Symposium in honor of Keh-Fei Liu on the occasion of his 60th Birthday

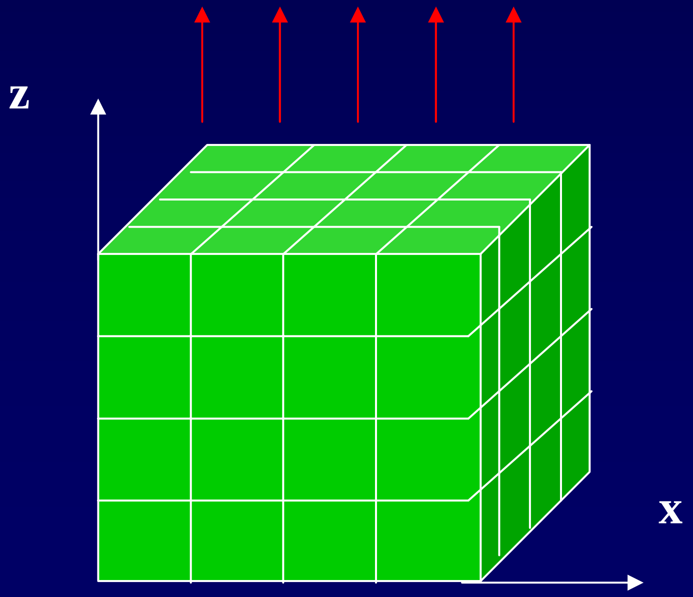
- A good physicist
  - wide knowledge, deep intuition, full of innovative ideas, up-to-date in theory and experiment
- Visionary
  - For example: implementation of the overlap operator (reached a record 180 MeV pion mass on a 3.2 fm lattice, exposed ghosts in the chiral regime)
- Quality over quantity
  - "A study of pentaquarks on the lattice with overlap fermions", Kentucky group, PRD70 (2004).
    - no evidence of bound state, consistent with KN scattering states
    - volume dependence of the spectral weight (1/V). G(t) = w exp(-m t)
    - described as the "most credible lattice calculation" at the plenary of Lattice05.

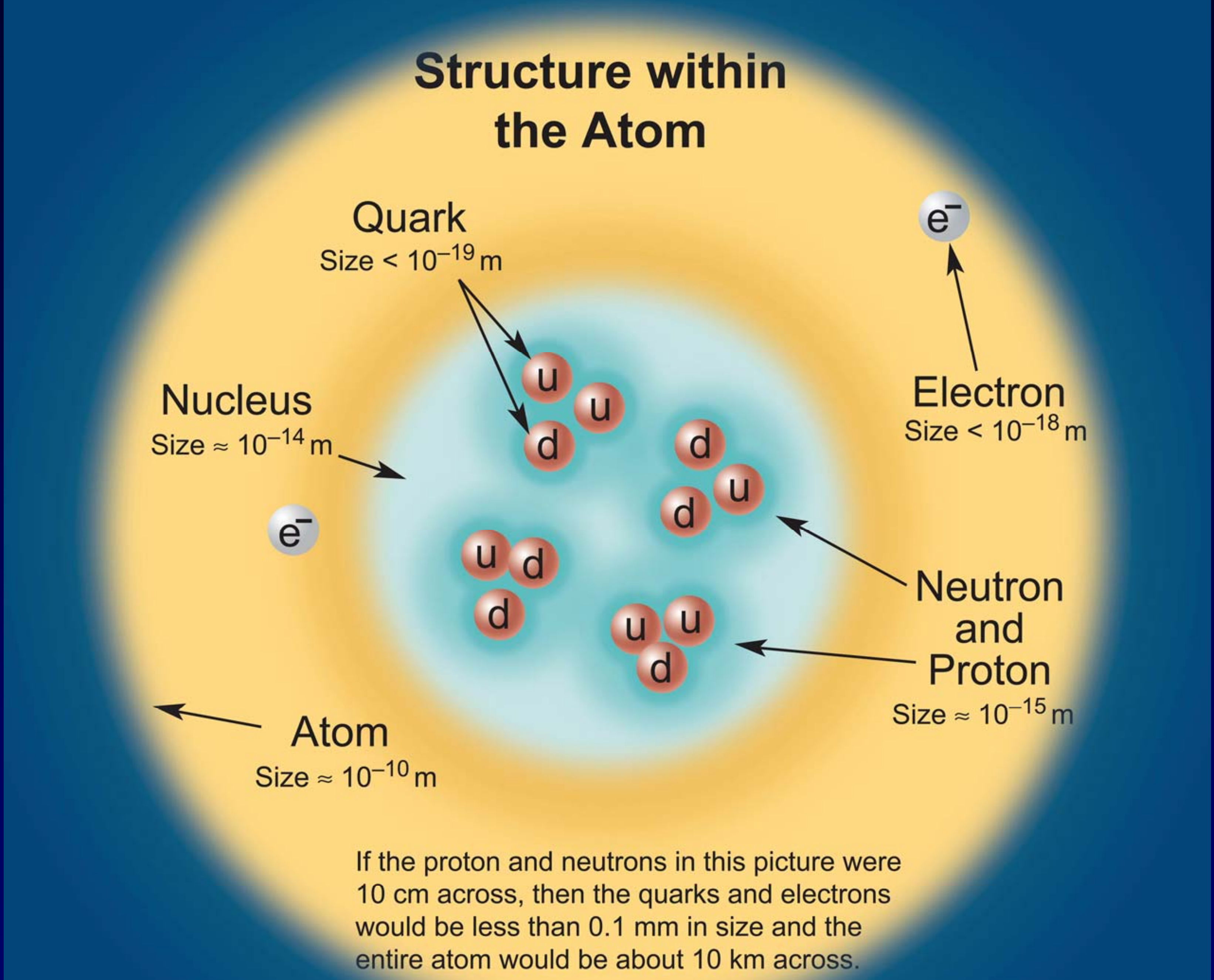
Background Field Calculations in Lattice QCD

Frank Lee, GWU

polarQCD Collaboration
http://eagle.phys.gwu.edu/~fxlee/polarQCD.html

- Physics motivation
- Background field method
- Some results
  - magnetic moments
  - magnetic polarizability
  - electric polarizability
- Outlook

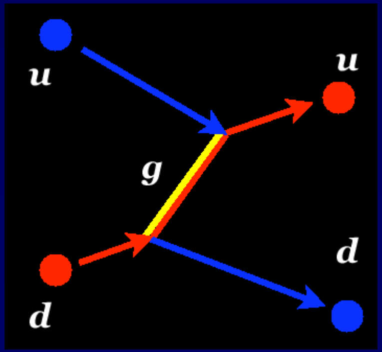




A journey deep into the heart of the atom

Quarks and Strong Interactions

	name	mass (GeV/c^2)	EM charge (e)
u	up	0.003	+2/3
d	down	0.006	–1/3
s	strange	0.08	–1/3
c	charm	1.3	+2/3
b	bottom	4.5	–1/3
t	top	174	+2/3



But, it is a long, hard struggle from quarks and gluons to …

The Particle Zoo (excitation spectrum of QCD)

Baryons (3 quarks)

p	P_{11}	****
n	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1,11}$	***
$N(2700)$	$K_{1,13}$	**

$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3,11}$	****
$\Delta(2750)$	$I_{3,13}$	**
$\Delta(2950)$	$K_{3,15}$	**
$\Theta(1540)^+$		***
$\Phi(1860)$		*

Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	**
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-		****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+		****
$\Lambda_c(2593)^+$		***
$\Lambda_c(2625)^+$		***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$		**
$\Sigma_c(2455)$		****
$\Sigma_c(2520)$		***
Ξ_c^+		***
Ξ_c^0		***
$\Xi_c'^+$		***
$\Xi_c'^0$		***
$\Xi_c(2645)$		***
$\Xi_c(2790)$		***
$\Xi_c(2815)$		***
Ω_c^0		***
Ξ_{cc}^+		*
Λ_b^0		***
Ξ_b^0, Ξ_b^-		*

Mesons (quark-antiquark)

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $h_1(1170)$	$0^-(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $b_1(1235)$	$1^+(1^{+-})$	$\rho(1900)$	$1^+(1^{--})$
• $a_1(1260)$	$1^-(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $f_2(1270)$	$0^+(2^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$f_0(2020)$	$0^+(0^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $a_4(2040)$	$1^-(4^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $f_4(2050)$	$0^+(4^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$f_0(2100)$	$0^+(0^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$f_2(2150)$	$0^+(2^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
• $a_0(1450)$	$1^-(0^{++})$		
• $\rho(1450)$	$1^+(1^{--})$	$\eta(2225)$	$0^+(0^{-+})$
• $\eta(1475)$	$0^+(0^{-+})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_0(1500)$	$0^+(0^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_1(1510)$	$0^+(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$h_1(1595)$	$0^-(1^{+-})$	$a_6(2450)$	$1^-(6^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_6(2510)$	$0^+(6^{++})$
$a_1(1640)$	$1^-(1^{++})$	**OTHER LIGHT**	
$f_2(1640)$	$0^+(2^{++})$	Further States	
• $\eta_2(1645)$	$0^+(2^{-+})$		
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{sJ}^*(2317)^\pm$	$0(0^+)$
• $D_{sJ}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

$c\bar{c}$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
$\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$\psi(3836)$	$0^-(2^{--})$
$X(3872)$	$?^?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

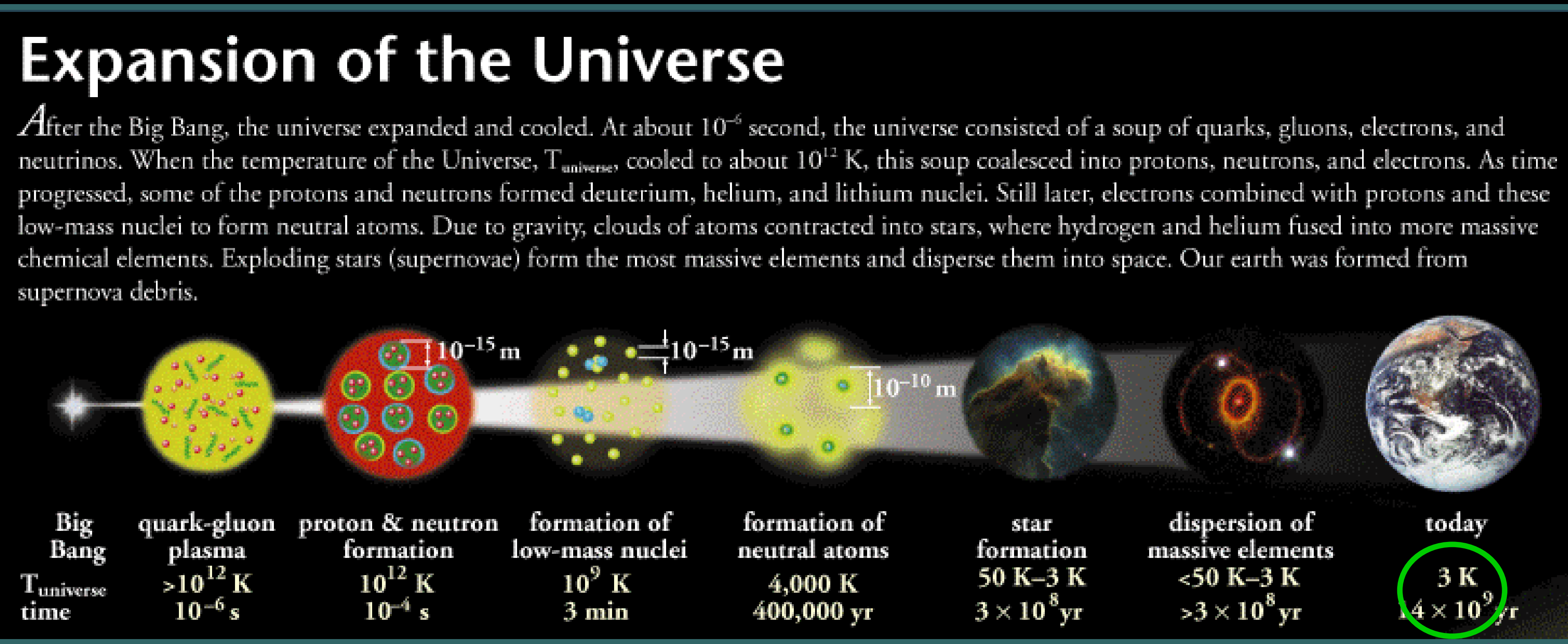
$b\bar{b}$

$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON-$q\bar{q}$ CANDIDATES

NON-$q\bar{q}$ CANDIDATES

$$L_{QCD} = \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}(\gamma^\mu D_\mu + m_q)q$$



2006 Nobel
Prize in Physics

- Study of the very small is closely related to the study of the very big.

