Hadron spectrum from Lattice QCD

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Jefferson Lab

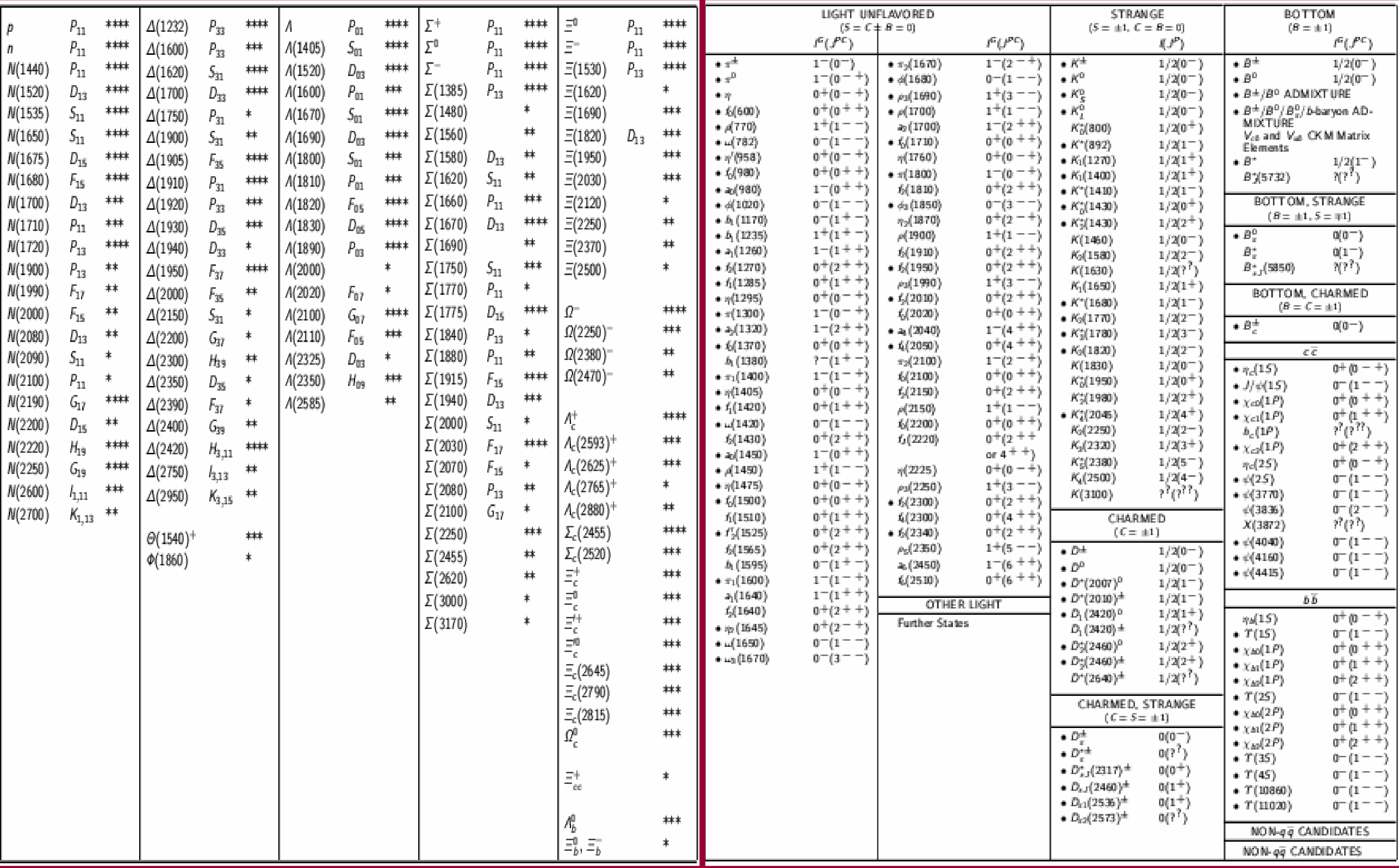
Outline

- Baryons
  - Some recent results on baryon spectrum
  - Parity ordering in baryon sector (Roper, S_{11} etc)
- Quenching artifacts -Unphysical ghost states
- Multiquark states
- Group theoretical baryon operators
- Mesons
  - Tetraquark
  - Exotics-Gluex expt.
  - Charmonium excited states and photocouplings
- Conclusions

The Particle Zoo

Baryons (3-quarks)

Mesons (2-quarks)



p	P_{11}	****
n	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1,11}$	***
$N(2700)$	$K_{1,13}$	**

$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3,11}$	****
$\Delta(2750)$	$I_{3,13}$	**
$\Delta(2950)$	$K_{3,15}$	**
$\Theta(1540)^+$		***
$\Phi(1860)$		*

Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	**
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-		****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+		****
$\Lambda_c(2593)^+$		***
$\Lambda_c(2625)^+$		***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$		**
$\Sigma_c(2455)$		****
$\Sigma_c(2520)$		***
Ξ_c^+		***
Ξ_c^0		***
$\Xi_c^{\prime+}$		***
$\Xi_c^{\prime0}$		***
$\Xi_c(2645)$		***
$\Xi_c(2790)$		***
$\Xi_c(2815)$		***
Ω_c^0		***
Ξ_{cc}^+		*
Λ_b^0		***
Ξ_b^0, Ξ_b^-		*

LIGHT UNFLAVORED
$(S = C = B = 0)$

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $h_1(1170)$	$0^-(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $b_1(1235)$	$1^+(1^{+-})$	$\rho(1900)$	$1^+(1^{--})$
• $a_1(1260)$	$1^-(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $f_2(1270)$	$0^+(2^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$f_0(2020)$	$0^+(0^{++})$
• $a_2(1320)$	$1^-(2^{++})$	• $a_4(2040)$	$1^-(4^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $f_4(2050)$	$0^+(4^{++})$
$h_1(1380)$	$?^-(1^{+-})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$f_0(2100)$	$0^+(0^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$f_2(2150)$	$0^+(2^{++})$
• $f_1(1420)$	$0^+(1^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $\omega(1420)$	$0^-(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
$f_2(1430)$	$0^+(2^{++})$	$f_J(2220)$	$0^+(2^{++}$
• $a_0(1450)$	$1^-(0^{++})$		or $4^{++})$
• $\rho(1450)$	$1^+(1^{--})$	$\eta(2225)$	$0^+(0^{-+})$
• $\eta(1475)$	$0^+(0^{-+})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_0(1500)$	$0^+(0^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_1(1510)$	$0^+(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$h_1(1595)$	$0^-(1^{+-})$	$a_6(2450)$	$1^-(6^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_6(2510)$	$0^+(6^{++})$
$a_1(1640)$	$1^-(1^{++})$		
$f_2(1640)$	$0^+(2^{++})$		
• $\eta_2(1645)$	$0^+(2^{-+})$		
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		

OTHER LIGHT

Further States

STRANGE
$(S = \pm1, C = B = 0)$

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED
$(C = \pm1)$

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE
$(C = S = \pm1)$

• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{sJ}^*(2317)^\pm$	$0(0^+)$
• $D_{sJ}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

BOTTOM
$(B = \pm1)$

	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

BOTTOM, STRANGE
$(B = \pm1, S = \mp1)$

• B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED
$(B = C = \pm1)$

• $B_c^\pm$	$0(0^-)$

$c\bar{c}$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
$\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$\psi(3836)$	$0^-(2^{--})$
$X(3872)$	$?^?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

$b\bar{b}$

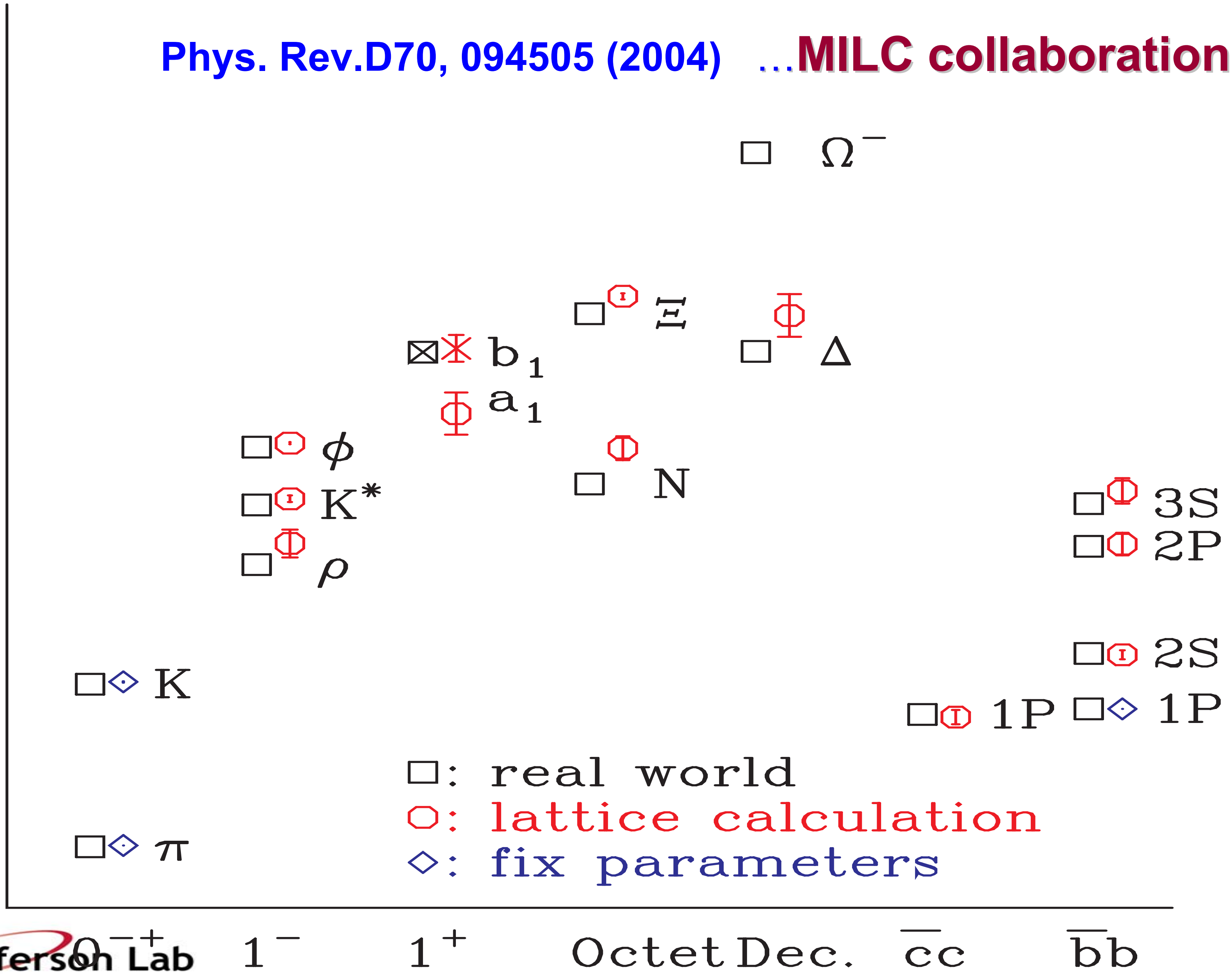
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON-$q\bar{q}$ CANDIDATES

