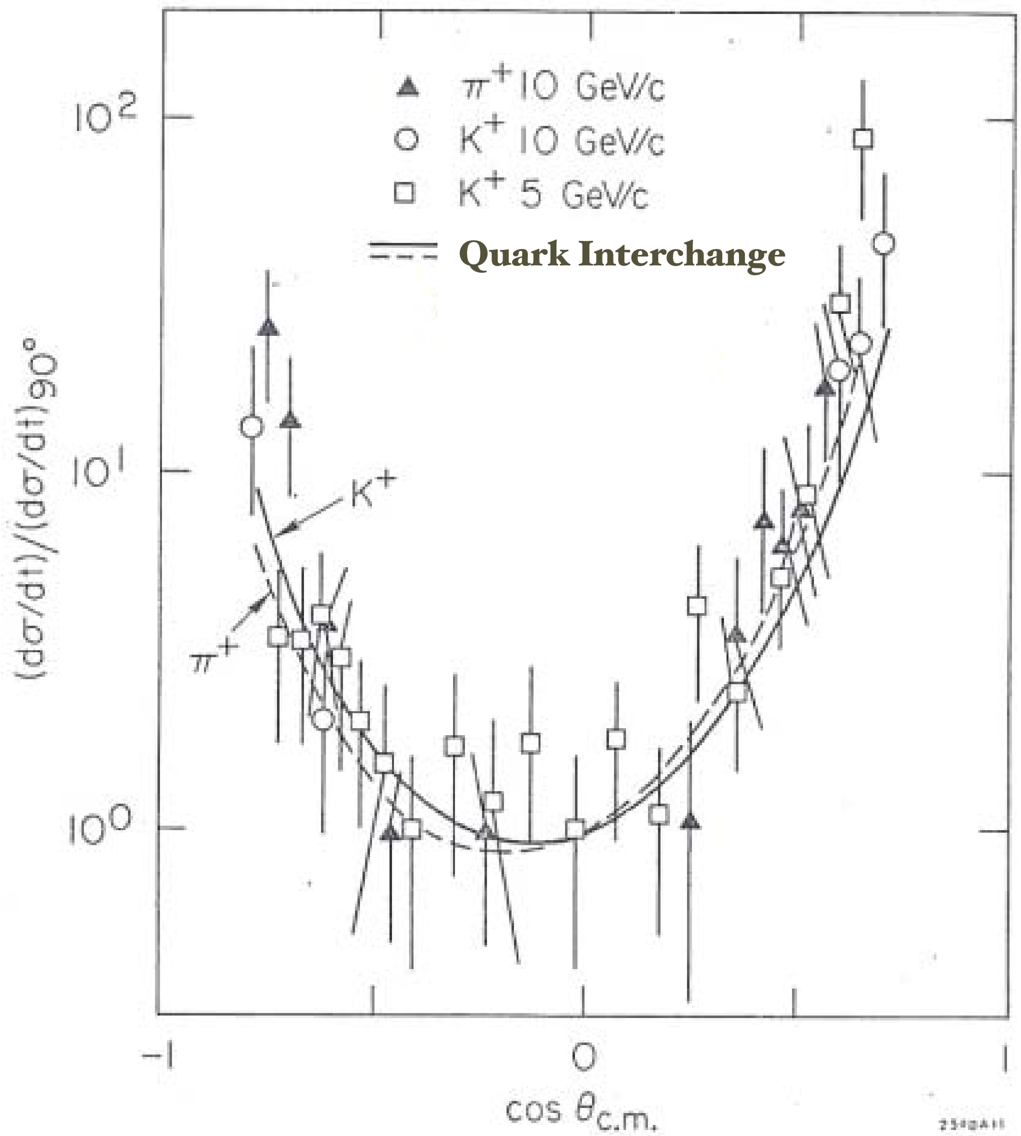
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx\ \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \rightarrow p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \rightarrow pK^{\pm}$; $p^{\pm}p \rightarrow pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \rightarrow p\pi^{\pm}$,