Quantum Chromodynamics (QCD)

--- the fundamental theory of the strong interaction (in terms of quarks and gluons)

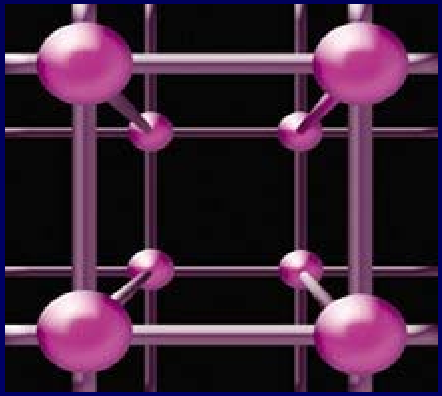
Action of QCD:

$$S_{QCD} = \int d^4x \left[\tfrac{1}{2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} + \bar{q}(\gamma^\mu D_\mu + m_q) q \right]$$

All physics is computed from path integrals

$$\langle \hat{O} \rangle \equiv \frac{\int DG_\mu Dq D\bar{q} e^{-S_{QCD}} \hat{O}(G_\mu, q, \bar{q})}{\int DG_\mu Dq D\bar{q} e^{-S_{QCD}}}$$

which can be evaluated on a space-time lattice.



Path Integral Method

(Richard Feynmann)

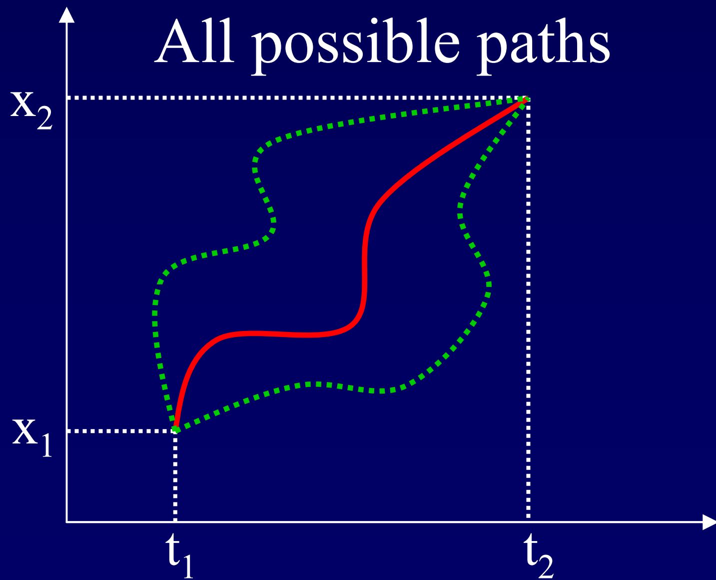
- Applicable to any problem that can be cast into the form

$$\langle O \rangle \equiv \frac{\int Dx\, e^{-S} O(x)}{\int Dx\, e^{-S}}$$

with an action, for example:

$$S = \int_{t_1}^{t_2} dt \left\{ \frac{1}{2} m \dot{x}^2(t) + V[x(t)] \right\}$$

- Successfully used in
  - statistical physics
  - quantum chemistry
  - condensed matter physics
  - biological physics
  - quantum field theories (QED,QCD, …)
  - and more



Background-Field Calculations

New Topic

Polarizabilities

Interaction energy of a hadron in the presence of external electromagnetic fields:

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$

$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$

$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$

$$-\frac{1}{12}\alpha_{E2}E_{ij}^2 - \frac{1}{12}\beta_{M2}B_{ij}^2 + \cdots$$

Time and spatial derivatives : $\dot{E} = \frac{\partial E}{\partial t},\ E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$, etc

Probe of internal structure of the system in increasingly finer detail.

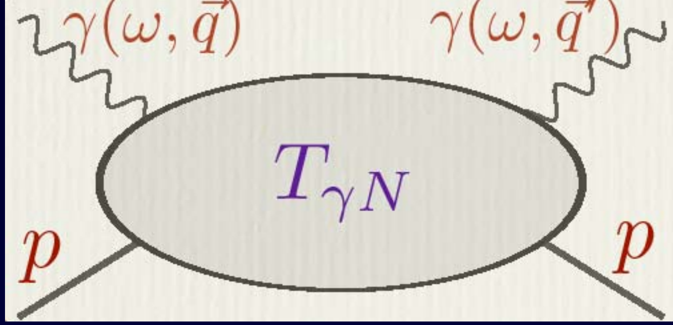
μ, α, β:

static bulk response

others :

spatial and time resolution

Compton Scattering



Low-energy expansion of real Compton scattering amplitude on the nucleon

$$T_{\gamma N} = A_1(\omega,\theta)\,\vec{\epsilon}\,'\cdot\vec{\epsilon} + A_2(\omega,\theta)\,\vec{\epsilon}\,'\cdot\hat{k}\,\vec{\epsilon}\cdot\hat{k}' + i\,A_3(\omega,\theta)\,\vec{\sigma}\cdot(\vec{\epsilon}\,'\times\vec{\epsilon}) + i\,A_4(\omega,\theta)\,\vec{\sigma}\cdot(\hat{k}'\times\hat{k})\,\vec{\epsilon}\,'\cdot\vec{\epsilon}$$
$$+i\,A_5(\omega,\theta)\,\vec{\sigma}\cdot\left[(\vec{\epsilon}\,'\times\hat{k})\,\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k}')\,\vec{\epsilon}\,'\cdot\hat{k}\right] + i\,A_6(\omega,\theta)\,\vec{\sigma}\cdot\left[(\vec{\epsilon}\,'\times\hat{k}')\,\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k})\,\vec{\epsilon}\,'\cdot\hat{k}\right]$$

$$\begin{aligned}
A_1(\omega,\theta) &= -Z^2\frac{e^2}{M_N} + \frac{e^2}{4M_N^3}\left(\mu^2(1+\cos\theta) - Z^2\right)(1-\cos\theta)\,\omega^2 + 4\pi(\alpha+\beta\,\cos\theta)\omega^2 + \mathcal{O}(\omega^4)\,,\\
A_2(\omega,\theta) &= \frac{e^2}{4M_N^3}(\mu^2 - Z^2)\omega^2\cos\theta - 4\pi\beta\omega^2 + \mathcal{O}(\omega^4)\,,\\
A_3(\omega,\theta) &= \frac{e^2\omega}{2M_N^2}\left(Z(2\mu - Z) - \mu^2\cos\theta\right) + 4\pi\omega^3(\gamma_1 - (\gamma_2 + 2\gamma_4)\,\cos\theta) + \mathcal{O}(\omega^5)\,,\\
A_4(\omega,\theta) &= -\frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_2 + \mathcal{O}(\omega^5)\,,\\
A_5(\omega,\theta) &= \frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_4 + \mathcal{O}(\omega^5)\,,\\
A_6(\omega,\theta) &= -\frac{e^2\omega}{2M_N^2}Z\mu + 4\pi\omega^3\gamma_3 + \mathcal{O}(\omega^5)\,,
\end{aligned}$$

polarizabilities: α, β, γ_1 , γ_2 , γ_3 , γ_4

Experimental Information on Polarizabilities

- Proton electric polarizability (α_p) is around 12 in units of 10^{-4} fm^3.
- Proton magnetic polarizability (β_p) is around 2 in units of 10^{-4} fm^3.
- α_n is about the same as α_p
- β_n is about the same as β_p
- Experiments are under way or planned for other polarizabilities at electron accelerators around the world

Polarizabilities from ChPT

	CHPT (leading order)		Δ-pole	Dispersion Relations				
				A_2^{as}	excitations + A_1^{as}			
					proton		neutron	
	π^0	loop			HDT	SAID	HDT	SAID
α_E		12.3				11.9[a,c]		13.3[b,c]
β_M		1.2	12.0			1.9[a,c]		1.8[b,c]
$(10^{-4}\,\mathrm{fm}^3)$								
$\alpha_{E\nu}$		2.2				-3.8		-2.4
$\beta_{M\nu}$		3.5	5.3			9.1		9.2
α_{E2}		20.7	0.2			27.5		27.2
β_{M2}		-8.9				-22.4		-23.5
$(10^{-4}\,\mathrm{fm}^5)$								
γ_{E1}	$11.3\tau_3$	-5.5		$11.2\tau_3$	-4.5	-3.4	-5.5	-5.6
γ_{M1}	$-11.3\tau_3$	-1.1	4.0	$-11.2\tau_3$	3.4	2.7	3.4	3.8
γ_{E2}	$-11.3\tau_3$	1.1	0.75	$-11.2\tau_3$	2.3	1.9	2.6	2.9
γ_{M2}	$11.3\tau_3$	1.1		$11.2\tau_3$	-0.6	0.3	-0.6	-0.7
γ		4.4	-4.8		-0.6	-1.5	-0.2	-0.4
γ_π	$-45.3\tau_3$	4.4	4.8	$-45.0\tau_3$	10.8	7.8[d]	12.1	13.0
$(10^{-4}\,\mathrm{fm}^4)$								

Babusci et al, PRC58, 1013 (1998)

Polarizabilities on the Lattice

Measure mass shifts in progressively-small external electric and magnetic fields, specially designed to isolate them:

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$
$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$
$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$
$$-\frac{1}{12}\alpha_{E2}E_{ij}^2 - \frac{1}{12}\beta_{M2}E_{ij}^2 + \cdots$$

Small field expansion:

$$\delta m(B) = m(B) - m(0) = c_1 B + c_2 B^2 + c_3 B^3 + c_4 B^4 + \cdots$$

Introduction of an external electromagnetic field on the lattice

- Minimal coupling in the QCD covariant derivative in Euclidean space

$$D_\mu \to \partial_\mu + gG_\mu + qA_\mu$$

- Recall that SU(3) gauge field is introduced by the link variables

$$U_\mu(x) = \exp(iagG_\mu)$$

- It suggests multiplying a U(1) phase factor to the links

$$U'_\mu(x) = \exp(iaqA_\mu)U_\mu$$

- This should be done in two places where the Dirac operator appears: both in the dynamical gauge generation and quark propagator generation