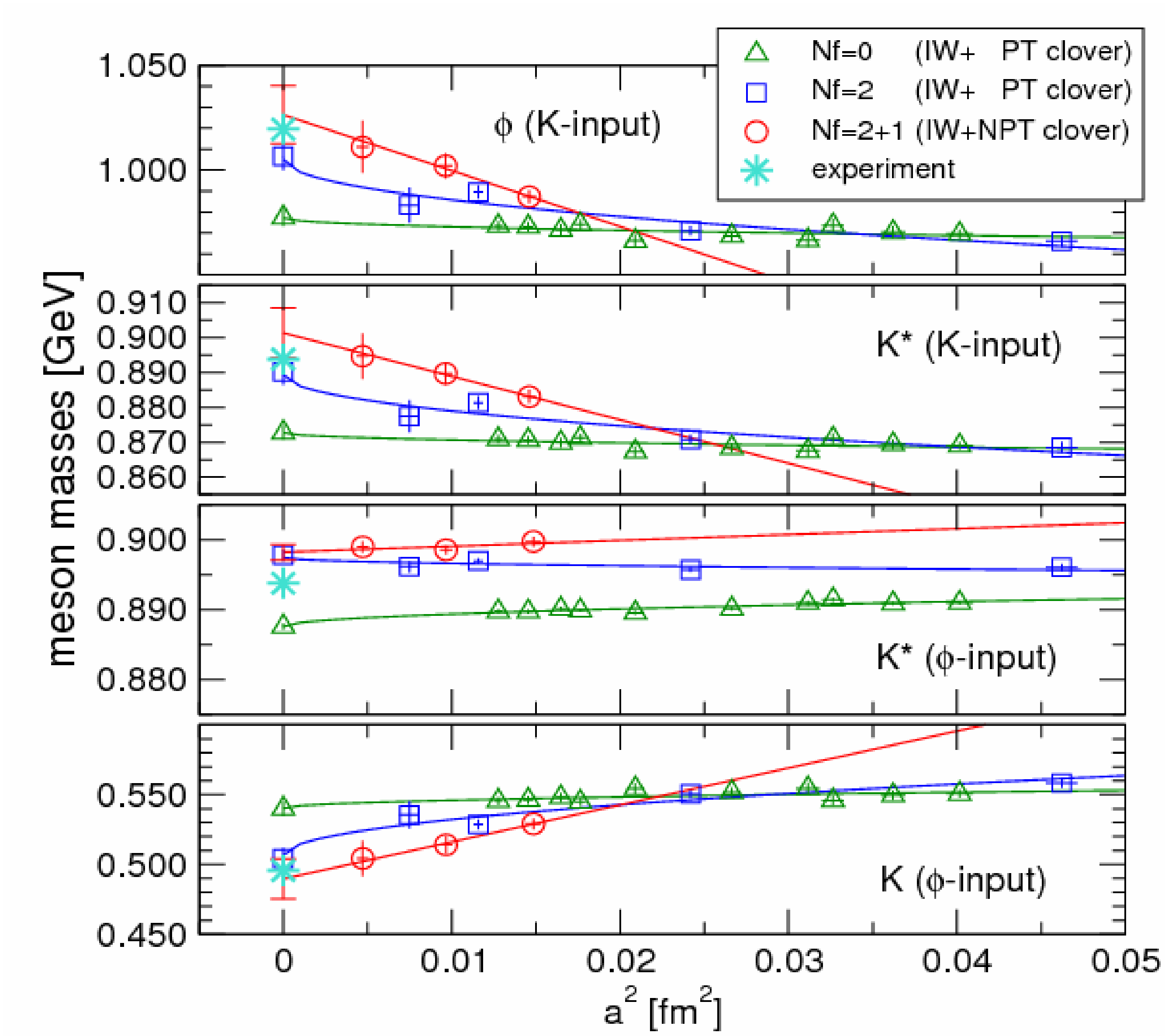
NON-$q\bar{q}$ CANDIDATES

HADRON SPECTRUM ...PDG

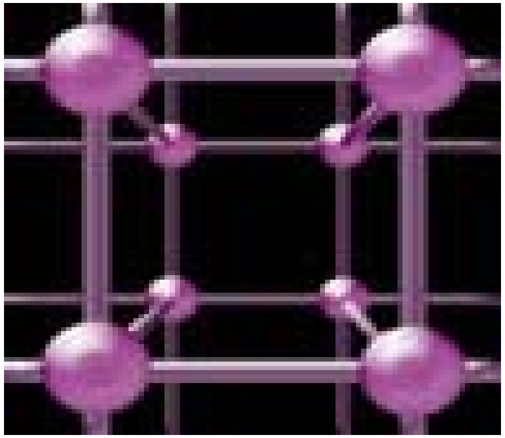
Phys. Rev.D70, 094505 (2004) …**MILC collaboration**



CP-PACS' RESULTS



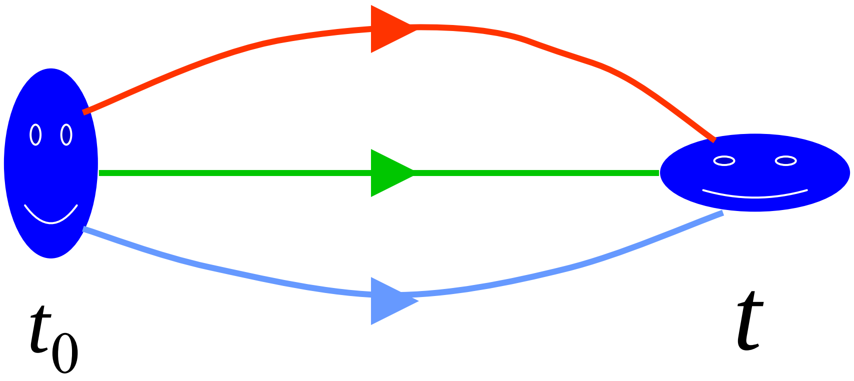
arXiv:0704.1937



How to calculate an observable??

From statistical mechanical correlation functions

Two Point Correlation Function



The two-point correlation function
decays exponentially at large separation of time

$$G^{\alpha\alpha}_{NN}(t,t_0,\vec{p}) \equiv \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle T(\chi^{\alpha}(x)\overline{\chi}^{\alpha}(x_0))\rangle$$

$$\xrightarrow{t-t_0>>1} \langle 0|\chi^{\alpha}|N^{\alpha}(\vec{p})\rangle\langle N^{\alpha}(\vec{p})|\overline{\chi}^{\alpha}|0\rangle \frac{e^{-E_p(t-t_0)}}{2E_pV_3} \equiv \frac{E_p+m}{E_p}|\phi|^2 e^{-E_p(t-t_0)}$$

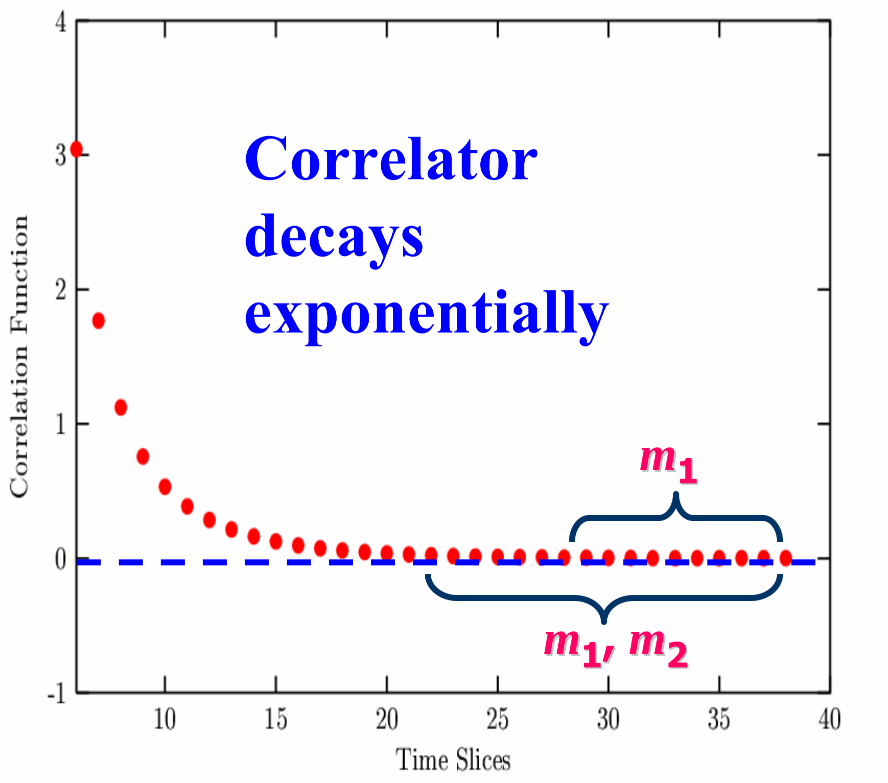
Analysis (Extraction of Mass)

$$G(t) = \sum_{i=1}^{N} W_i \, e^{-m_i t} \underset{t \to \infty}{\approx} W_1 \, e^{-m_1 t}$$

How to extract m_2 m_3… : excited states.
Non linear fitting.

Need help with statistics.