$K^{\pm}p \rightarrow pK^{\pm}$,

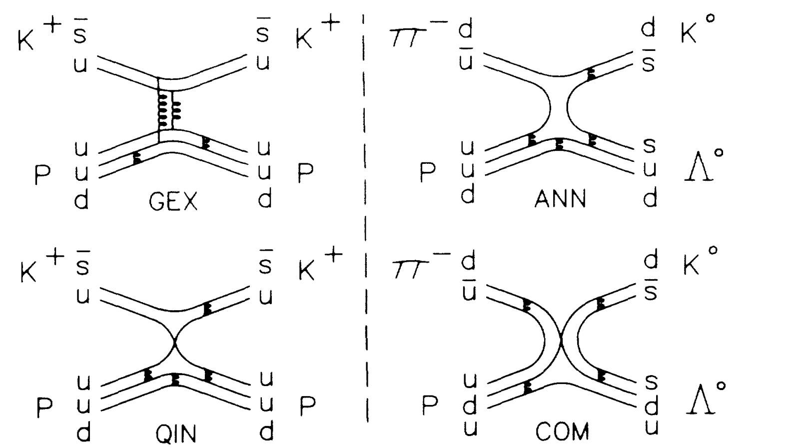
$\pi^{\pm}p \rightarrow p\rho^{\pm}$,

$\pi^{\pm}p \rightarrow \pi^{+}\Delta^{\pm}$,

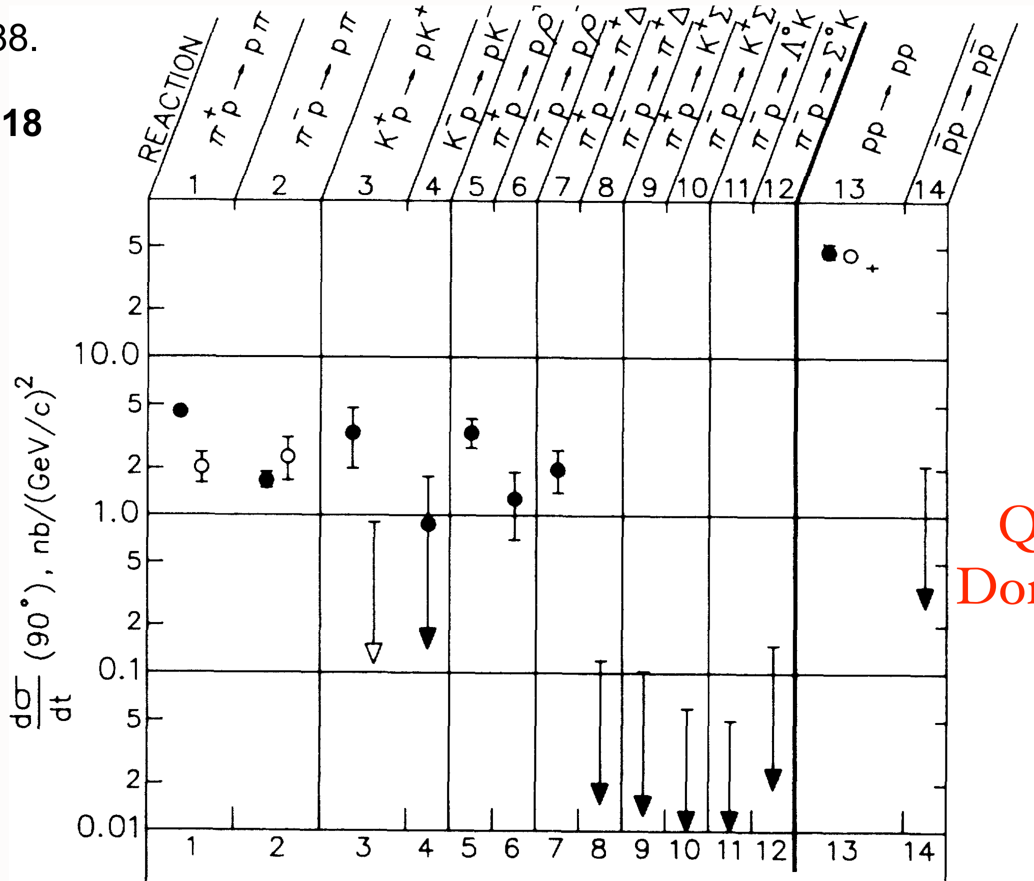
$\pi^{\pm}p \rightarrow K^{+}\Sigma^{\pm}$,

$\pi^{-}p \rightarrow \Lambda^0K^0, \Sigma^0K^0$,

$p^{\pm}p \rightarrow pp^{\pm}$.



