Remarks on the QCD Vacuum Structure

(at Keh-Fei fest)

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All Corners of Physics are not Equal...

- Developed Subject vs Undeveloped Subject
  - well-defined goals
  - relevant merits
  - standard methods
  - standard language
- Live Subject vs Dead Subject

If at any point of time the four items above were accepted by most practitioners (or highly contested by most practitioners) the subject lived at that time

When I came to UK I considered “QCD vacuum structure” (at least in its lattice guise) an <u>undeveloped</u> subject that perhaps <u>half-lived</u> some time ago…

The Most Prominent Issue...

VACUUM ⟺ STATISTICAL ENSEMBLE OF CONFIGURATIONS

UNDERLYING ASSUMPTION of path-integral approach to vacuum structure:

The statistical sum is dominated by a specific kind of configurations with high degree of space-time order (typical configurations)!

$$\langle \Omega \rangle = \sum_A P(A)\, \Omega(A)$$

⟹ VACUUM STRUCTURE is associated with SPACE-TIME STRUCTURE in typical configurations.

BUT THERE WAS NO SIGN OF IT ANYWHERE IN EQUILIBRIUM CONFIGURATIONS OF LATTICE-REGULARIZED THEORY!

The Story of the Double-Sheet Structure

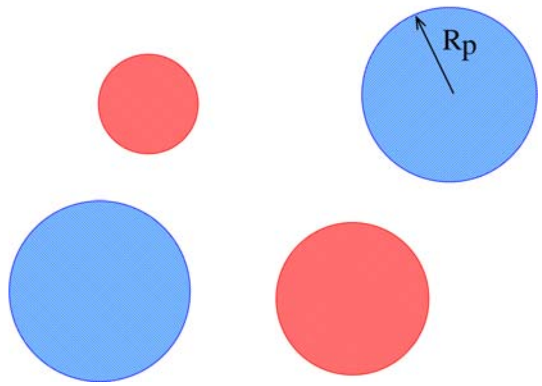
I. E-mail from Tamas Kovacs – Fall 2001
(telling me about the negativity of the topological density correlator)

$$\langle\, q(x)q(0)\,\rangle \;\leq\; 0, \quad |x|>0$$

Seiler & Stamatescu, 1987

Two effects:

(1) Changed my way of thinking about instantons



Vacuum cannot be dominated by sign-coherent lumps at all scales (instantons at best an approximation for low-energy behavior)

The Story of the Double-Sheet Structure...

(2) Assuming the existence of observable order in typical configurations, how could this constrain be satisfied?

If charge organizes on a lower-dimensional sign-coherent objects with objects of opposite sign highly spatially correlated with one another.

The Story of the Double-Sheet Structure...

II. The availability of new topological density operator based on chirally symmetric Dirac kernel

$$q(x) \equiv \frac{1}{2} \mathrm{tr}\, \gamma_5 D(x,x) = -\frac{1}{2} \mathrm{tr}\, \gamma_5 \left(1 - D(x,x)\right)$$

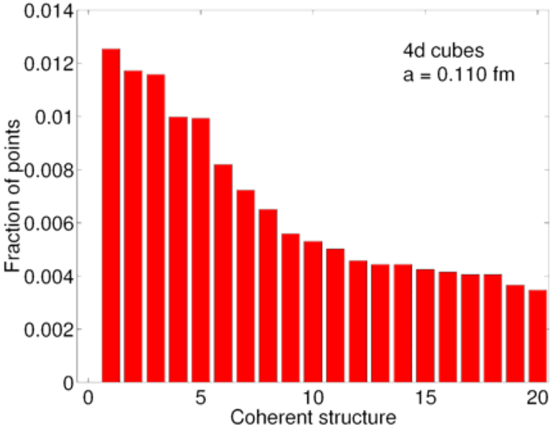
Hasenfratz, Laliena, Niedermayer 1998

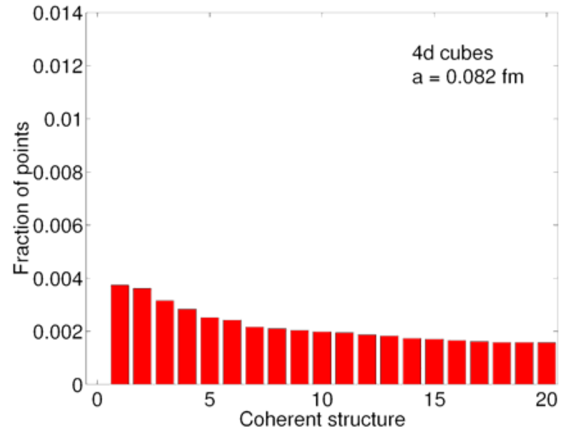
(a) strictly topological
$$\frac{\delta}{\delta U_\mu(z)} \sum_x q(x) = \frac{\delta}{\delta U_\mu(z)} Q = 0$$