For Example

$$\vec{B} = \nabla \times \vec{A}$$

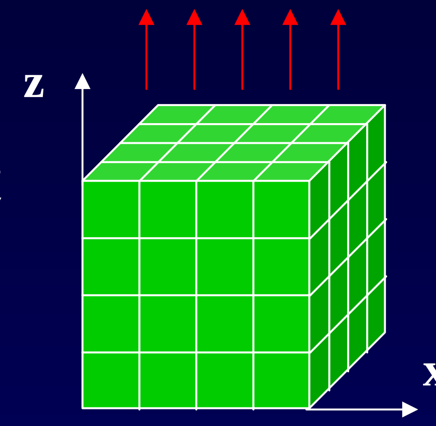
$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

- To apply magnetic field **B** in the z-direction, one can choose the 4-vector potential

$$A_\mu \equiv (\phi, \vec{A}) = (0, 0, Bx, 0)$$

then the y-link is multiplied by a x-dependent phase factor

$$U_y \to \exp(iqaBx) U_y$$



- To apply electric field **E** in the x-direction, one can choose the 4-vector potential

$$A_\mu = (0, Et, 0, 0)$$

then the x-link is multiplied by a t-dependent phase factor

$$U_x \to \exp(iqaEt) U_x$$

Background Field Method

Wilson action, 6^3x10 lattice L=1 fm, pion mass about 1400 and 900 MeV, 9 configurations. done on a VAX

- "Lattice quantum-chromodynamics calculation of some baryon magnetic moments", Bernard, Draper, Olynyk, PRL49 (1982) 1076; NPB220 (1983) 508
- "Dependence of lattice hadron masses on external magnetic fields", Rubinstein, Solomon and Wittlich, NPB457 (1995) 577
- "A study of hadron electric polarizability in quenched lattice QCD", Fiebig, Wilcox, Woloshyn, NPB324, 47 (1989)
- "Electric Polarizability of Neutral Hadrons from Lattice QCD", Christensen, Wilcox, Lee, Zhou, PRD72, 034503 (2005)
- "Baryon magnetic moments in the background field method", Lee, Kelly, Zhou, Wilcox, PLB627, 71 (2005)
- "Magnetic polarizability of hadrons from lattice QCD in the background field method", Lee, Zhou, Wilcox, Christensen, PRD73, 034503 (2006)
- "Electricmagnetic and spin polarizabilitites in lattice QCD", Detmold, Tiburzi, Walker-Loud, PRD73 (2006) 114505
- "Neutron electric dipole moment with external electric field method in lattice QCD", Shintani et al, CP-PACS collaboration, PRD75, 034507 (2007)

Computational Demands

$$U_y \to \exp(iqaBx)U_y$$

- Consider quark propagator generation

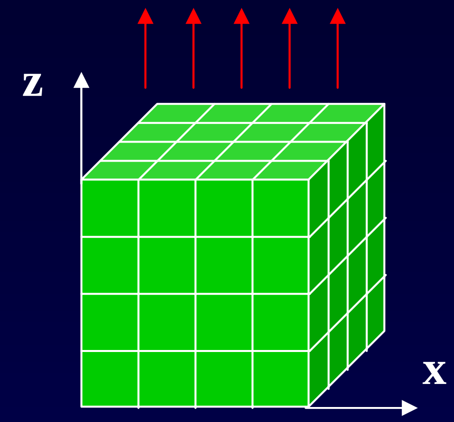
$$\frac{\int DG_\mu \det(\not{D} + m_q)\, e^{-S_G} (\not{D} + m_q)^{-1}}{\int DG_\mu \det(\not{D} + m_q)\ e^{-S_G}}$$

$$D_\mu \to \partial_\mu + gG_\mu + qA_\mu$$

- For each value of external field, a new dynamical ensemble is needed that couple to u-quark (q=1/3) and d- and s-quark (q=-2/3).
  - quark propagator is then computed on the ensembles with matching values
- Cost is much reduced if no field is applied in the vacuum: any gauge ensemble can be used to compute valence quark propagators.

Lattice details

$U_y \rightarrow \exp(iqaBx)U_y$



- Standard Wilson gauge action
  - 24^4 lattice, β=6.0 (or a ≈ 0.1 fm)
  - 150 configurations
- Standard Wilson fermion action
  - κ=0.1515, 0.1525, 0.1535, 0.1540, 0.1545, 0.1555
  - Pion mass about 1015, 908, 794, 732, 667, 522 MeV
  - Strange quark mass corresponds to κ=0.1535 (or m_π~794 MeV)
  - Fermion boundary conditions: periodic in y and z, fixed in x and t
  - Source location (t,x,y,z)=(2,12,1,1)
- The following 5 dimensionless numbers $\eta \equiv qBa^2$ =+0.00036, -0.00072, +0.00144, -0.00288, +0.00576 correspond to 4 small B fields

eBa^2 = -0.00108, 0.00216, -0.00432, 0.00864 for both u and d (or s) quarks.

  - Small in the sense that the mass shift is only a fraction of the proton mass: μB/m ~ 1 to 5% at the smallest pion mass. In physical units, B ~ 10^{13} Tesla.

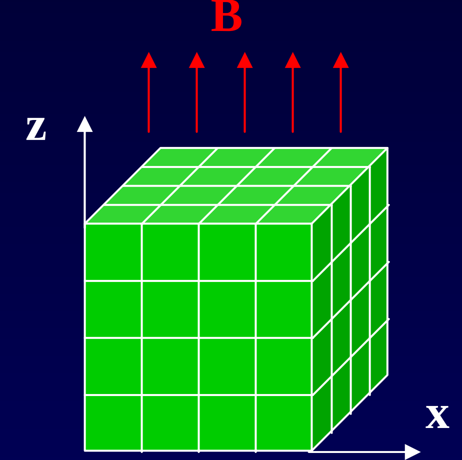
What about boundary conditions?

$$U_y^B = \exp(iqa^2 Bx)$$

- On a finite lattice with periodic boundary conditions, to get a constant magnetic field, B has to be quantized by

$$qBa^2 = \frac{2\pi n}{N_x}, \quad n = 1,2,3,\cdots$$

to ensure that the magnetic flux through plaquettes in the x-y plane is constant.



- But, for N_x=24 and 1/a=2 GeV, the lowest allowed field would give the proton a mass shift of about 500 MeV, which is unacceptably large (proton is severely distorted). So we have to abandon the quantization condition, and work with much smaller fields.
- To minimize the boundary effects, we work with fixed b.c. in x-direction, so that quarks originating in the middle of the lattice has little chance of propagating to the edge.

A computational trick

- We generate two sets of quark propagators, one with the original set of fields, the other with the fields reversed.
- The mass shift in the presence of small fields is

$$\boxed{\delta m(B) = m(B) - m(0) = c_1 B + c_2 B^2 + c_3 B^3 + c_4 B^4 + \cdots}$$

- At the cost of a factor of two,
  - by taking the average, $[\delta m(B) + \delta m(-B)]/2$, we get the leading quadratic response with the odd-powered terms eliminated. (magnetic polarizability)
  - by taking the difference, $[\delta m(B) - \delta\, m(-B)]/2$, we get the leading linear response with the even-powered terms eliminated. (magnetic moment)
- Our calculation is equivalent to 11 mass spectrum calculations.
  - 5 original fields, 5 reversed, plus the zero-field to set the baseline

Magnetic Moments

New Topic

Magnetic moment

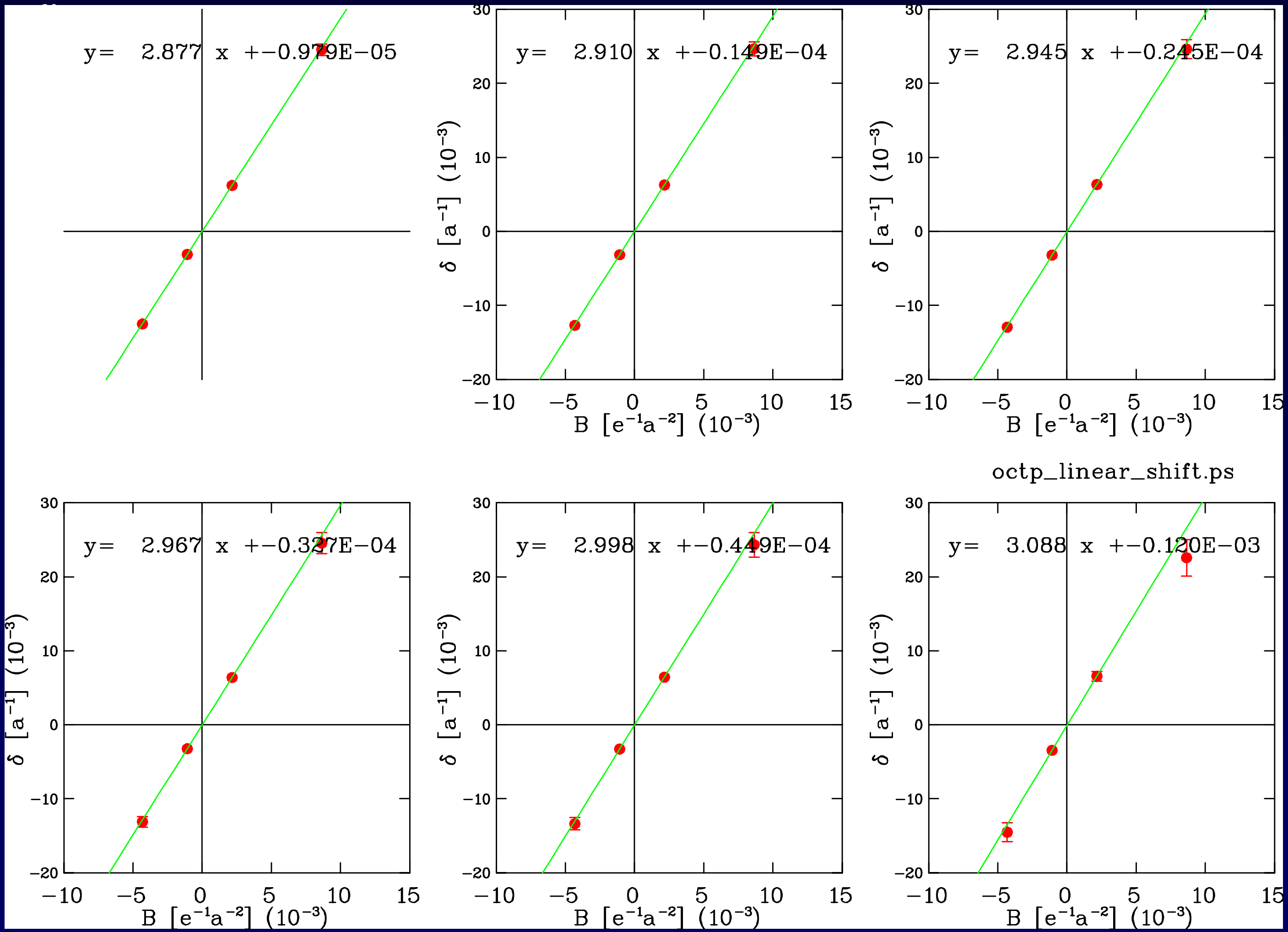
- For a Dirac particle of spin s in small fields, $E_{\pm} = m \pm \mu B$ where upper sign means spin-up and lower sign spin-down, and $\mu = g\frac{e}{2m}s$
- g factor is extracted from

$$\boldsymbol{g = m\frac{(E_{+} - m) - (E_{-} - m)}{eBs}}$$

- Look for the slope (g-factor) on the straight line of the form $\boldsymbol{\delta m = g(eB)}$