B.R. Baller *et al.*. 1988.
Published in
Phys.Rev.Lett.60:1118-1121,1988



Comparison of Exclusive Reactions at Large *t*

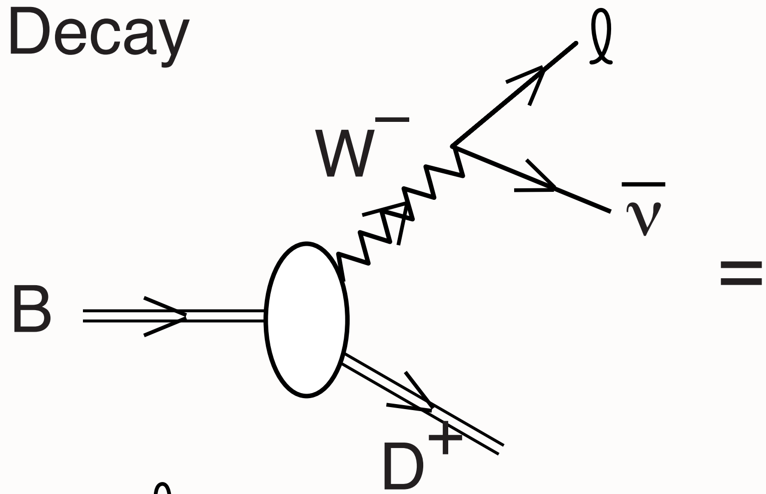
The cross section and upper limits (90% confidence level) measured by this experiment are indicated by the filled circles and arrowheads. Values from this experiment and from previous measurements represent an average over the angular region of $-0.05 < \cos\theta_{c.m.} < 0.10$. The other measurements were obtained from the following references: π^+p and K^+p elastic, Ref. 5; $\pi^-p \to p\pi^-$, Ref. 6; $pp \to pp$, Ref. 7: Allaby, open circle; Akerlof, cross. Values for the cross sections [(Reaction), cross section in nb/(GeV/c)^2] are as follows: (1), 4.6 ± 0.3; (2), 1.7 ± 0.2; (3), 3.4 ± 1.4; (4), $0.9^{+0.7}_{-0.9}$; (5), 3.4 ± 0.7; (6), 1.3 ± 0.6; (7), 2.0 ± 0.6; (8), < 0.12; (9), < 0.1; (10), < 0.06; (11), < 0.05; (12), < 0.15; (13), 48 ± 5; (14), < 2.1.