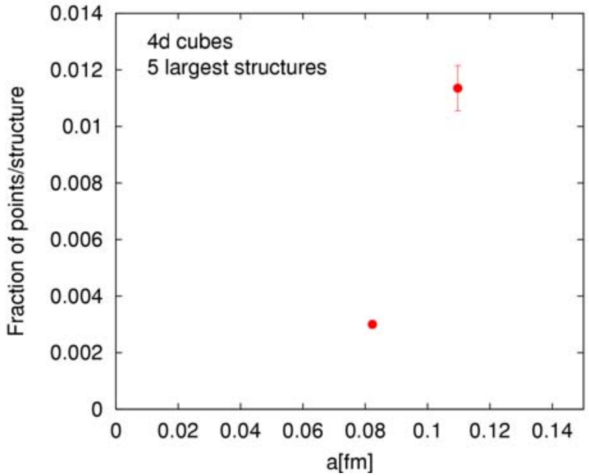
(b) index relation by construction
$$Q = n_- - n_+$$

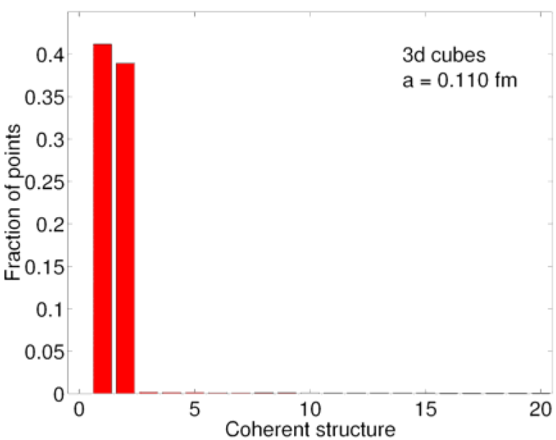
The Story of the Double-Sheet Structure...

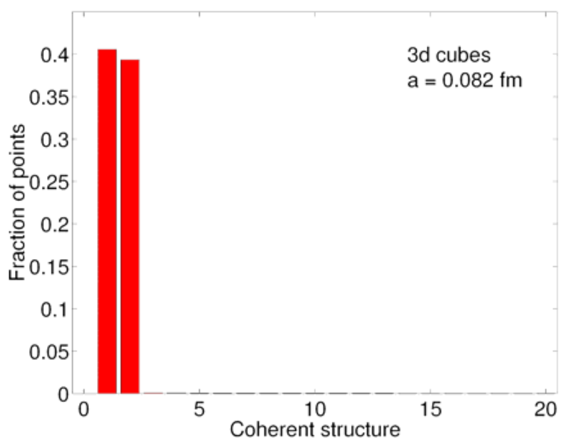
III. Just do it! (Summer 2002) (I.H. et al. PRD 68 (2003) 114505)

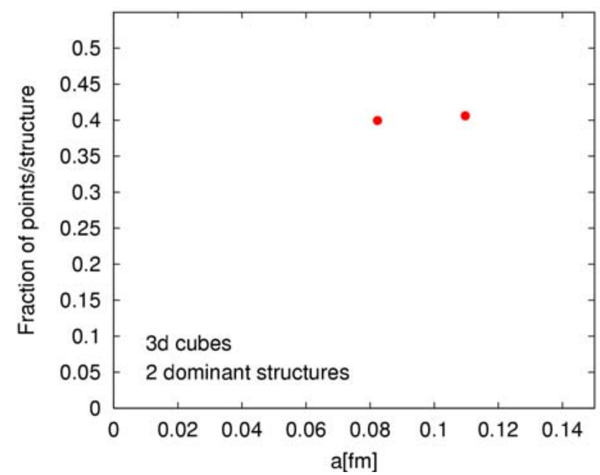












The Story of the Double-Sheet Structure...

(i) We really observed an ordered structure!