Sequential Baysian Method.
….hep-lat/0405001



Alternative approach :
Solving generalized
cross correlator eigenvalue problem

$$G(t)v = \lambda(t,t_0)G(t_0)v$$

Overlap Fermions

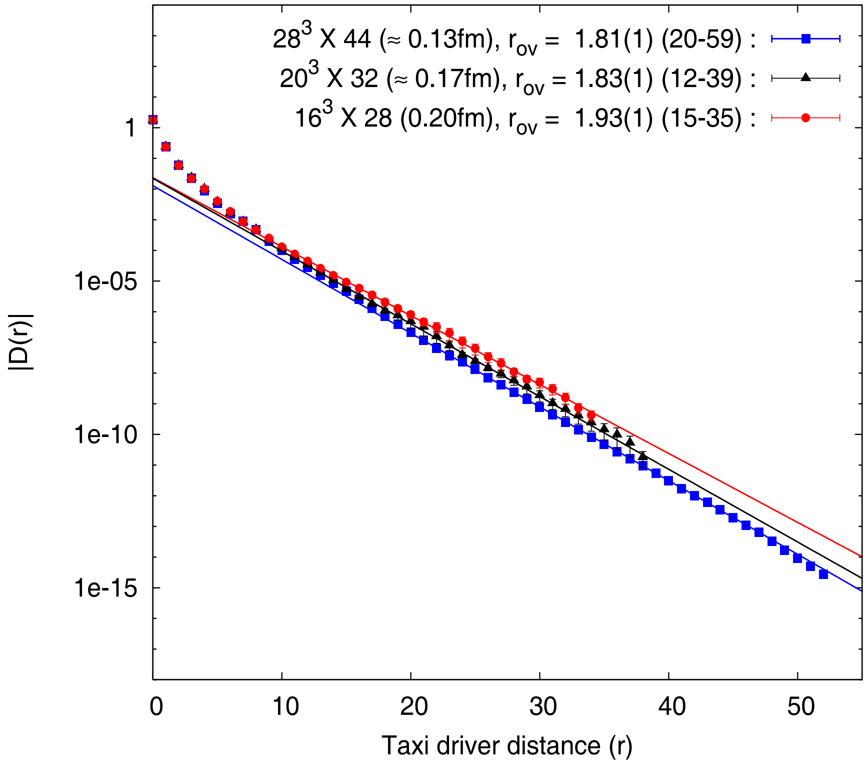
❖ Expected features:

- Numerically intensive (~100 times of Wilson fermion to invert)
  Zolotarev approximation of matrix sign function (< 10^{-9} accurancy)
- No $O(a)$ errors (no dimension 5 chirally symmetric action)
- No additive quark mass renormalization
- No exceptional configuration (eigenvalues on a circle)
- Well defined topology both globally and locally
- No mixing of operators in different chiral sectors

❖ Unexpected desirable features:

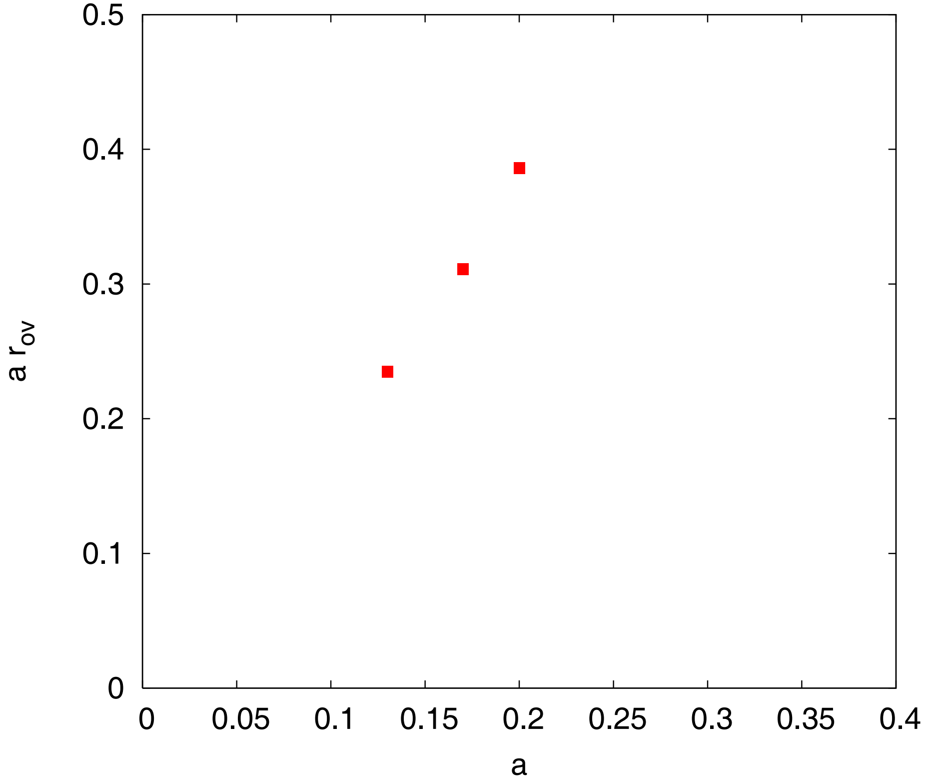
- ✓ $O(a^2)$ error are small (π and ρ masses on 3 lattices).
- ✓ $O(m^2a^2)$ errors are small (dispersion relation, renormalization constants). This justifies its application on both heavy and light quarks.
- ✓ Critical slowing down is gentle (m_π ~160 MeV).
- ✓ Topological charge density void of large ultra-violet fluctuation (overlap operator is exponentially local)

Locality of overlap action



Expectation value of |D(r)|
as a function of the taxi-driver distance

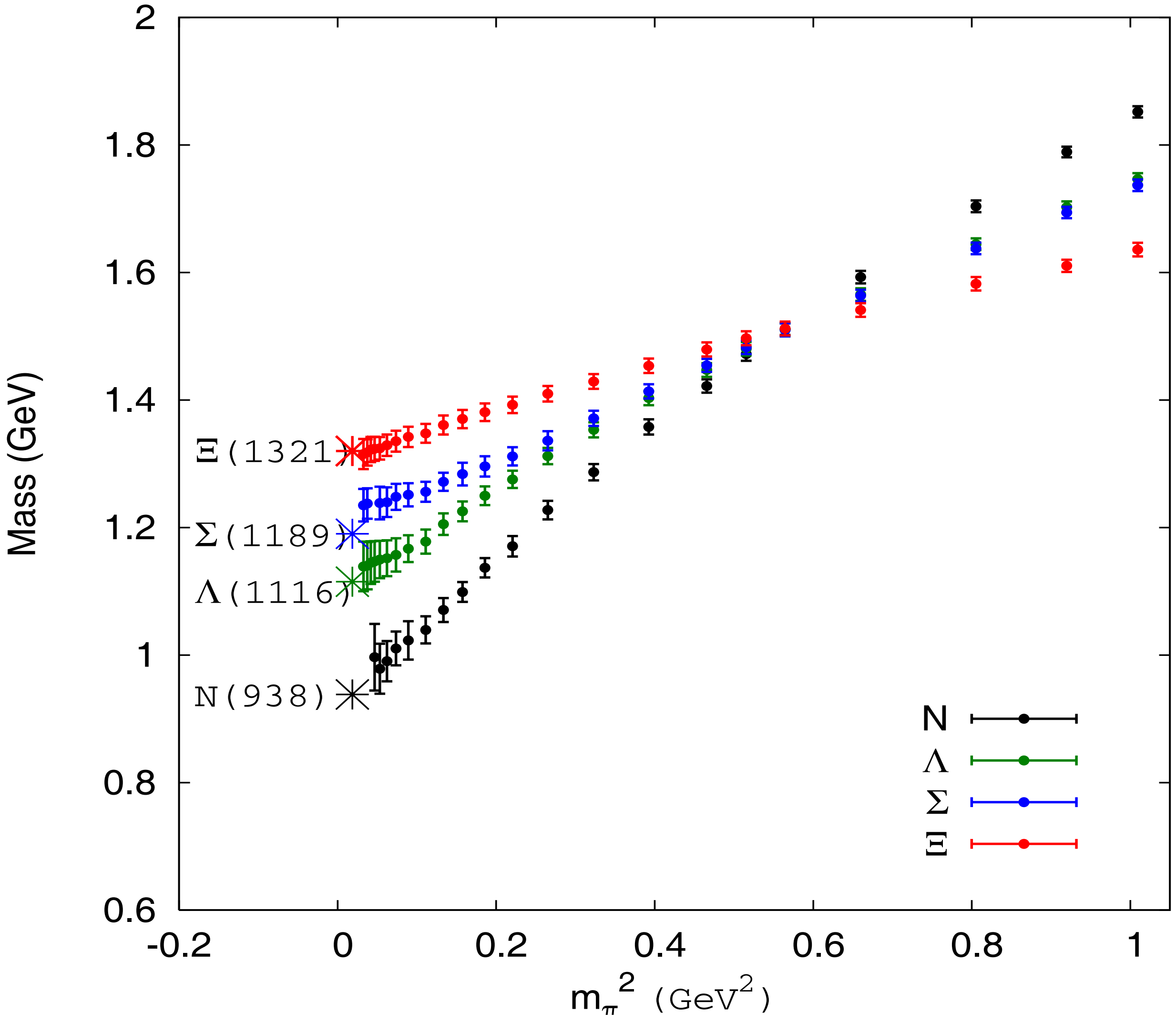
$$r = \|x-y\|_1 = \sum_{\mu} |x_\mu - y_\mu|$$