Quark Interchange: Dominant Dynamics at large t, u

Relative Rates Correct

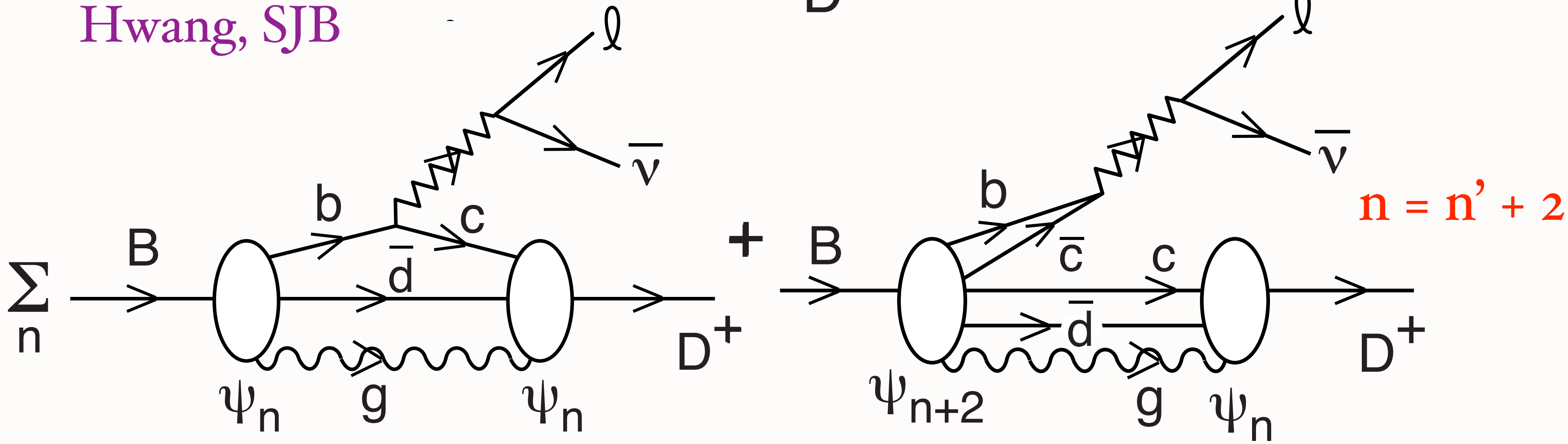
Weak Exclusive Decay

$\langle D \,|\, J^{+}(0) \,|\, B \rangle$



Exact Formula

Hwang, SJB

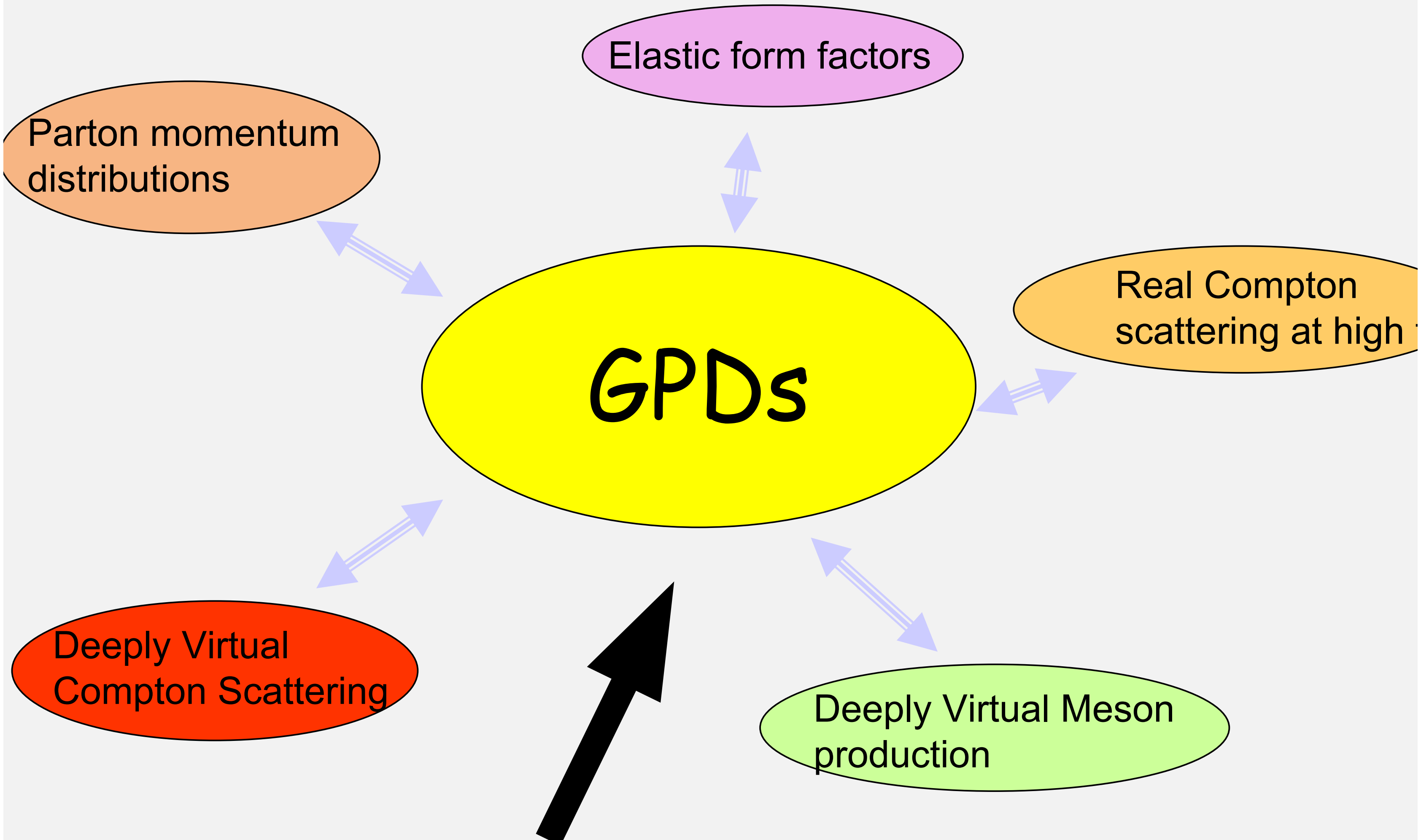


Annihilation amplitude needed for Lorentz Invariance

Light-Front Formalism

- Renormalization of LF Hamiltonian Theory (Wilson, Hiller, McCartor et al.)
- Color Transparency, Opaqueness (Mueller, Frankfurt, Strikman, sjb)
- Explicit 1+1 solutions from DLCQ (Pauli, Hornbostel, sjb)
  Supersymmetric DLCQ (Hiller, Pinsky, Trittman)
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)