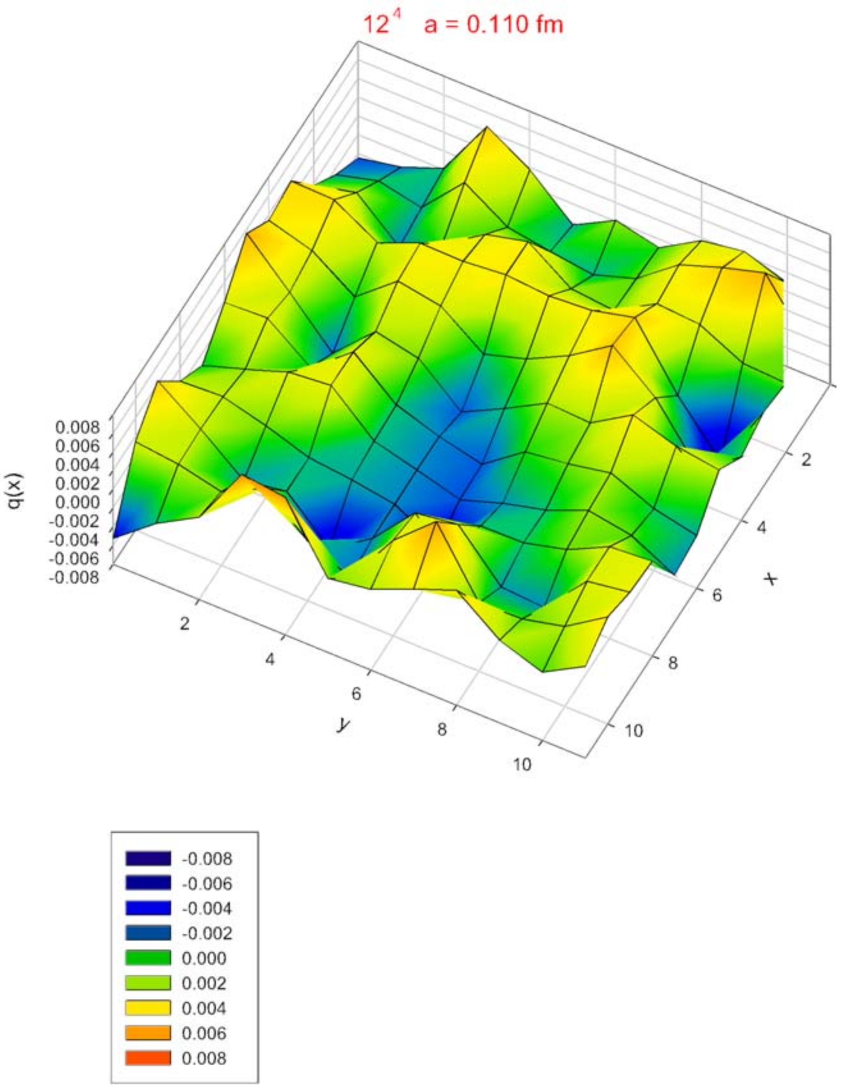
(ii) Low-dimensional (embedding dimension $d<4$; $d\sim3$ most preferred)

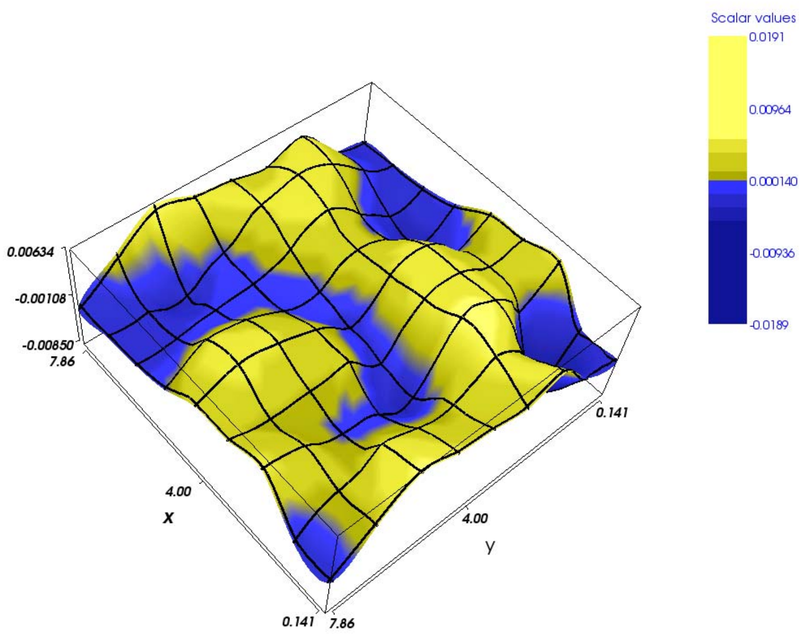
(iii) Global 2-part object (if broken into pieces correlator (susceptibility) not reproduced)