Taxi-driver range in physical units as a function of lattice spacing

hep-lat/0510075

Ground state Octet Baryons



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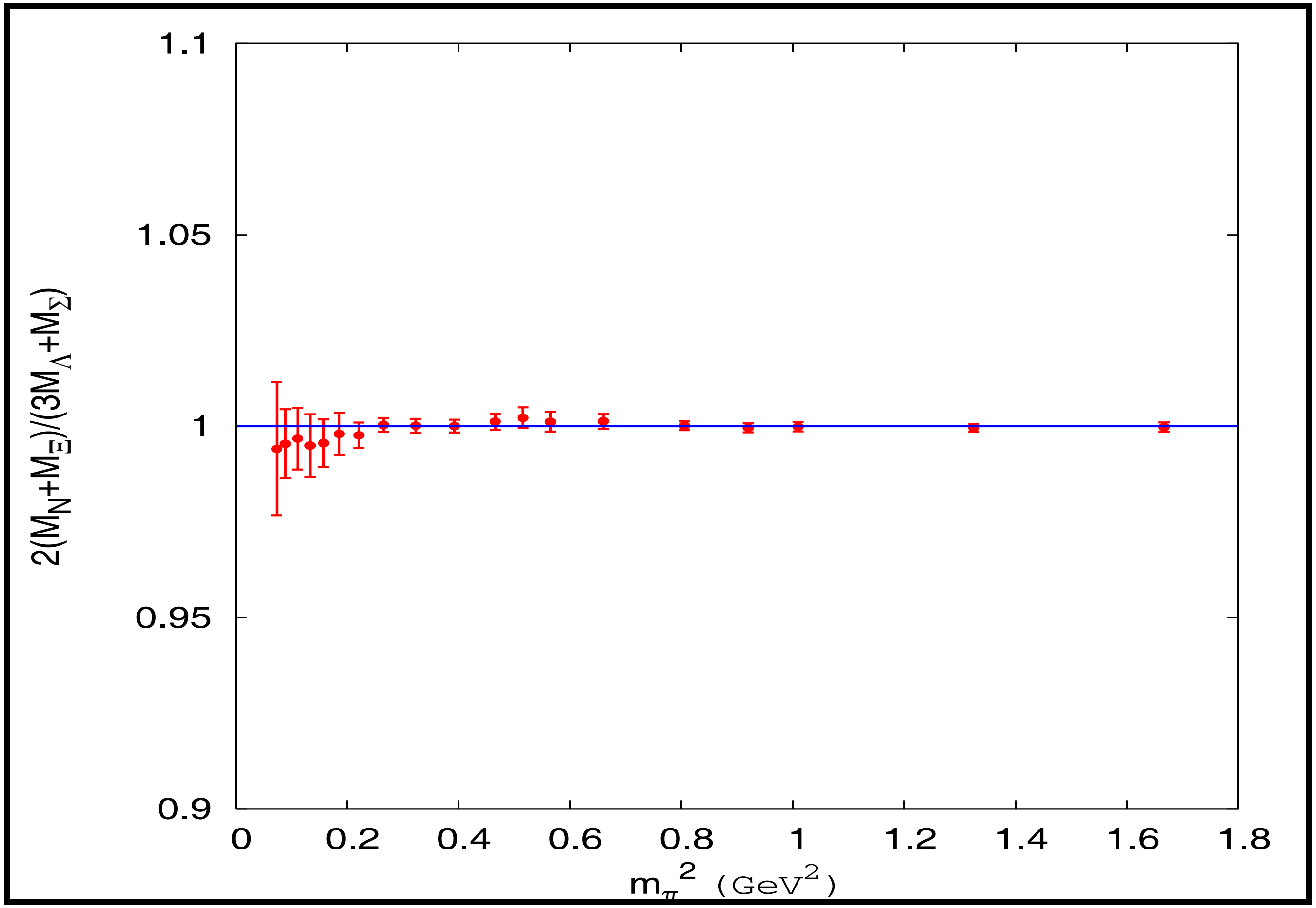
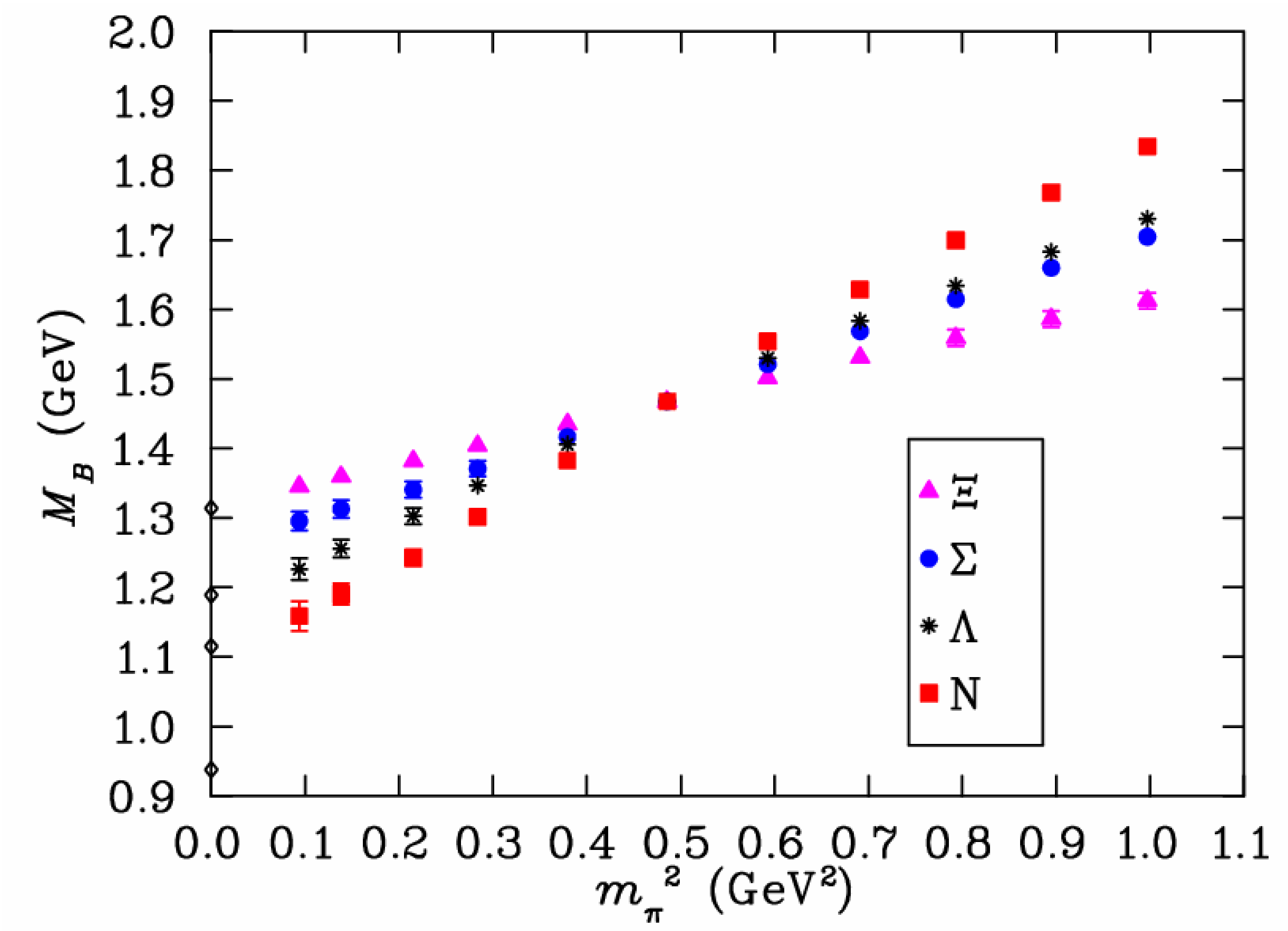
Gell-Mann Okubo Relation

$$2\left(M_N + M_\Xi\right) = 3M_\Lambda + M_\Sigma$$





Boinepalli et al. **hep-lat/0604022**

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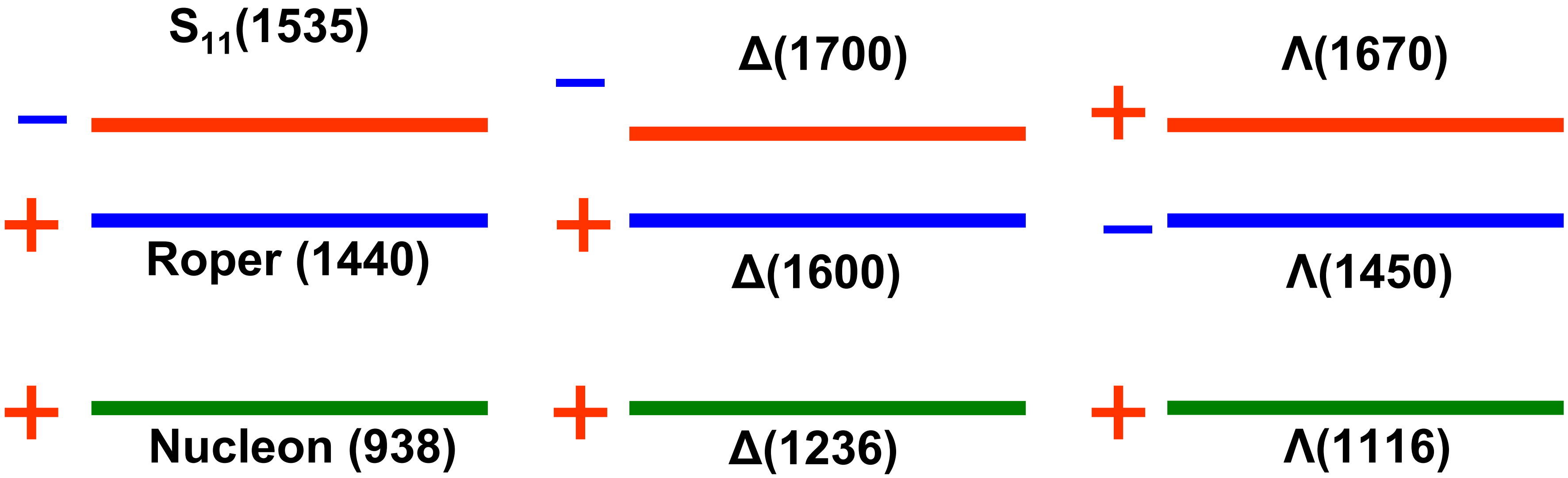


− $S_{11}(1535)$

+ Roper (1440)

+ Nucleon (938)

− Δ(1700)

+ Δ(1600)

+ Δ(1236)

+ Λ(1670)

− Λ(1450)

+ Λ(1116)

What is the nature of the
Roper ((1440) $1/2^+$) resonance?

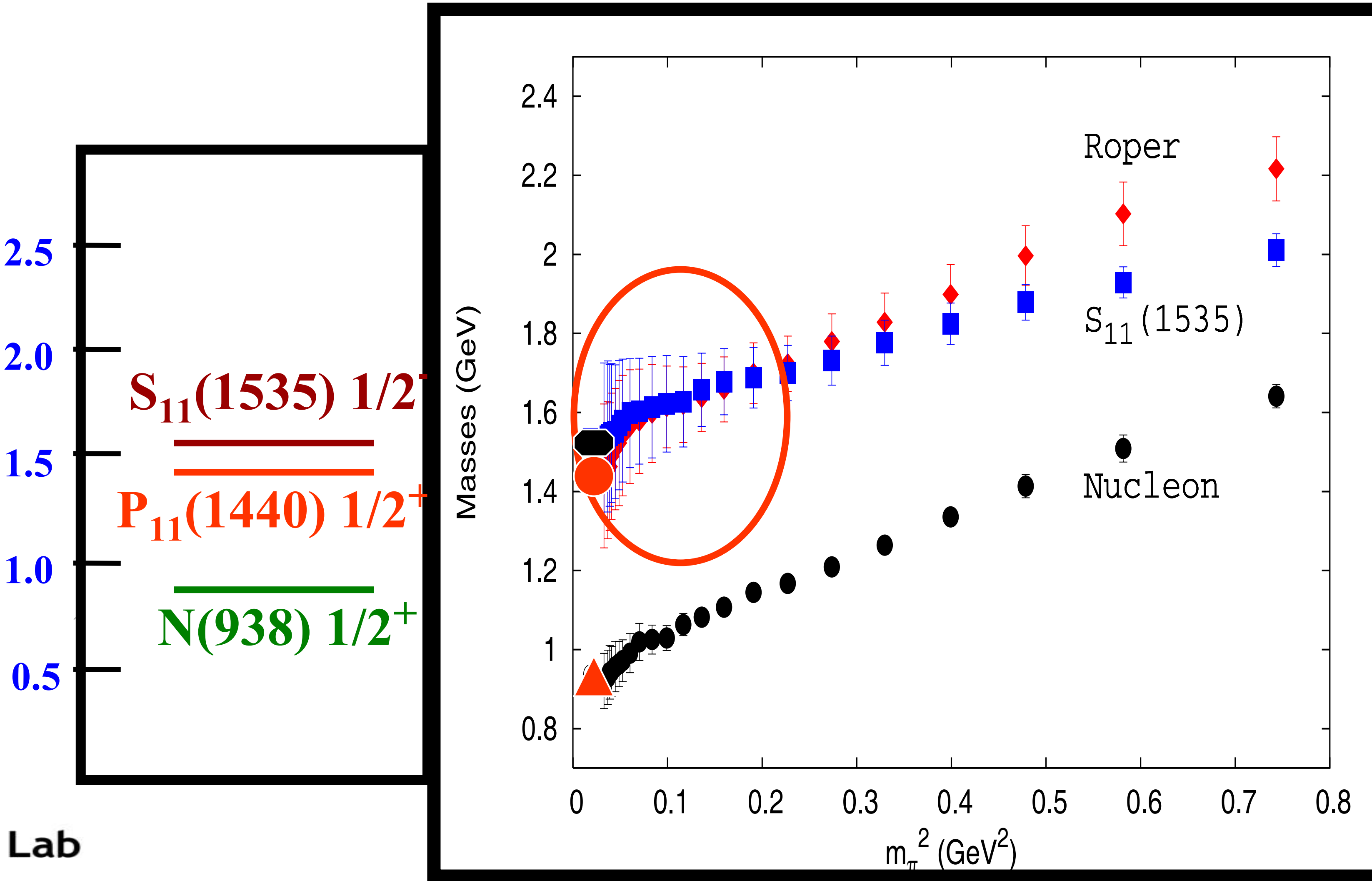
Radial excitation? $q^4\underline{q}$ state?