A Unified Description of Hadron Structure

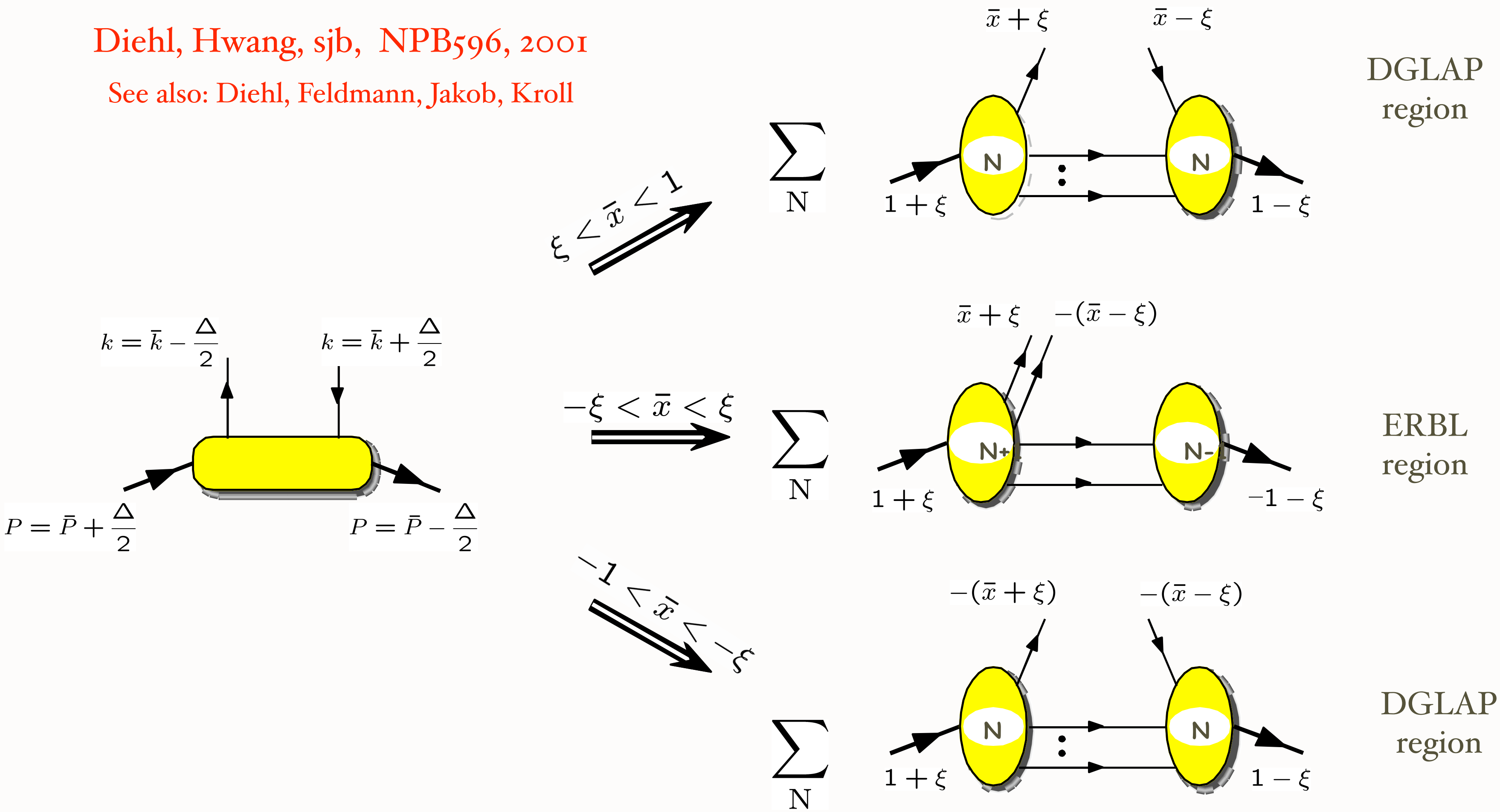


Light Front Wavefunctions

Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



N=3 VALENCE QUARK ⇒ Light-cone Constituent quark model

N=5 VALENCE QUARK + QUARK SEA ⇒ Meson-Cloud model

Liu, Pasquini

Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Example of LFWF representation of GPDs (n+1 => n-1)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M} E_{(n+1\to n-1)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1-\sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1}\vec{k}_{\perp j}\right)$$

$$\times 16\pi^3 \delta(x_{n+1}+x_1-\zeta)\delta^{(2)}(\vec{k}_{\perp n+1}+\vec{k}_{\perp 1}-\vec{\Delta}_\perp)$$

$$\times \delta(x-x_1)\psi^{\uparrow *}_{(n-1)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\psi^{\downarrow}_{(n+1)}(x_i,\vec{k}_{\perp i},\lambda_i)\delta_{\lambda_1 -\lambda_{n+1}},$$

where $i = 2, \ldots, n$ label the $n-1$ spectator partons which appear in the final-state hadron wavefunction with

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp .$$

The Generalized Parton Distribution $E(x, \zeta, t)$

The generalized form factors in virtual Compton scattering $\gamma^*(q) + p(P) \to \gamma^*(q') + p(P')$ with $t = \Delta^2$ and $\Delta = P - P' = (\zeta P^+, \boldsymbol{\Delta}_\perp, (t + \boldsymbol{\Delta}_\perp^2)/\zeta P^+)$, have been constructed in the light-front formalism. [Brodsky, Diehl, Hwang, 2001]

We find, under $\boldsymbol{q}_\perp \to \boldsymbol{\Delta}_\perp$, for $\zeta \le x \le 1$,

$$\frac{E(x,\zeta,0)}{2M} = \sum_a (\sqrt{1-\zeta})^{1-n} \sum_j \delta(x - x_j) \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \times \psi_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i) \,,$$

with $x_j' = (x_j - \zeta)/(1-\zeta)$ for the struck parton j and $x_i' = x_i/(1-\zeta)$ for the spectator parton i.

The E distribution function is related to a $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}}$ matrix element at finite ζ as well.

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int_{-1}^{1} dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \quad \text{Dirac f.f.}$$

$$\int_{-1}^{1} dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \quad \text{Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

AdS/CFT and Integrability

- Conformal Symmetry plus Confinement: Reduce AdS/QCD Equations to Linear Form
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations
- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).