(iv) Two coherent parts are strongly correlated

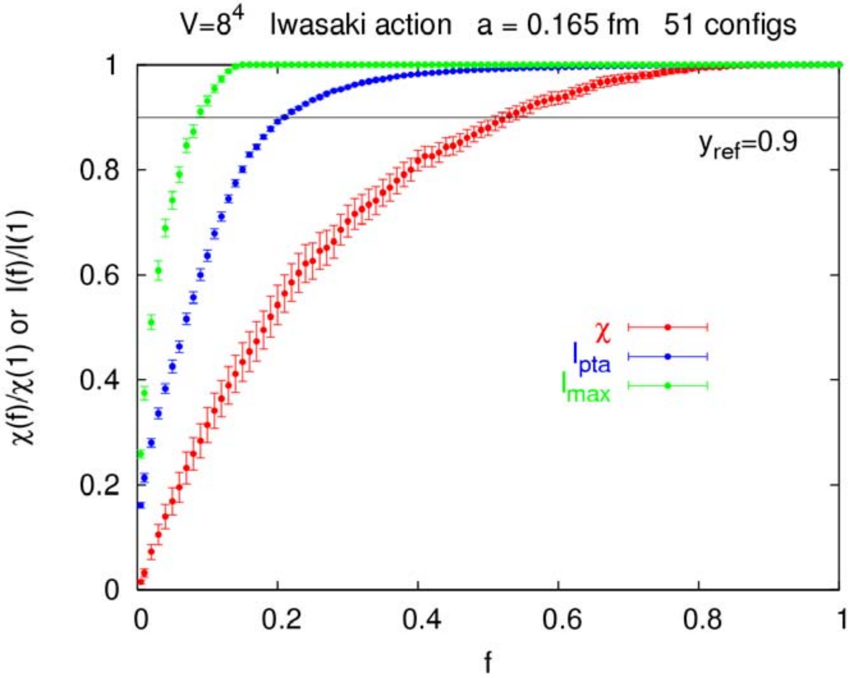
(v) Fills a macroscopic (finite) fraction of space-time

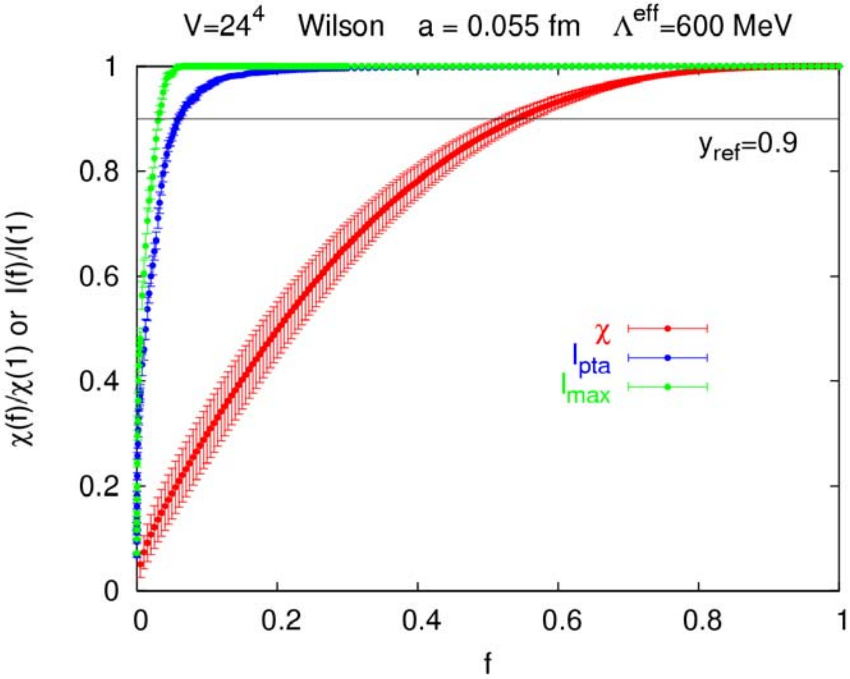
The Story of the Double-Sheet Structure...





The Story of the Double-Sheet Structure...