- Hybrid state (qqq*g*)?
- Dynamical meson-baryon state?

Roper

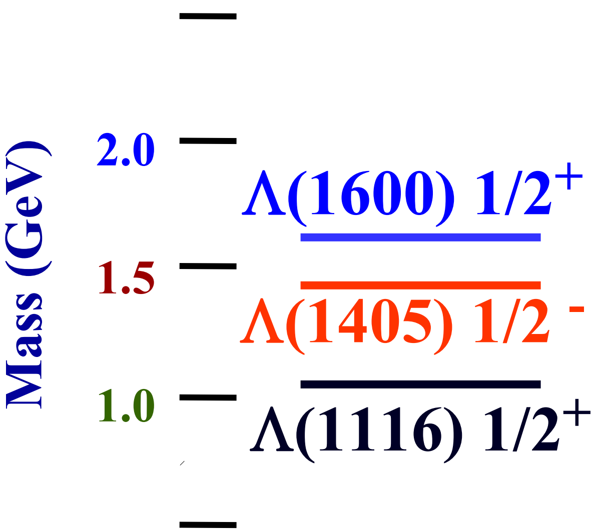
Roper is seen on the lattice at the right mass with <u>three quark</u> interpolation field …..Mathur et.al. **Phys. Lett. B605, 137 (2005)**

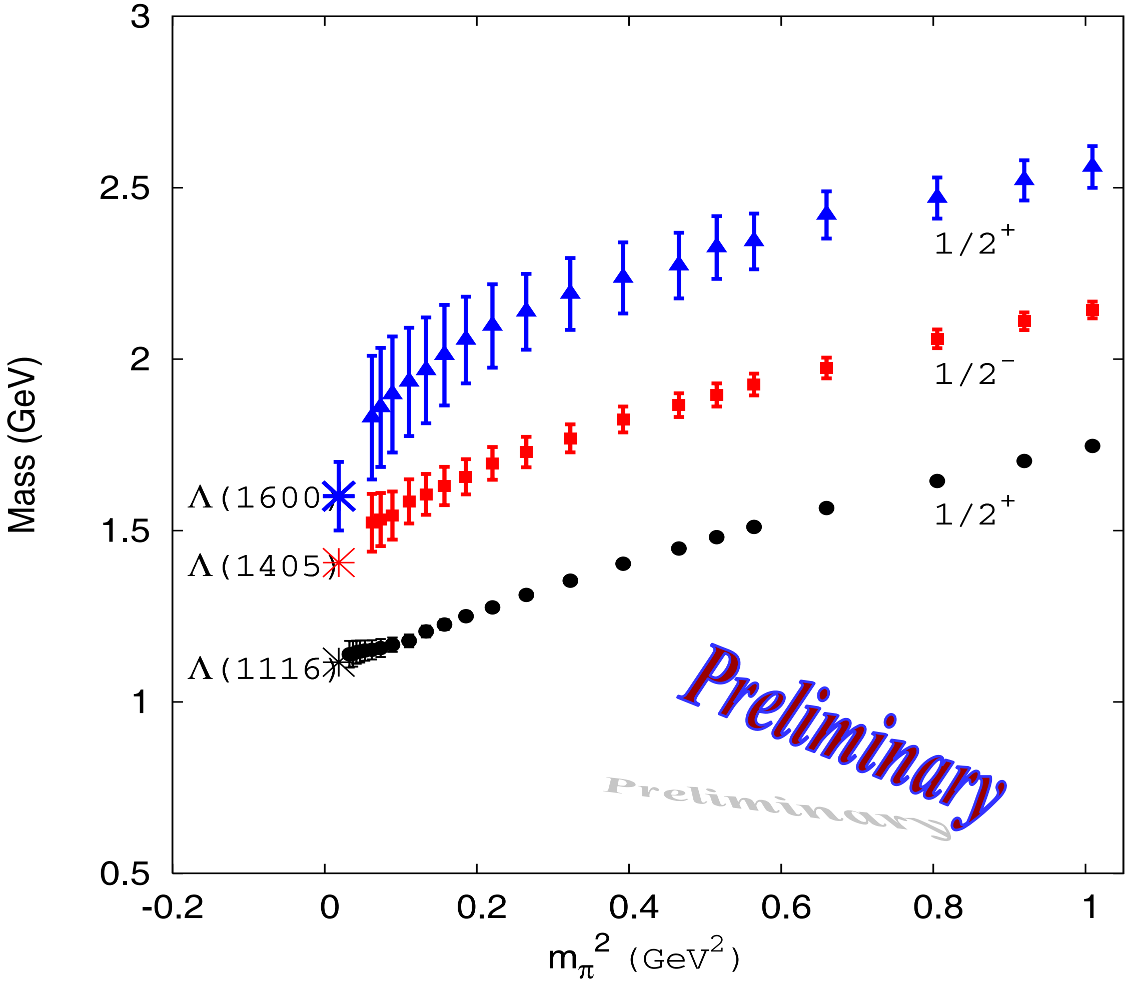
Cross over occurs in chiral domain

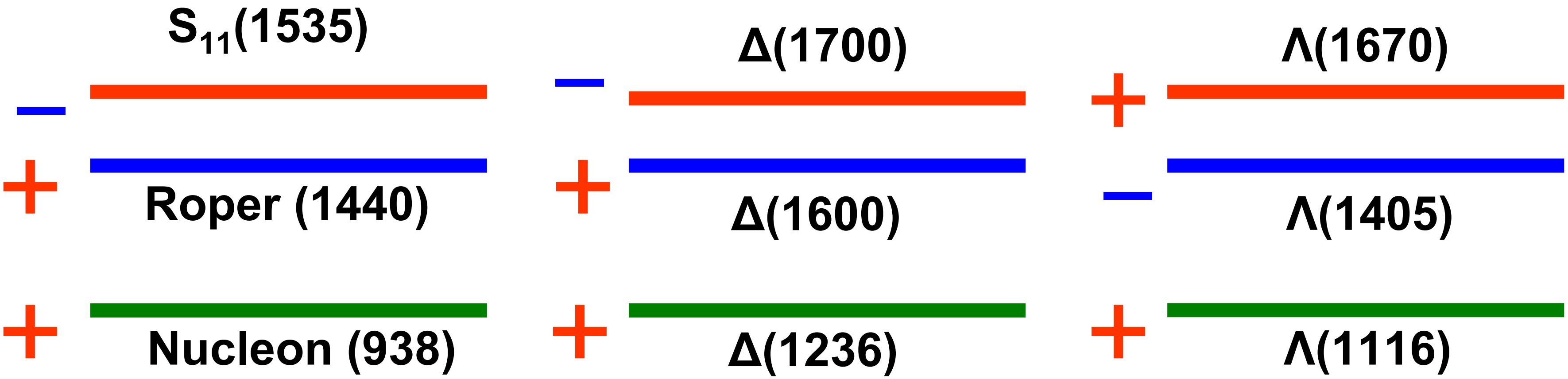


What about Hyperons?
The Λ(1405)?

...different **story!!**







– $S_{11}(1535)$

+ Roper (1440)

+ Nucleon (938)

– Δ(1700)

+ Δ(1600)

+ Δ(1236)

+ Λ(1670)

– Λ(1405)

+ Λ(1116)

Hyperfine Interaction of quarks in Baryons

$$\lambda_c^1 . \lambda_c^2 \vec{\sigma}_1 . \vec{\sigma}_2$$

Color-Spin Interaction

Excited positive > Negative ..Isgur

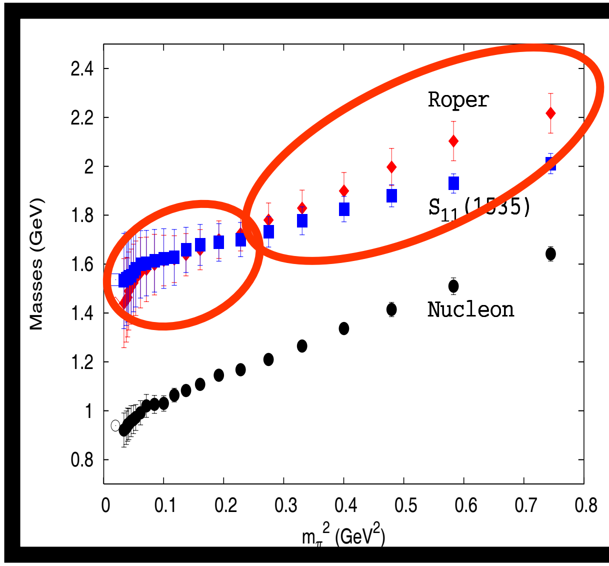
$$\lambda_F^1 . \lambda_F^2 \vec{\sigma}_1 . \vec{\sigma}_2$$