Proton mass shifts

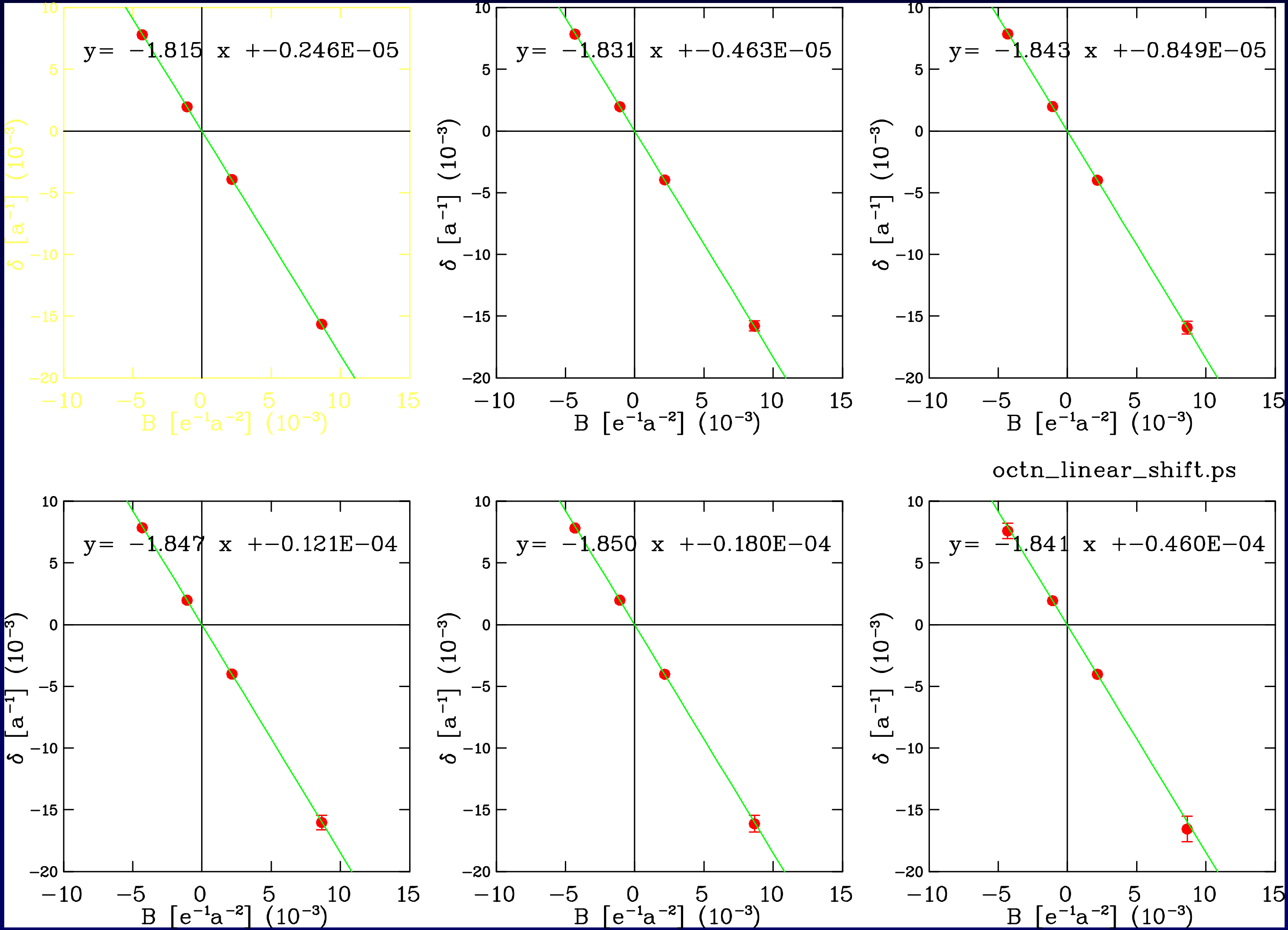
$\delta m = g(eB$



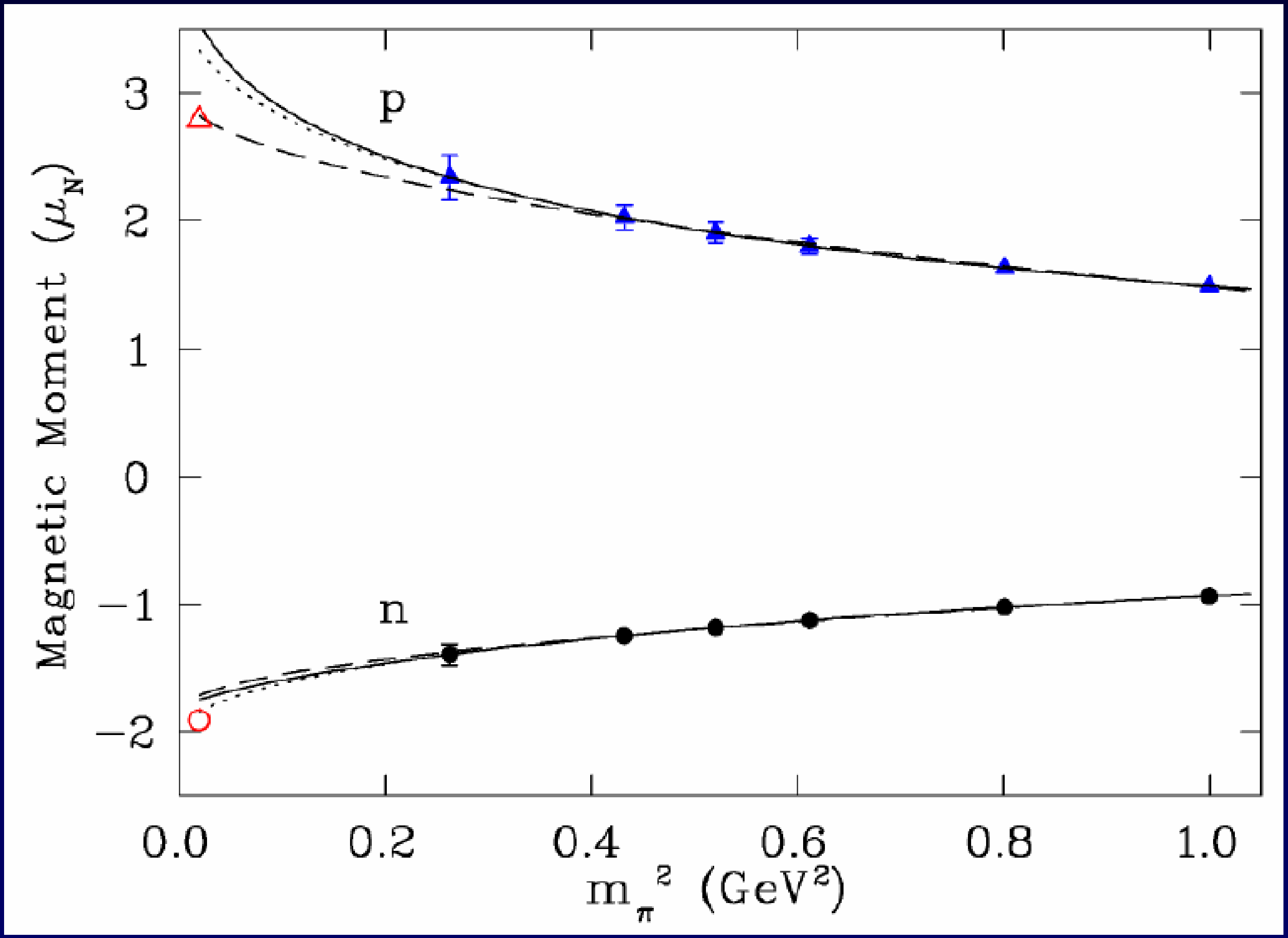
- We use the 2 smallest fields to fit the line.

$\delta m = g(eB)$

Neutron mass shifts



Proton and neutron magnetic moments



Chiral Extrapolation

- To one meson loop, χPT predicts

$$\mu = \mu_0 + c_1 m_\pi + c_2 m_\pi^2 \log m_\pi^2 + c_3 m_\pi^2 + \cdots$$

but only applicable in small mass region.

- Encapsulating form (Leinweber, Lu and Thomas, PRD60 (1999) 034014)