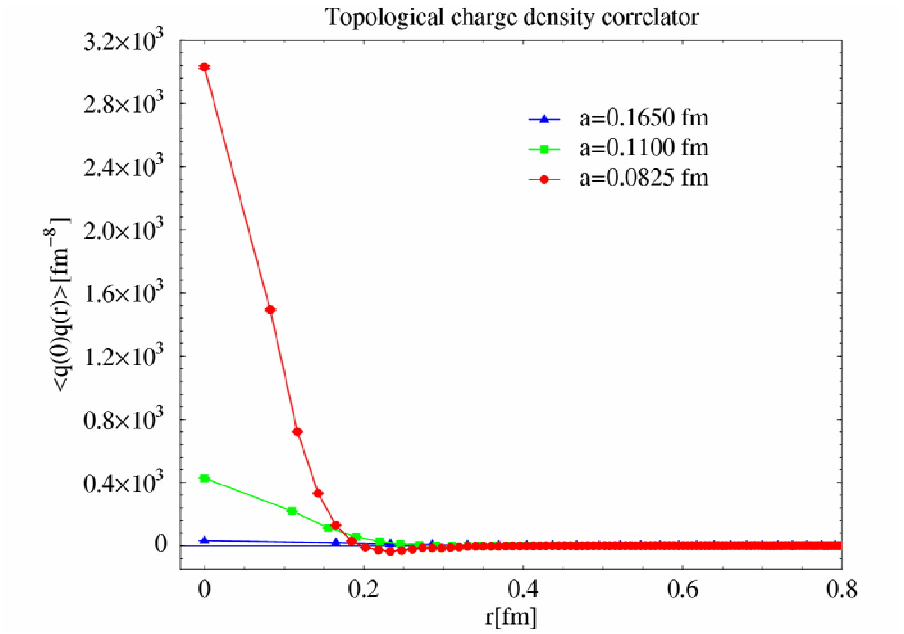


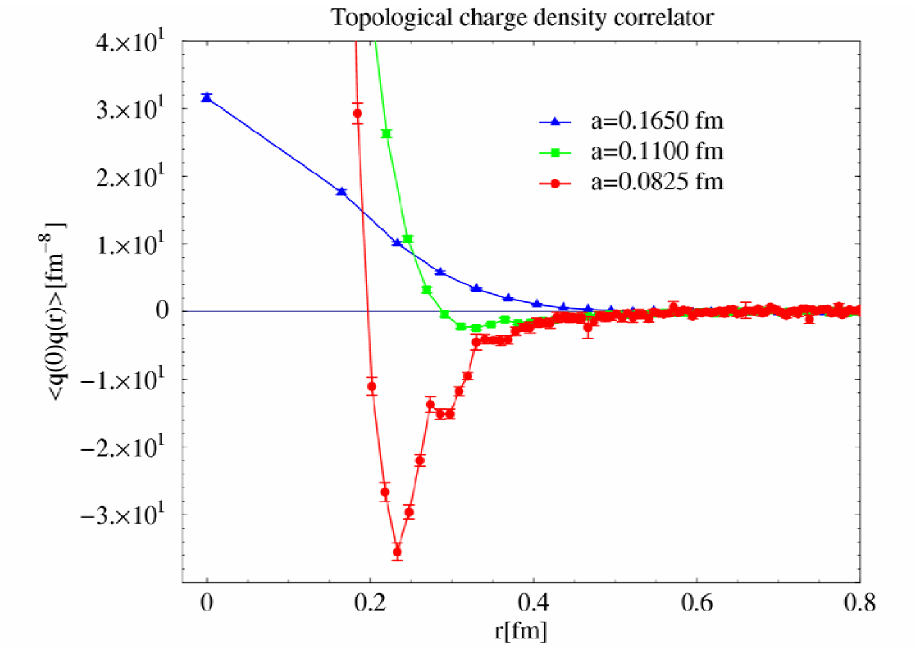


Inherently global

I.H. et al, PLB 612 (2005) 21

The Story of the Double-Sheet Structure...



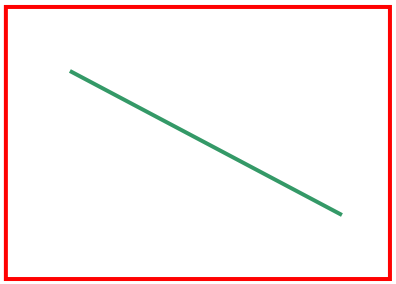


Negativity – sheets are strongly correlated

I.H. et al, PLB 617 (2005) 49

The Double-Sheet Story... Space-Filling Feature

- Two seemingly contradictory facts:
  - Coherent topological structure is low-dimensional
  - Occupies finite fraction of space-time