Flavor-Spin interaction

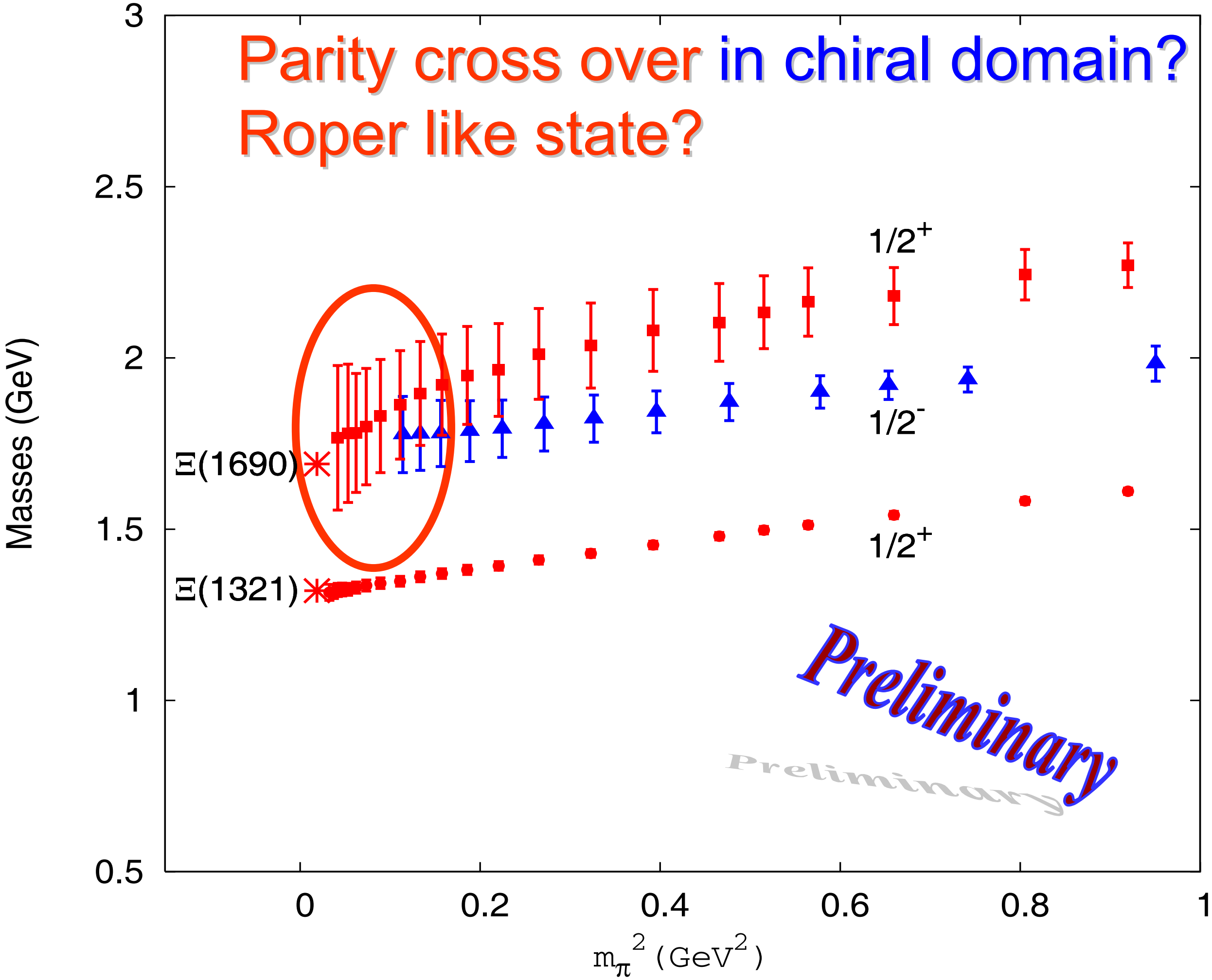
Chiral symmetry plays major role

Negative > Excited positive

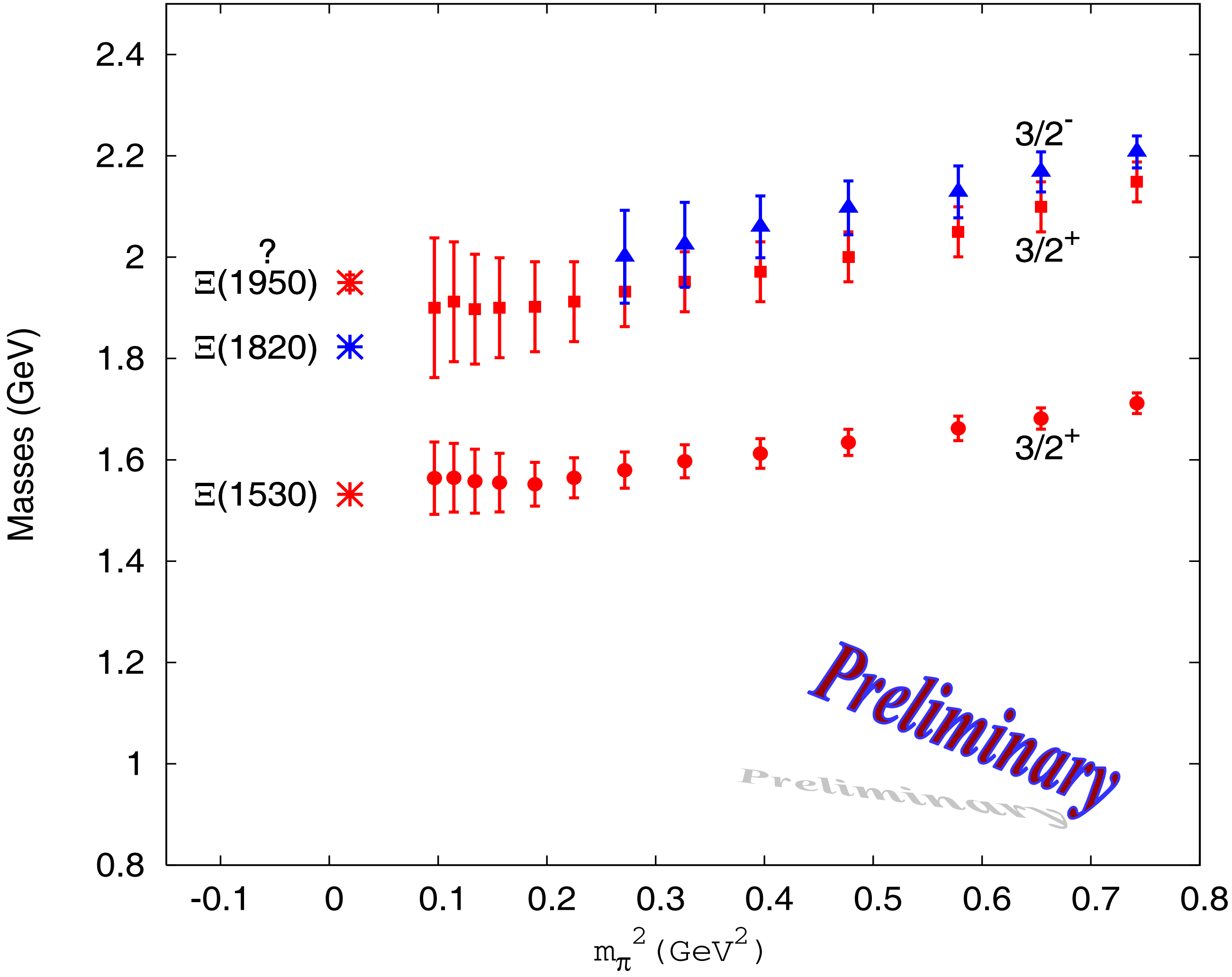
Glozman & Riska
Phys. Rep. 268,263 (1996)



Octet Ξ Baryons



Decuplet Ξ Baryons



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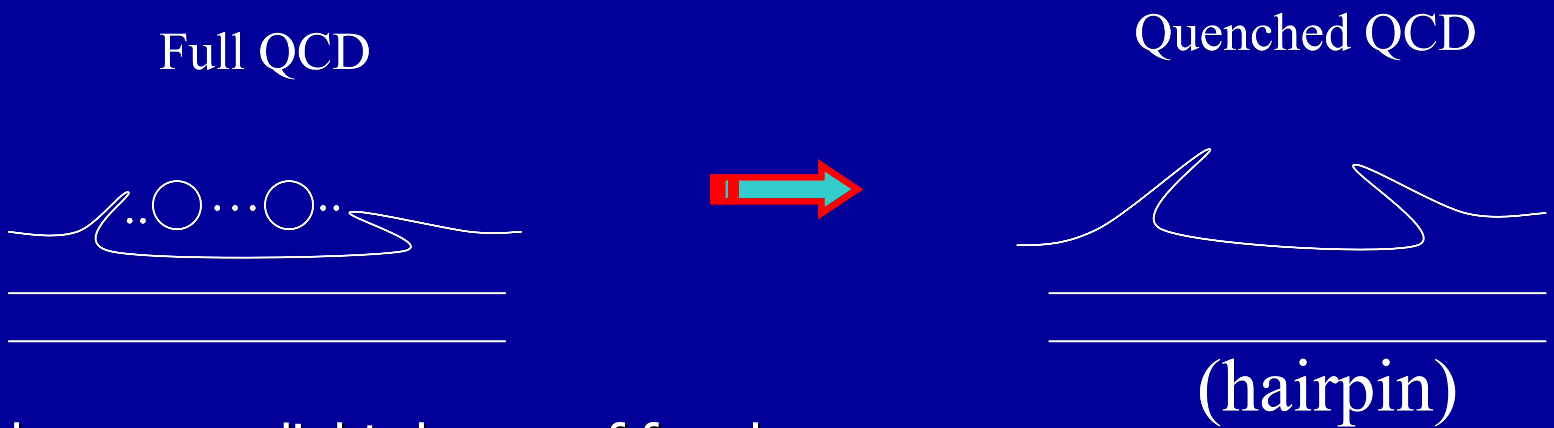
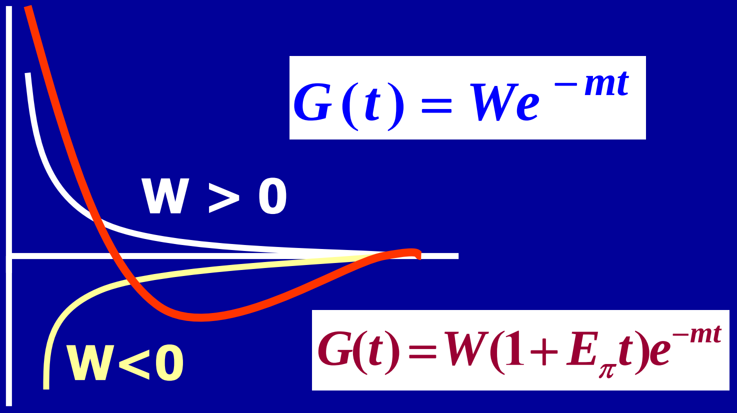
The η′ ghost in quenched QCD



- It becomes a light degree of freedom
  - with a mass degenerate with the pion mass.
- It is present in all hadron correlators G(t).
- It gives a negative contribution to G(t).
  - It is unphysical (thus the name ghost).