AdS/CFT LF Equation for Mesons with HO Confinement

$$\nu = L$$

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

LF Hamiltonian

$$H^\nu_{LF}\phi_\nu = \mathcal{M}^2_\nu\phi_\nu$$ **Bilinear** $$H^\nu_{LF} = \Pi^\dagger_\nu\Pi_\nu,$$

where

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

de Teramond, sjb

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

AdS/CFT LF Equation for Mesons with HO Confinement

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

Define $b_\nu^\dagger = -i\Pi_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta$

$$b_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \qquad\qquad b_\nu^\dagger b_\nu = b_{\nu+1} b_{\nu+1}^\dagger$$

Ladder Operator $b_\nu^\dagger|\nu\rangle = c_\nu|\nu+1\rangle$

$$\left(-\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)\phi_\nu(\zeta) = c_\nu\phi_{\nu+1}(\zeta)$$

$$\phi_\nu(z) = C z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} G_\nu(\zeta),$$

$$2xG_\nu(x) - G'(x) = xG_{\nu+1}(x)$$

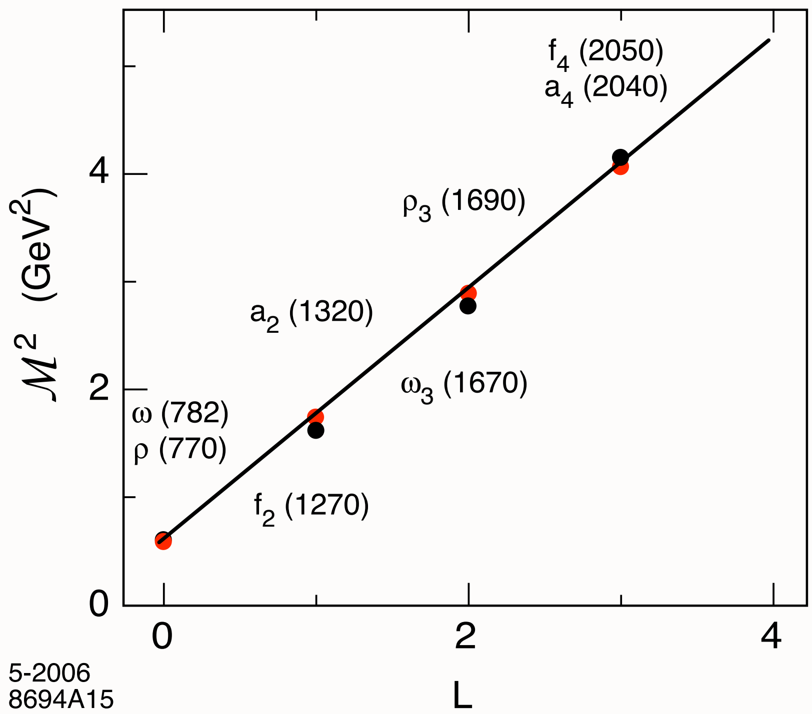
defines the associated Laguerre function $L_n^{\nu+1}(x^2)$

$$\phi_\nu(z) = C_\nu z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2).$$

Subtract Vacuum Energy

$$\mathcal{M}^2 \to \mathcal{M}^2 - 2\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + \frac{1}{2}).$$



$$J = L + 1 \text{ vector meson Regge trajectory for } \kappa \simeq 0.54 \text{ GeV}$$

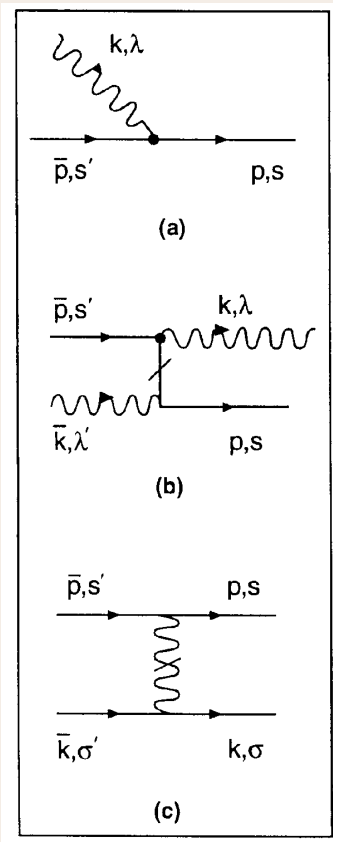
Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