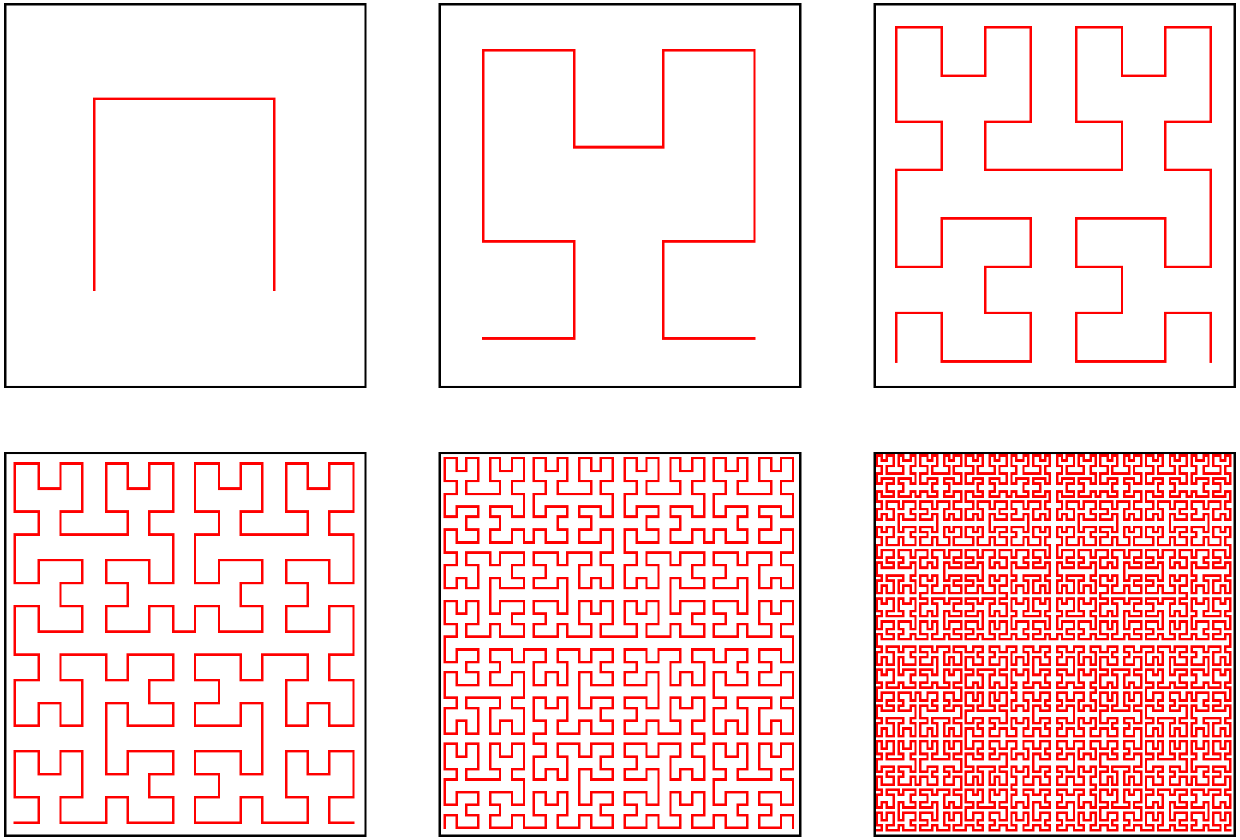


Finite line occupies zero fraction of a surface

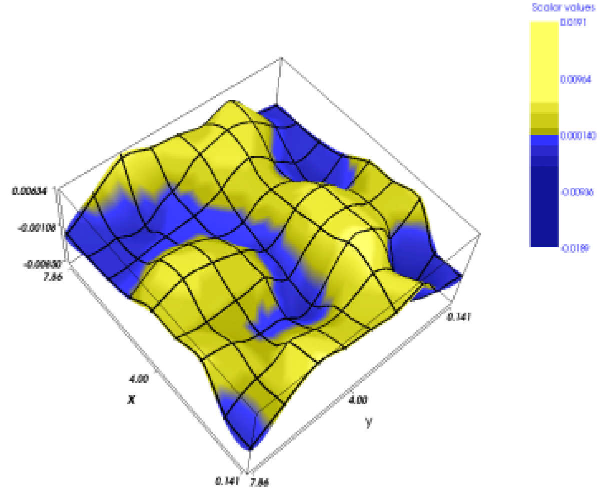
- In geometry there are intriguing objects defying this

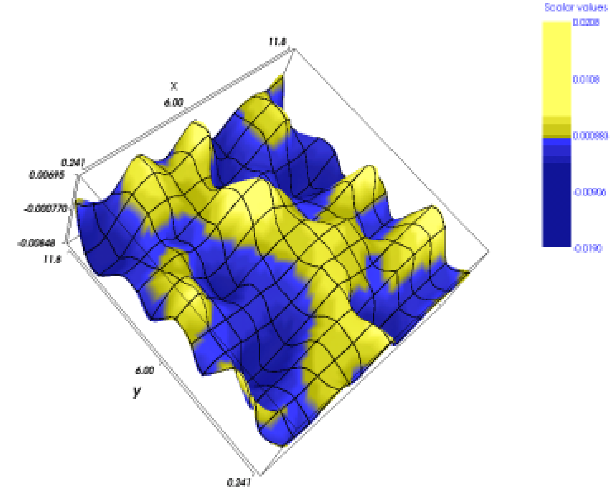
  space-filling curves (Peano, 1890)