$G(t) = We^{-mt}$

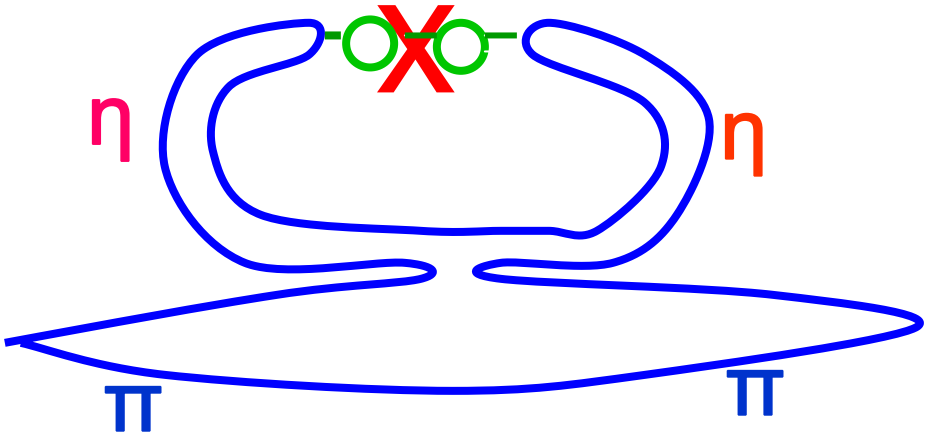
$G(t) = W(1 + E_\pi t)e^{-mt}$

Quenched Artifacts

Chiral *log* in m_π^2

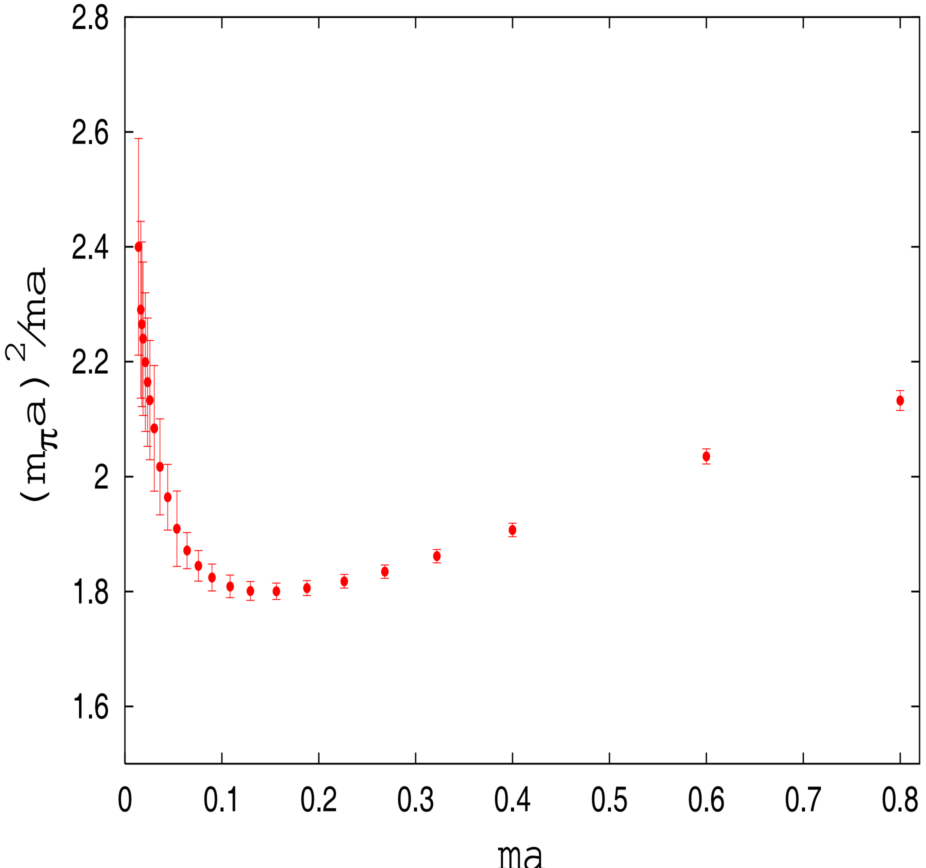
$$m_\pi^2 = Am\left\{1-\delta\left[\ln(Am/\Lambda_\chi^2)+1\right]\right\}+Bm^2$$

Quenched QCD

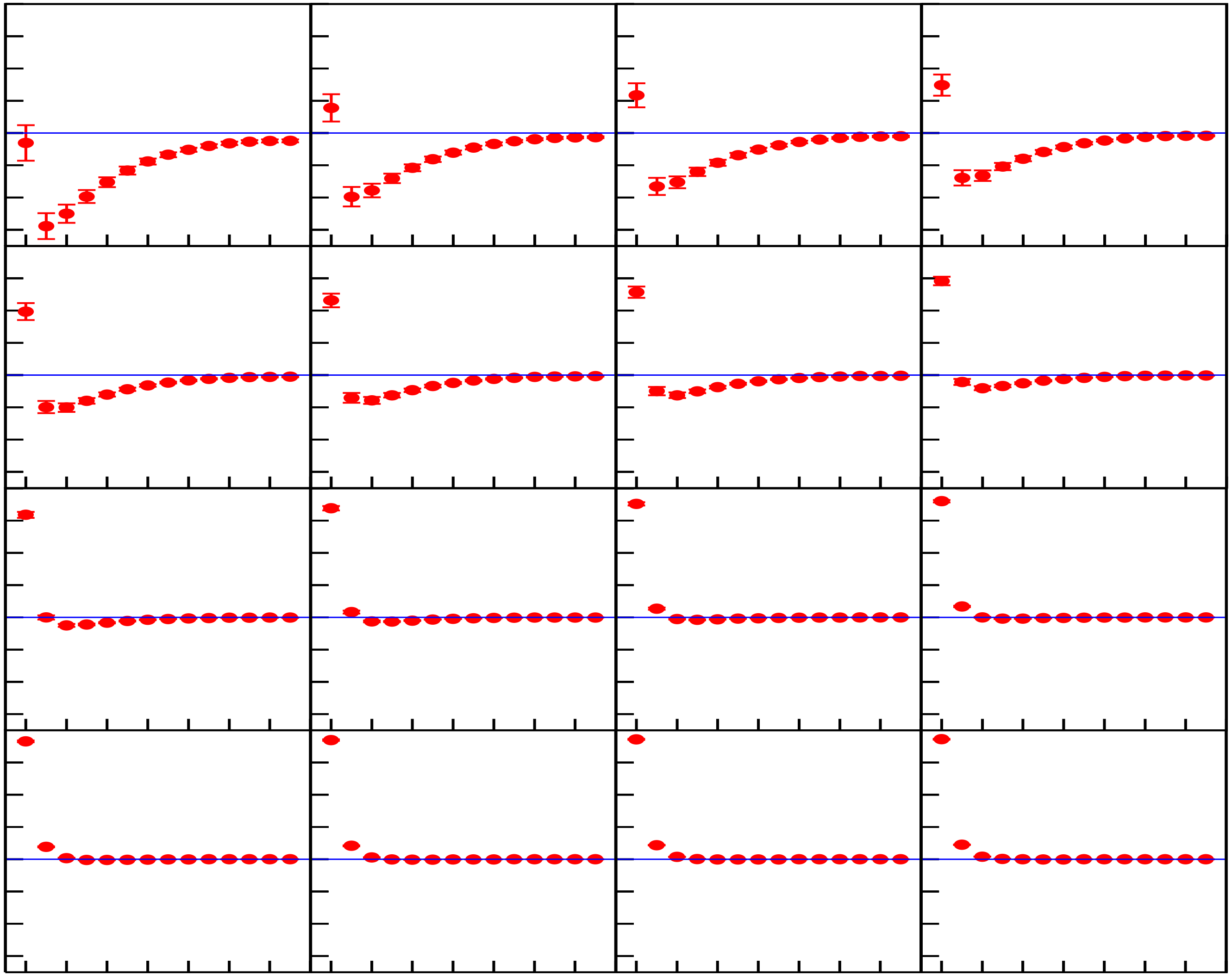


δ = 0.2 ± 0.03

Phys. Rev. D70, 034502 (2004)



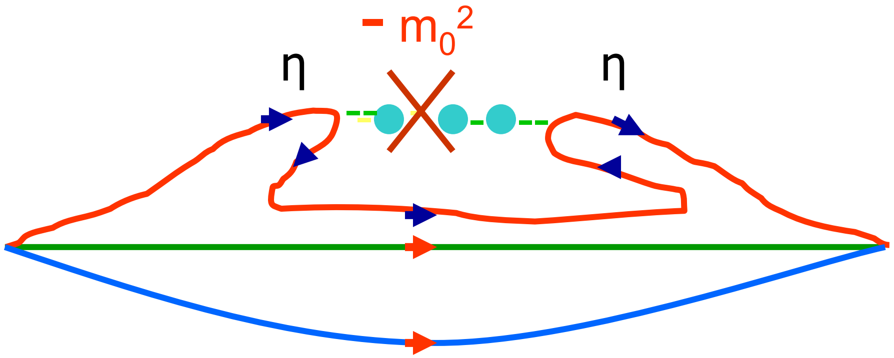
Is (0++) a two quark state?