$$\mu = \frac{\mu_0}{1 + \alpha m_\pi + \beta m_\pi^2}$$

Pade ansatz

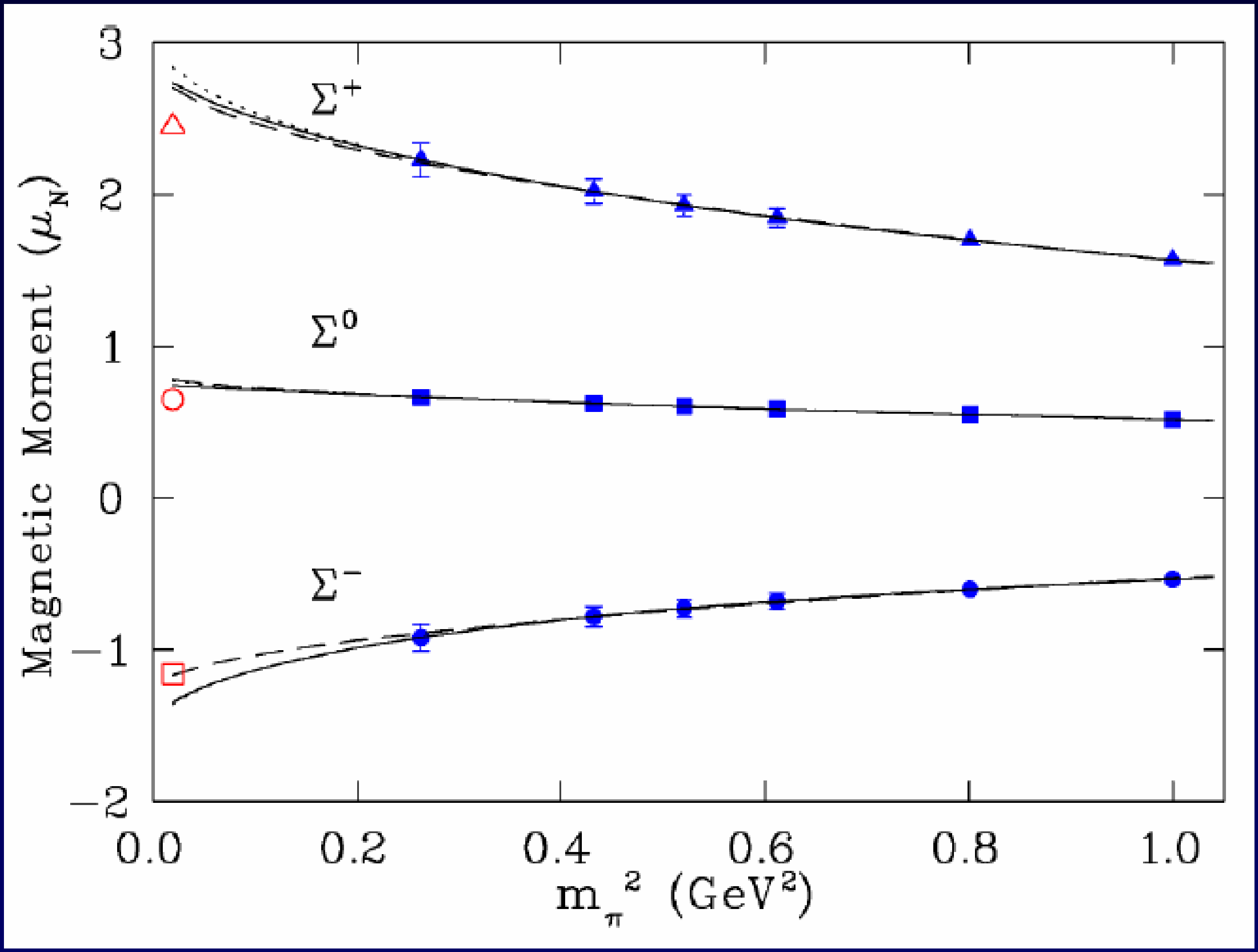
- For small mass,

$$\mu = \mu_0 [1 - \alpha m_\pi + (\alpha^2 - \beta) m_\pi^2 + \cdots]$$

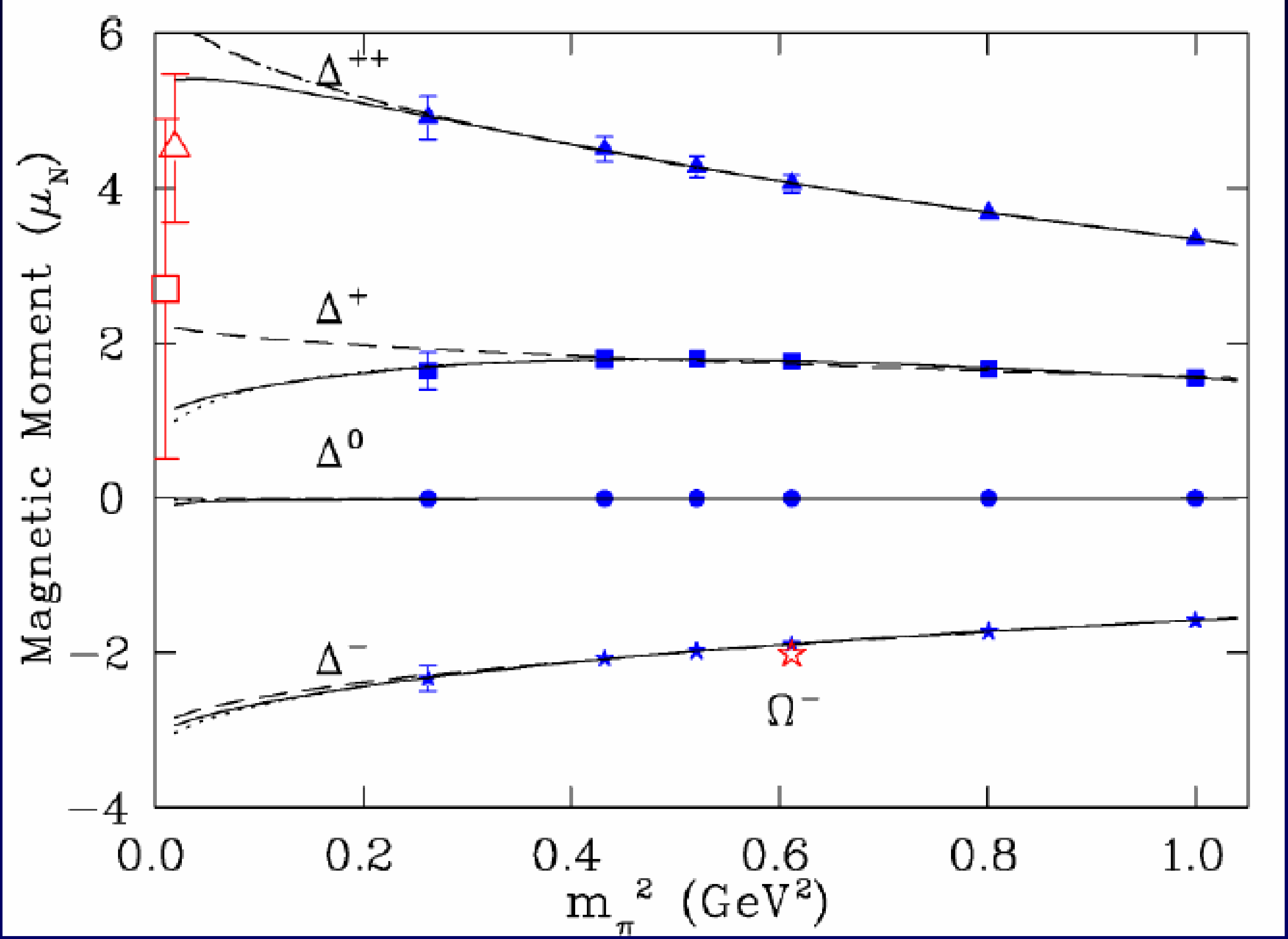
- For large mass,

$$\mu = \frac{\mu_0}{\beta m_\pi^2}\left(1 - \frac{\alpha}{\beta m_\pi} + \cdots\right) \propto \frac{1}{m_q}$$

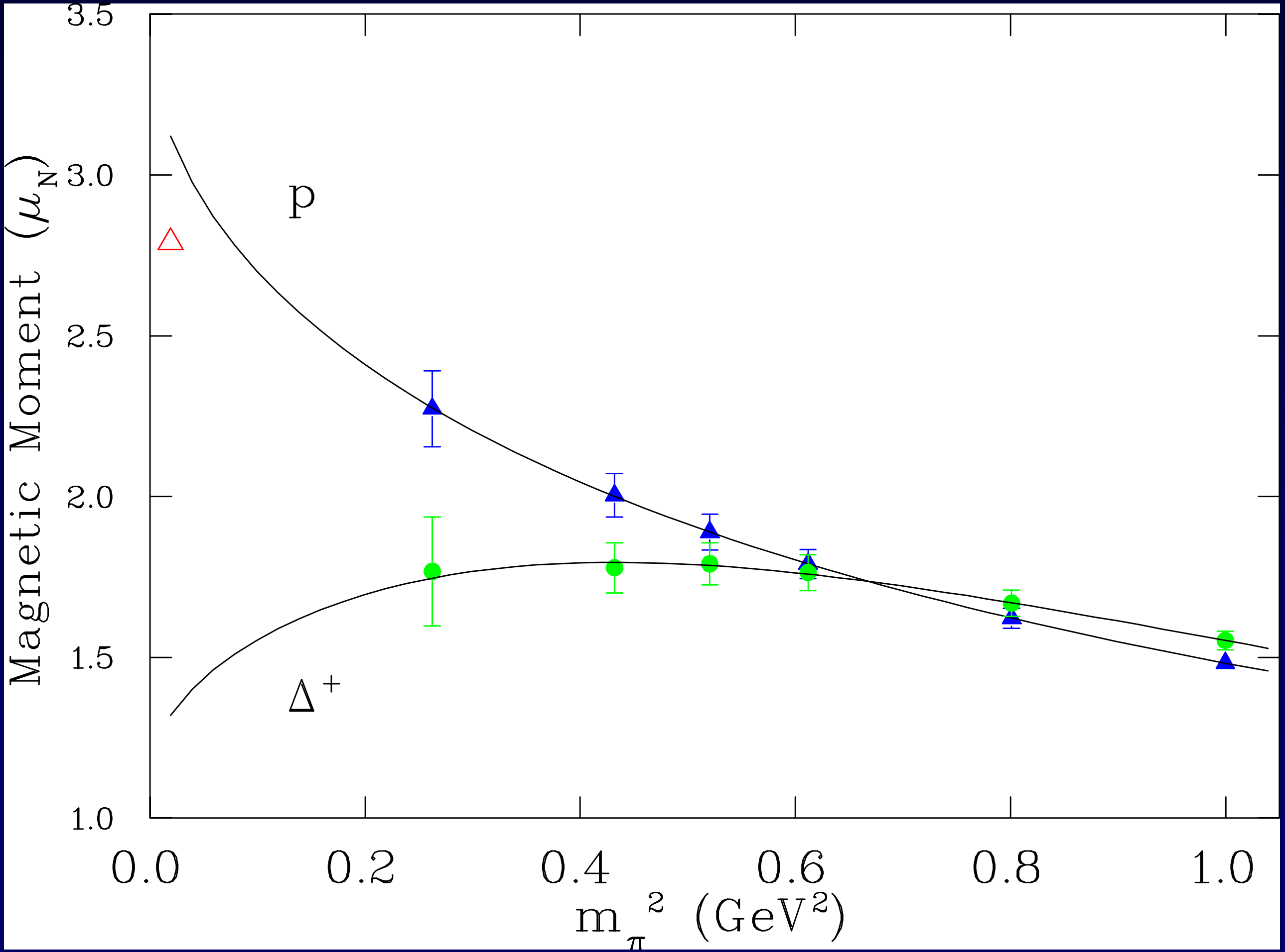
Octet Sigma magnetic moments



Delta magnetic moments



Proton and Δ^+ magnetic moments



Curvatures expected from ChPT.