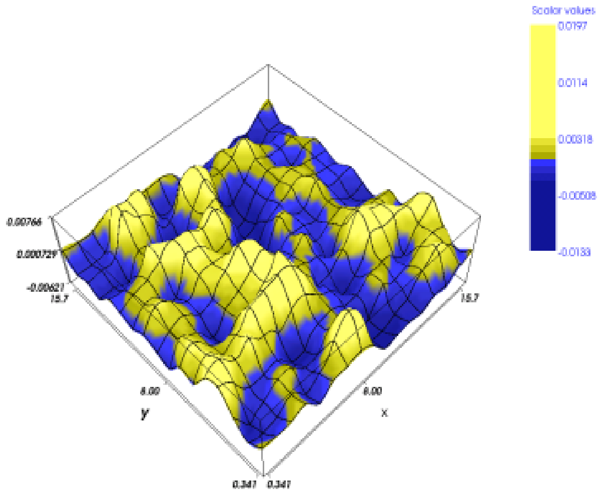
The Story... Space-Filling Feature

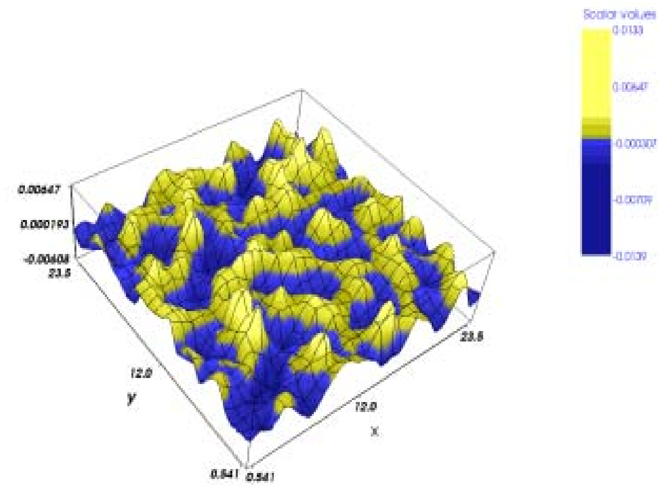


Space-Filling Feature continued...









Andrei's graphics software!

The Double-Sheet Story... Space-Filling Feature

- Peano curve: continuous surjection $[0,1] \to [0,1]^2$
- QCD structure: continuous surjection $[0,1]^d \to \text{dense } \Omega \subset T^4$
- d is the embedding dimension of the structure $1 \le d < 4$
- QCD topological structure is a quantum analog of space-filling object!

The Double-Sheet Story...

Topological structure of QCD vacuum behaves as a <u>charge-polarized, space-filling brane!</u>

Remarks on Randomness & Structure

How do we quantify degree of space-time order in a configuration?

I.H. hep-lat/0605008

$U(x) \longrightarrow$ 01011001011010101110...

binary string S

Kolmogorov complexity of S is a measure of order in $U(x)$

P(S) $\longrightarrow$ Universal Turing machine $\longrightarrow$ S

Minimal length of P(S) in bits is the Kolmogorov complexity of $U(x)$

The Story of Coherent QCD

How to generalize this? In which Directions?

(I) Fall 2001 - want operators expandable in Dirac eigenmodes!

$$D = S \times I + V_{\mu} \times \gamma_{\mu} + T_{\mu\nu} \times \sigma_{\mu\nu} + A_{\mu} \times (i\gamma_{\mu}\gamma_5) + P \times \gamma_5$$

Double sheet structure took priority…

The Story of Coherent QCD...

In case of scalar component:

Conjecture:

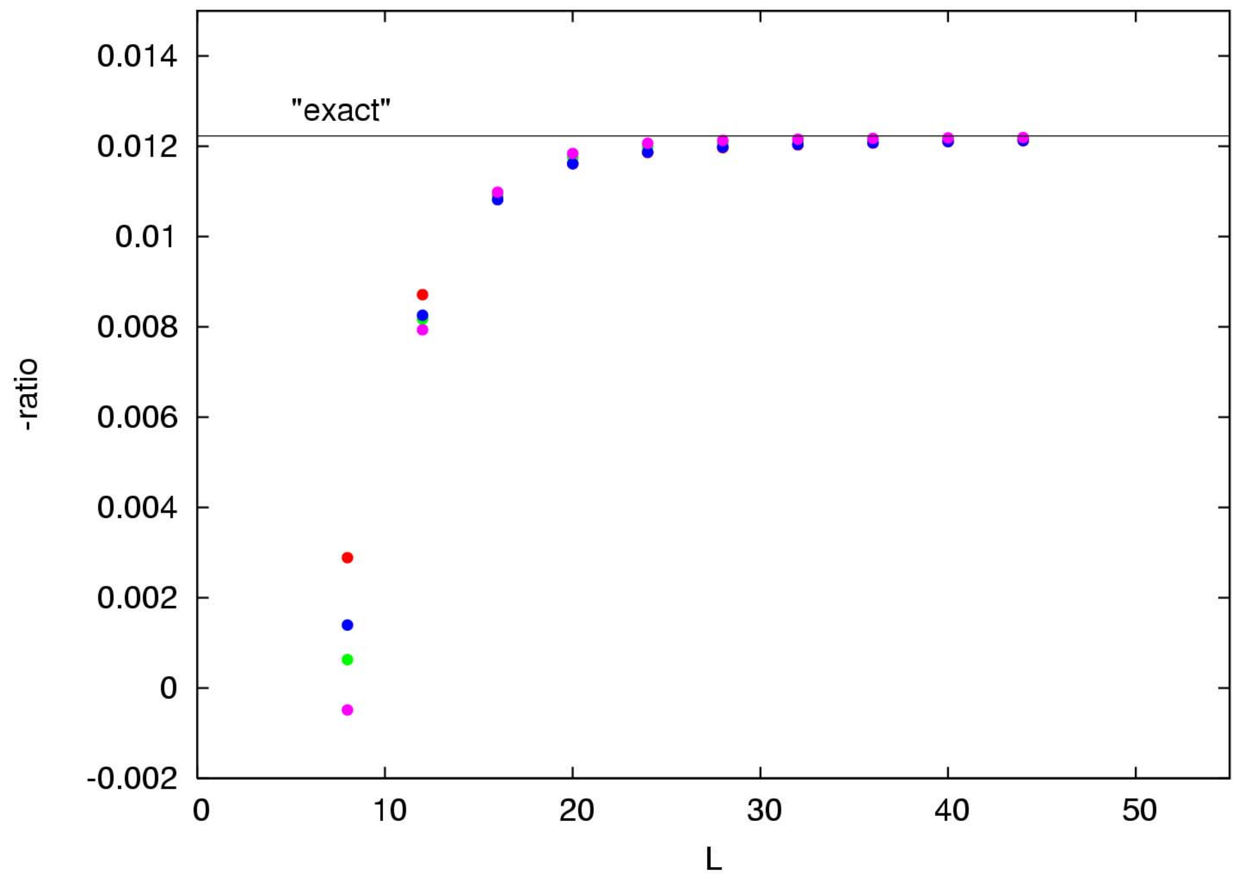
I.H., hep-lat/0607031

If *A(x)* is smooth su(3) gauge potential and *U(n,a)* is its transcription to the hypercubic lattice with classical lattice spacing *a* then

$$\mathrm{tr}\left(\mathrm{D}_{0,0}(U) - D_{0,0}(I)\right) = -c^S a^4 \ \mathrm{tr}\, F_{\mu\nu}(0) F_{\mu\nu}(0) + \mathrm{O}(a^6)$$

for generic chirally symmetric D. The proportionality constant is non-zero and independent of *A(x).*

The Story of Coherent QCD...



It works!

A.Alexandru, I.H., K.F.Liu, in prep.

The Story of Coherent QCD...

In case of tensor component:

Conjecture:

If *A(x)* is smooth su(3) gauge potential and *U(n,a)* is its transcription to the hypercubic lattice with classical lattice spacing *a* then

$$\mathrm{tr}^s \ \sigma_{\mu\nu} \ \mathrm{D}_{0,0}(U) = -c^T a^2 \ F_{\mu\nu}(0) + \mathrm{O}(a^4)$$

for generic chirally symmetric D. The proportionality constant is non-zero and independent of *A(x).*

This was recently derived explicitly:

K.F.Liu, A.Alexandru, I.H., hep-lat/0703010

The Story of Coherent QCD...

(II) Fall 2002 - want the whole theory expandable in Dirac eigenmodes!

$$S = \mathrm{Tr}\left(\bar{\beta} - i\bar{\theta}\gamma_5\right) D + \sum_{\mathrm{f}=1}^{\mathrm{N_f}} \bar{\psi}^{\mathrm{f}} \left(D + m_f\right) \psi^f$$

I.H., hep-lat/0607031

$$P \propto \det\left[(D+m)^{N_f} \exp\left(-\bar{\beta} D\right)\right]$$

But the double-sheet structure took priority…

The Story of Coherent QCD...

(III) Spring 2006 - want the theory where the gauge part maximally resembles the fermionic part

$$S = \sum_{f=0}^{N_f} \bar{\psi}^f \left(D + m_f \right) \psi^f$$

I.H., hep-lat/0607031

symmetric logarithmic LQCD

$$P \propto \det \prod_{f=0}^{N_f} (D + m_f)$$

Gauge dynamics can be viewed as due to the infinitely heavy fermion!

Conclusion

Happy Birthday, Keh-Fei!

And thanks for those few nice years!