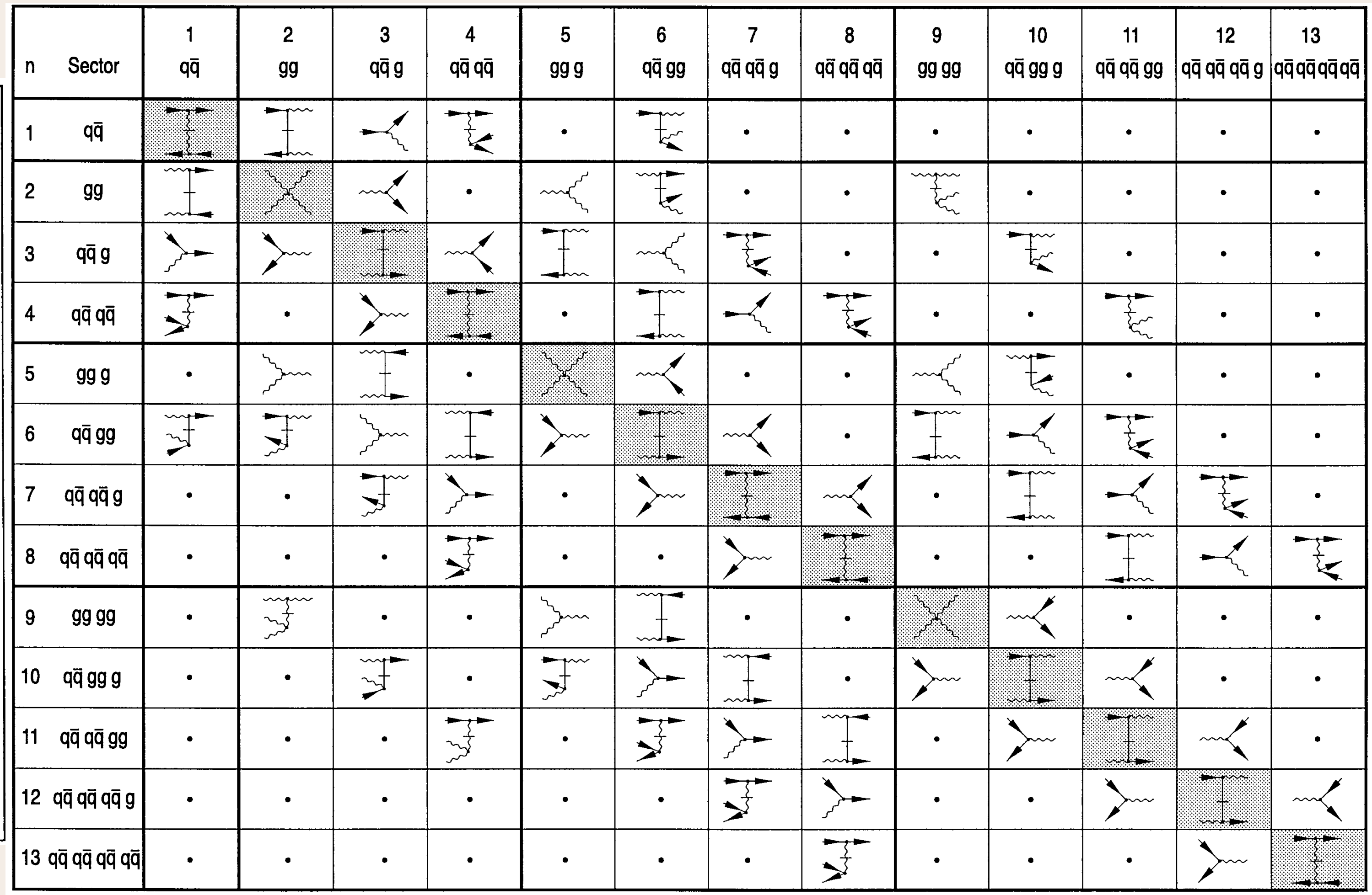
- Good initial approximant
- Better than plane wave basis — Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion — Vary, Harinandrath, sjb
- Similar to Shell Model calculations

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

DLCQ





Use AdS/QCD basis functions

Pauli, Pinsky, sjb

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$ and 4 states $\overline{q}q$, qqq, and gg appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.

Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227. E. van Beveren et al.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

SCIENCE VOL 265 15 SEPTEMBER 1995

A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

Frank and Ernest

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