Magnetic moments

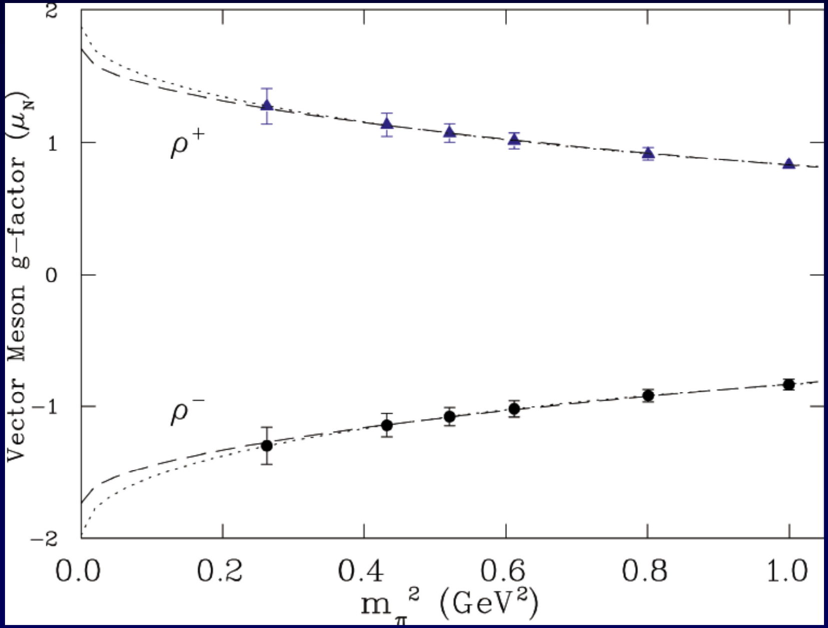
Table 1

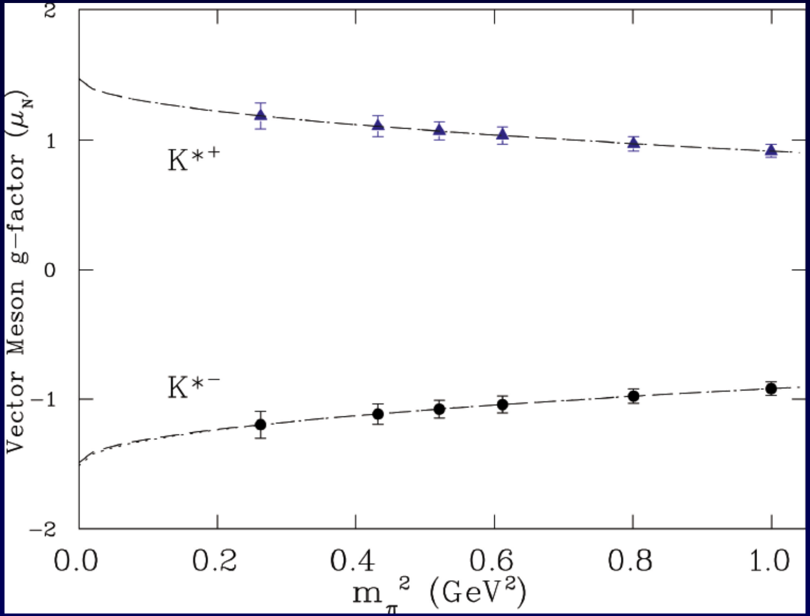
The computed magnetic moments for the baryon octet and decuplet in nuclear magnetons (μ_N) as a function of the pion mass. The extrapolated values are based on Eq (10). The experimental values are taken from the PDG [16]

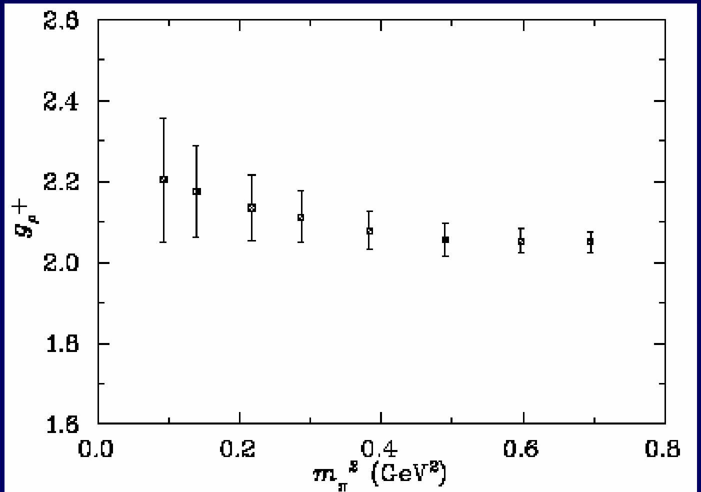
κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	Extrap.	Expt.
m_π (GeV)	1.015	0.908	0.794	0.732	0.667	0.522		
p	1.48(3)	1.63(4)	1.80(6)	1.90(8)	2.02(10)	2.34(17)	3.04(6)	2.79
n	−0.94(2)	−1.02(2)	−1.13(3)	−1.18(4)	−1.25(5)	−1.39(8)	−1.84(3)	−1.91
Σ^+	1.57(4)	1.70(5)	1.85(6)	1.93(7)	2.02(8)	2.23(11)	2.87(3)	2.45
Σ^0	0.52(1)	0.55(1)	0.59(2)	0.61(2)	0.63(2)	0.67(3)	0.76(1)	0.65
Σ^-	−0.54(4)	−0.60(4)	−0.68(6)	−0.73(6)	−0.78(7)	−0.92(9)	−1.48(5)	−1.16
Ξ^0	−1.07(4)	−1.10(4)	−1.13(4)	−1.15(4)	−1.17(5)	−1.21(7)	−1.37(1)	−1.25
Ξ^-	−0.76(6)	−0.77(7)	−0.77(8)	−0.77(8)	−0.77(9)	−0.78(11)	−0.82(1)	−0.65
Λ	−0.59(2)	−0.60(2)	−0.60(2)	−0.61(2)	−0.61(2)	−0.62(3)	−0.70(1)	−0.61
Δ^{++}	3.35(7)	3.68(9)	4.06(12)	4.28(14)	4.50(16)	4.92(28)	5.24(18)	4.52(1.00)
Δ^+	1.55(4)	1.67(6)	1.77(8)	1.80(10)	1.80(11)	1.64(24)	0.97(8)	
Δ^0	−0.002(0)	−0.003(0)	−0.004(1)	−0.005(1)	−0.007(1)	−0.011(6)	−0.035(2)	
Δ^-	−1.58(4)	−1.73(5)	−1.89(7)	−1.98(8)	−2.07(10)	−2.34(17)	−2.98(19)	
$\Sigma*^+$	1.40(5)	1.56(6)	1.72(8)	1.80(10)	1.86(11)	1.88(17)	1.27(6)	
$\Sigma*^0$	−0.13(1)	−0.09(1)	−0.03(0)	−0.01(0)	0.03(0)	0.08(2)	0.33(5)	
$\Sigma*^-$	−1.68(5)	−1.76(7)	−1.83(9)	−1.87(10)	−1.89(11)	−1.93(16)	−1.88(4)	
$\Xi*^0$	−0.09(2)	−0.06(1)	−0.022(5)	−0.002(1)	0.018(6)	0.05(3)	0.16(4)	
$\Xi*^-$	−0.59(7)	−0.61(8)	−0.62(9)	−0.62(10)	−0.63(10)	−0.63(11)	−0.62(1)	

F.X. Lee, R. Kelly, L. Zhou, W. Wilcox, Phys. Lett. B 627, 71 (2005)

Our new results on vector meson g factors (preliminary)







$$<\rho\,|\,J_\mu\,|\,\rho>\sim\{G_C(q^2),G_M(q^2),G_Q(q^2)\}$$

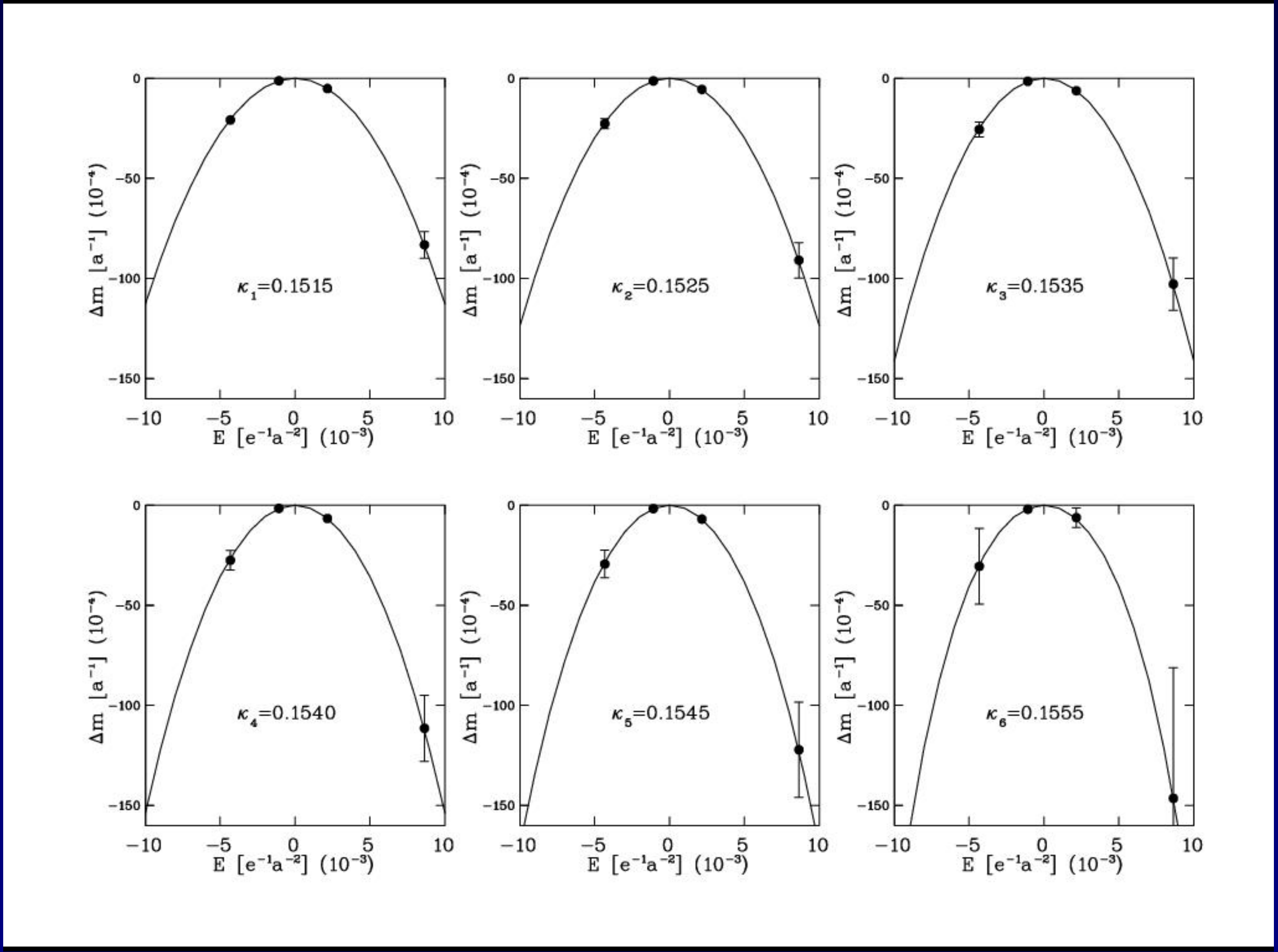
hep-lat/0703014, Adelaide group

Polarizabilities

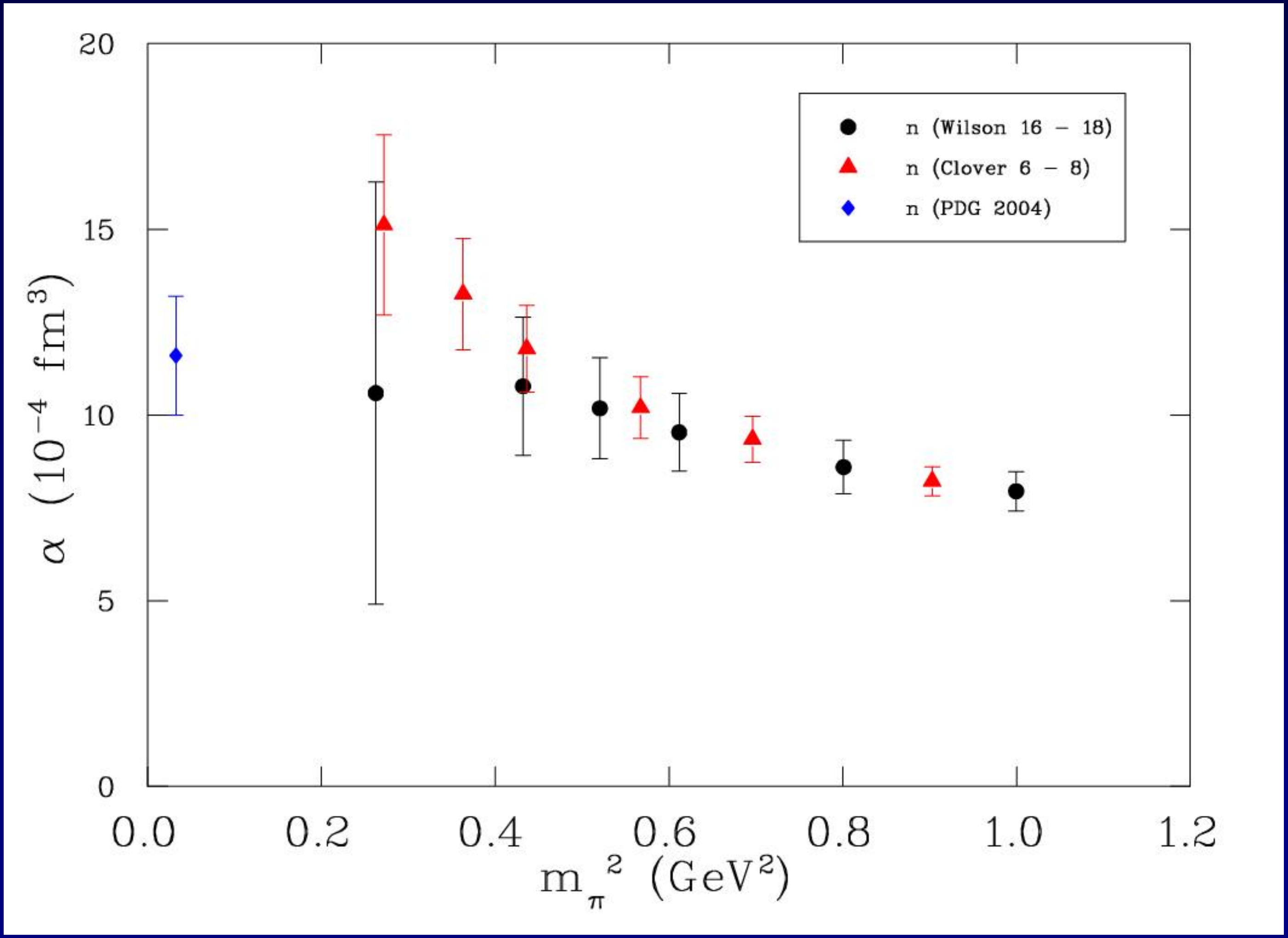
New Topic

Neutron Mass Shift in Electric Field

$$\Delta m = -\frac{1}{2}\alpha E^2$$



Electric Polarizability of neutron



Electric Polarizabilities of Neutral Particles

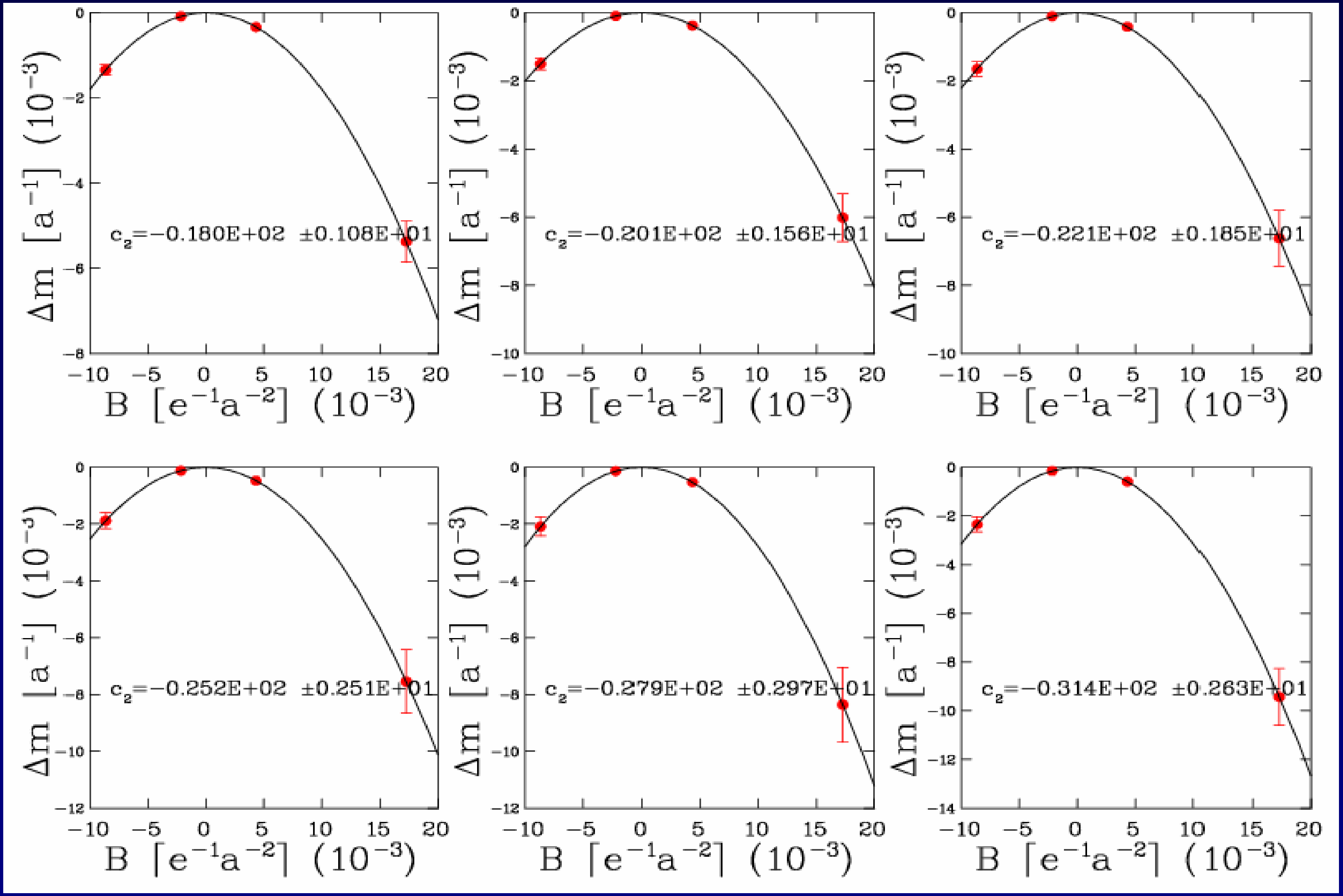
TABLE II: The electric polarizabilities from the calculation with Wilson action using six κ values. The units of the electric polarizability are 10^{-4} fm^3. The pion masses were fit on time steps $11-13$ from the propagator origin and are given in GeV.

Christensen, Wilcox, Lee, Zhou, Phys.Rev. D72 (2005) 034503

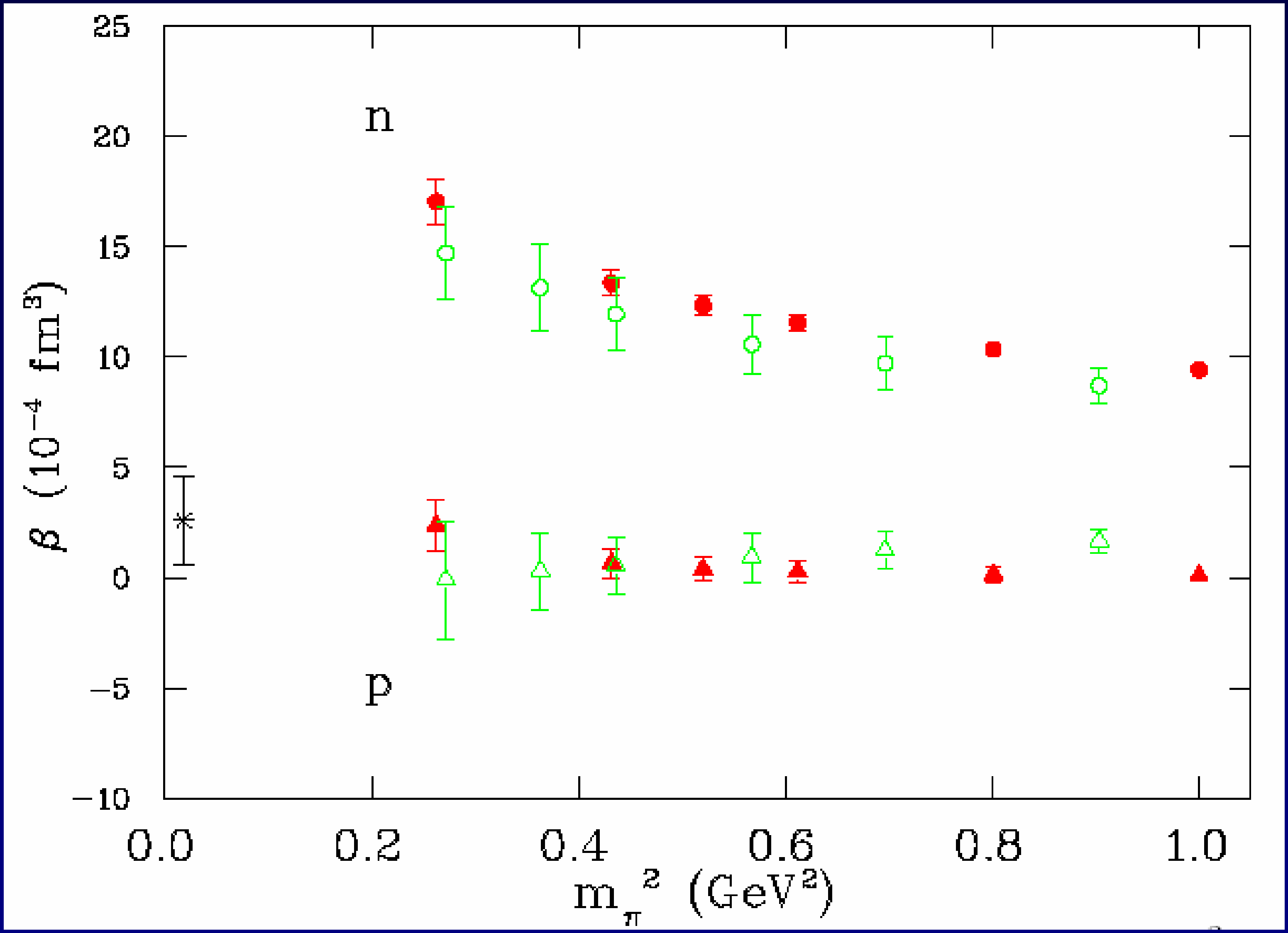
κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	
m_π	$1.000 \pm .005$	$0.895 \pm .006$	$0.782 \pm .006$	$0.721 \pm .006$	$0.657 \pm .007$	$0.512 \pm .007$	
Meson							fit range
ρ^0	5.0 ± 0.5	4.8 ± 0.7	4.4 ± 0.9	4.0 ± 1.2	3.5 ± 1.6	2.8 ± 4.8	13-15
K^{*0}	1.2 ± 0.2	1.2 ± 0.2	1.3 ± 0.3	1.3 ± 0.3	1.3 ± 0.4	1.2 ± 0.6	10-12
Baryon octet							
n	7.9 ± 0.5	8.6 ± 0.7	9.5 ± 1.0	10.2 ± 1.4	10.8 ± 1.9	10.6 ± 5.7	14-16
Σ^0	7.7 ± 0.7	8.1 ± 0.9	8.8 ± 1.3	9.3 ± 1.5	10.0 ± 1.9	11.5 ± 4.0	14-16
Λ^0_o	8.2 ± 0.7	8.8 ± 0.8	9.7 ± 1.1	10.2 ± 1.2	10.8 ± 1.6	13.2 ± 3.2	14-16
Ξ^0	9.3 ± 0.9	9.7 ± 1.0	10.0 ± 1.2	10.2 ± 1.4	10.3 ± 1.6	10.1 ± 2.3	14-16
Baryon decuplet							
Δ^0	1.7 ± 0.1	1.6 ± 0.2	1.5 ± 0.3	1.5 ± 0.4	1.6 ± 0.6	2.3 ± 1.2	9-11
Σ^{*0}	1.7 ± 0.2	1.6 ± 0.2	1.5 ± 0.4	1.5 ± 0.4	1.6 ± 0.5	2.0 ± 0.9	9-11
Ξ^{*0}	1.7 ± 0.2	1.6 ± 0.3	1.5 ± 0.4	1.5 ± 0.4	1.5 ± 0.5	1.6 ± 0.8	9-11

Neutron Mass Shifts in Magnetic Field

$$\Delta m = -\frac{1}{2}\beta B^2$$



Magnetic Polarizability of the Nucleon



Magnetic Polarizabilities: baryon octet

TABLE III. The calculated magnetic polarizabilities for the octet baryons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
p	0.09 ± 0.29	0.14 ± 0.37	0.26 ± 0.48	0.40 ± 0.56	0.64 ± 0.67	2.36 ± 1.20	12-14
n	9.4 ± 0.2	10.4 ± 0.3	11.6 ± 0.4	12.3 ± 0.5	13.4 ± 0.6	17.0 ± 1.1	12-14
Σ^+	-0.15 ± 0.36	0.09 ± 0.42	0.24 ± 0.50	0.40 ± 0.56	0.61 ± 0.64	1.60 ± 1.00	12-14
Σ^0	8.0 ± 0.3	8.5 ± 0.3	9.1 ± 0.4	9.6 ± 0.5	10.1 ± 0.6	11.9 ± 0.9	12-14
Σ^-	-11.8 ± 0.3	-12.6 ± 0.3	-13.6 ± 0.4	-14.2 ± 0.4	-14.7 ± 0.4	-16.1 ± 0.5	12-14
Ξ^0	10.7 ± 0.3	11.3 ± 0.4	11.9 ± 0.4	12.3 ± 0.4	12.8 ± 0.5	13.9 ± 0.7	12-14
Ξ^-	-12.6 ± 0.3	-13.1 ± 0.3	-13.8 ± 0.4	-14.1 ± 0.4	-14.6 ± 0.4	-15.6 ± 0.5	12-14
Λ^8	9.1 ± 0.4	9.9 ± 0.5	10.8 ± 0.6	11.4 ± 0.7	12.1 ± 0.8	14.0 ± 1.2	12-14
Λ^C	9.3 ± 0.4	10.2 ± 0.5	11.2 ± 0.6	11.9 ± 0.7	12.6 ± 0.9	15.0 ± 1.4	12-14
Λ^S	3.6 ± 0.1	3.4 ± 0.2	3.3 ± 0.2	3.3 ± 0.3	3.2 ± 0.4	3.2 ± 1.0	5-7

F.X. Lee, L. Zhou, W. Wilcox, J. Christensen, Phys. Rev. D73 (2006) 034503

Magnetic Polarizabilities: baryon decuplet

TABLE IV. The calculated magnetic polarizabilities for the decuplet baryons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
Δ^{++}	-39.9 ± 0.7	-43.9 ± 0.8	-48.7 ± 1.0	-51.5 ± 1.1	-54.7 ± 1.3	-63.1 ± 1.9	10-12
Δ^{+}	-2.5 ± 0.2	-3.0 ± 0.3	-3.5 ± 0.5	-3.9 ± 0.6	-4.3 ± 0.7	-5.1 ± 1.1	10-12
Δ^{0}	7.6 ± 0.2	8.2 ± 0.3	8.8 ± 0.4	9.2 ± 0.5	9.6 ± 0.6	10.9± 1.0	10-12
Δ^{-}	-10.1 ± 0.2	-11.1 ± 0.2	-12.4 ± 0.2	-13.1 ± 0.3	-14.0 ± 0.3	-16.2 ± 0.5	10-12
Σ^{*+}	-2.9 ± 0.3	-3.2 ± 0.4	-3.6 ± 0.5	-3.9 ± 0.6	-4.2 ± 0.6	-5.1 ± 0.9	10-12
Σ^{*0}	7.9 ± 0.2	8.4 ± 0.3	8.9 ± 0.4	9.2 ± 0.5	9.5 ± 0.6	10.3 ± 0.8	10-12
Σ^{*-}	-10.9 ± 0.2	-11.7 ± 0.2	-12.6 ± 0.3	-13.1 ± 0.3	-13.7 ± 0.3	-15.0 ± 0.4	10-12
Ξ^{*0}	8.1 ± 0.3	8.5 ± 0.3	9.0 ± 0.4	9.2 ± 0.5	9.4 ± 0.5	9.8 ± 0.7	10-12
Ξ^{*-}	-11.9 ± 0.2	-12.4 ± 0.2	-12.8 ± 0.3	-13.1 ± 0.3	-13.4 ± 0.3	-14.0 ± 0.3	10-12
Ω^{-}			-12.4 ± 0.2				10-12

PRD73 (2006) 034503

Magnetic Polarizabilities: mesons

TABLE V. The calculated magnetic polarizabilities for the selected mesons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
$\pi^\pm$	-16.0 $\pm$ 0.3	-17.8 $\pm$ 0.4	-19.8 $\pm$ 0.5	-21.1 $\pm$ 0.5	-22.5 $\pm$ 0.6	-26.4 $\pm$ 0.8	16-18
π^0	8.4 $\pm$ 0.2	9.1 $\pm$ 0.3	10.1 $\pm$ 0.4	10.8 $\pm$ 0.5	11.6 $\pm$ 0.5	14.1 $\pm$ 0.8	16-18
$K^\pm$	-18.4 $\pm$ 0.4	-19.4 $\pm$ 0.4	-20.5 $\pm$ 0.5	-21.1 $\pm$ 0.5	-21.7 $\pm$ 0.6	-23.1 $\pm$ 0.6	16-18
K^0	3.7 $\pm$ 0.1	3.8 $\pm$ 0.2	4.0 $\pm$ 0.2	4.1 $\pm$ 0.2	4.3 $\pm$ 0.2	4.7 $\pm$ 0.3	16-18
$\rho^\pm$	-11.8 $\pm$ 0.3	-12.5 $\pm$ 0.3	-13.1 $\pm$ 0.4	-13.3 $\pm$ 0.5	-13.3 $\pm$ 0.6	-12.5 $\pm$ 1.2	14-16
ρ^0	5.3 $\pm$ 0.2	5.6 $\pm$ 0.2	5.9 $\pm$ 0.3	6.0 $\pm$ 0.4	6.1 $\pm$ 0.4	6.5 $\pm$ 0.7	9-11
$K^{*\pm}$	-13.0 $\pm$ 0.3	-13.2 $\pm$ 0.4	-13.3 $\pm$ 0.5	-13.3 $\pm$ 0.5	-13.2 $\pm$ 0.6	-12.7 $\pm$ 0.8	14-16
K^{*0}	3.0 $\pm$ 0.2	3.2 $\pm$ 0.2	3.3 $\pm$ 0.3	3.5 $\pm$ 0.3	3.6 $\pm$ 0.3	4.1 $\pm$ 0.5	14-16

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What's Next ?

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$
$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$
$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$
$$-\frac{1}{12}\alpha_{E2} E_{ij}^2 - \frac{1}{12}\beta_{M2} E_{ij}^2 + \cdots$$

- Compute higher-order polarizabilities
  - Need non-uniform, sourceless fields
  - For example, to extract α_{E2} and γ_{E2}, choose

$$A_\mu = (0, -cxt, 0, -czt), \quad \vec{E} = (cz, 0, cx)$$

  - For example, to extract β_{M2} and γ_{M2}, choose

$$A_\mu = (0, -bxy, 0, bzy), \quad \vec{B} = (bz, 0, bx)$$

  - α and β must be re-measured and subtracted

- The path to unquenched calculations
  - Use CP-PACS 2+1 flavor dynamical gauge ensembles (Iwasaki glue + clover). But still U(1) quenched
  - Introduce U(1) fields in the dynamical gauge generation

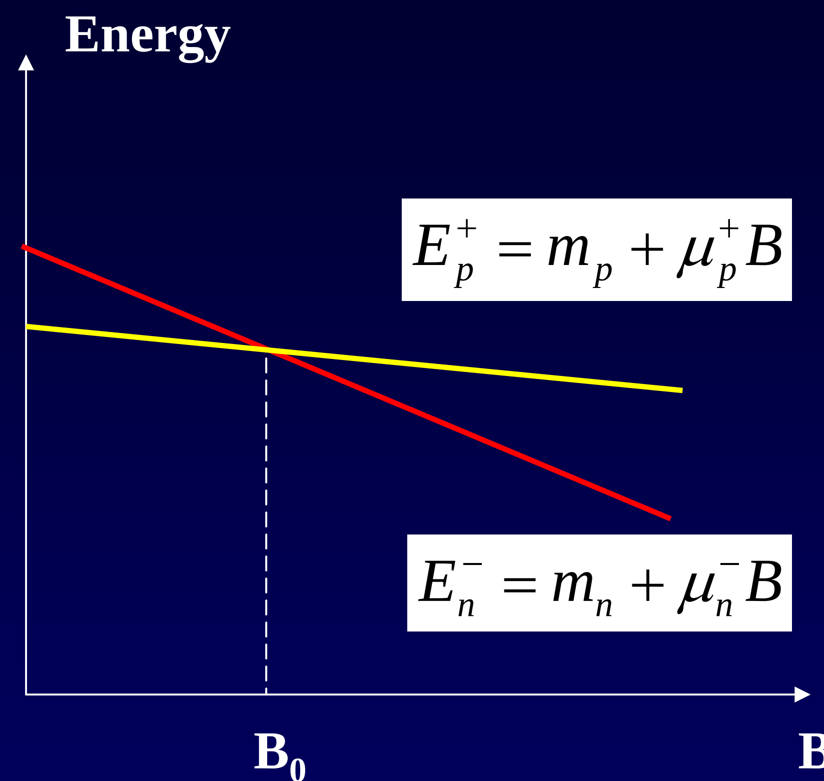
Conclusion

- The background field method in lattice QCD is a viable way of probing hadron internal structure
  - Magnetic moments
  - Electric and magnetic polarizabilities
  - Neutron electric dipole moment
  - Proton beta-decay
  - and more
- A nice complement to experiments

Beta-decay of proton in magnetic field

- At sufficiently large B fields (**10^{16} Tesla**), proton can become heavier than neutron, allowing the 'β-decay' of the proton:

$$p \rightarrow n + e^{+} + \nu_e$$



- As compared to the natural neutron β-decay:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Such process can take place in stars where extremely strong magnetic field exists.