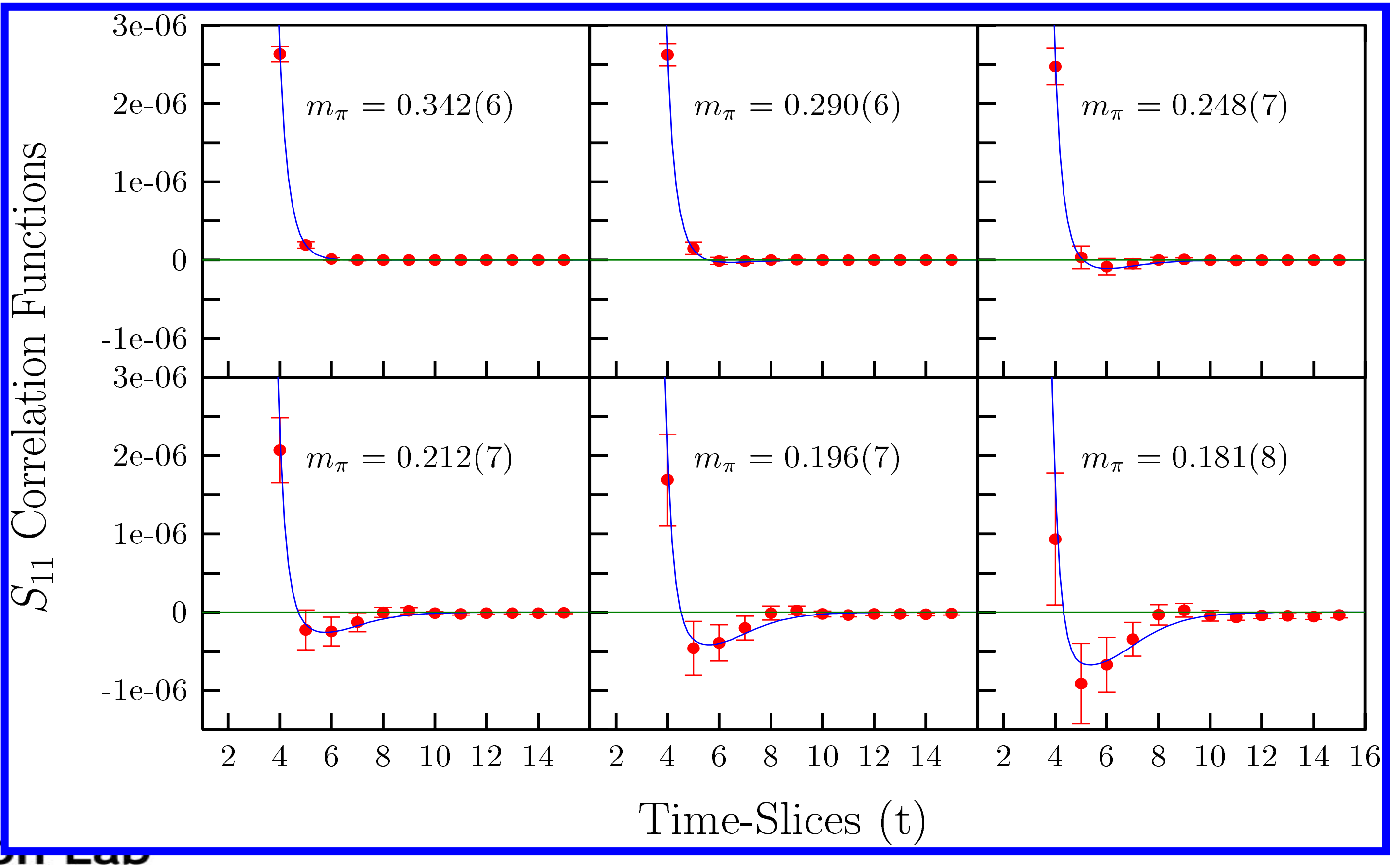


Correlation function for Scalar channel

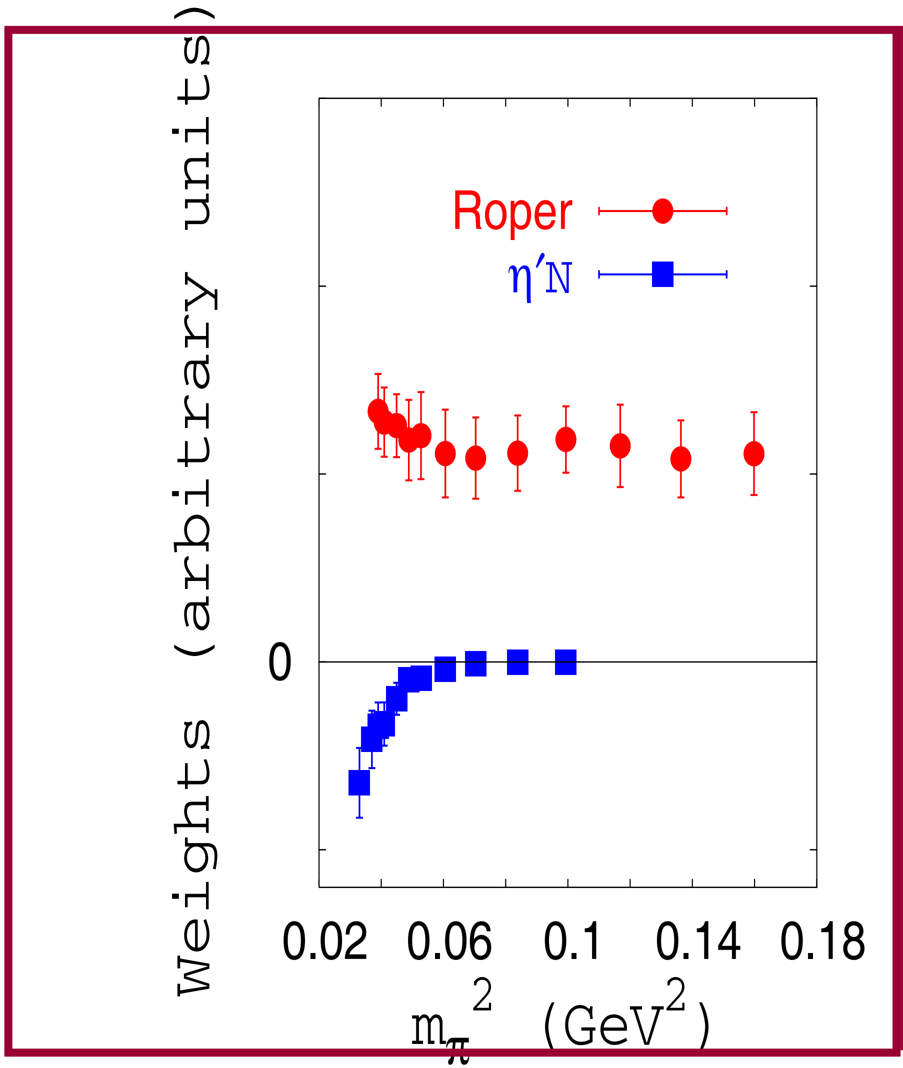
Ground state : πη′ ghost state, Excited state : a_0

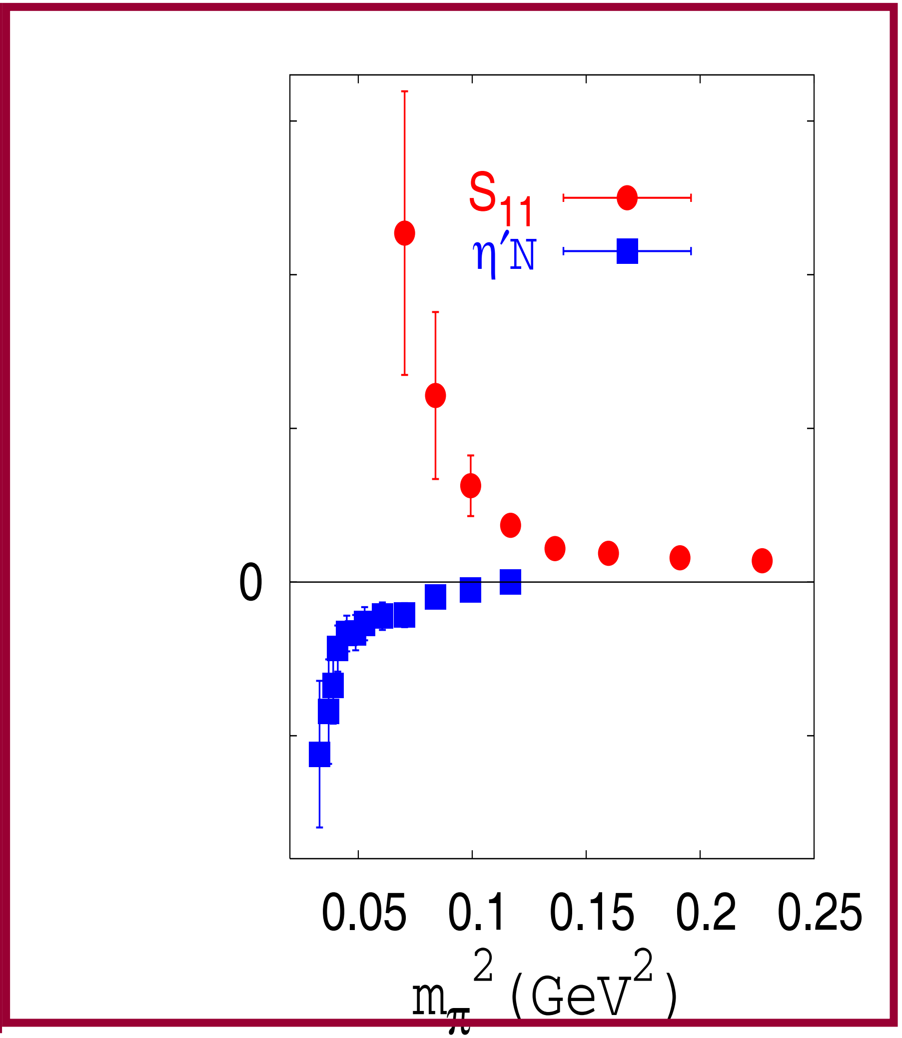
Evidence of η′N GHOST State in S_{11} (1535) Channel





Roper and S_{11}(1535)





Ghost States in Hadron Spectrum

Particle	Ghost State	Presence
a_0 (1450) 0^{++}	S-wave $\pi\ \eta'$ $2m_\pi < 1450$	Ground state $\pi\ \eta'$, a_0 . .
1^{-+} (1600?)	S-wave $a_1\ \eta'$ (1230 +140 < 1600)	Ground state $a_1\ \eta'$, 1^{-+}. .
N*(1440) $1/2^+$	P-wave N η' $\sqrt{m_\pi^2+p_L^2}+\sqrt{m_N^2+p_L^2} < m_N^*$?	1st or 2nd excited state N, N η' , N* ...
N*(1535) $1/2^-$	S-wave N η' (940 +140 < 1535)	Ground state N η' , N* . . .

Ghosts surround low pion mass quenched QCD

- We have isolated total 18 ghosts!
- Unless a good fitting algorithm, quenched analysis at low pion mass region will fall apart.

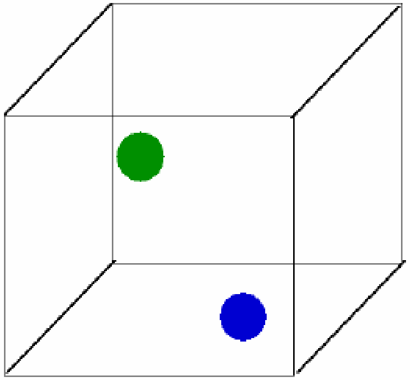
Properties of ghost states

- They contribute negatively to hadron correlators.
- They are multi-particle states. Weight depends on volume. Smaller volume have more dominant ghost states.
- They decouple from the spectrum around 250-350 MeV pion mass (for a_0, higher).
- Chiral fermions are more susceptible to ghost states than Wilson fermion.

Multiquark

Multi-particle states

A problem for finite box lattice



- ✓ Finite box : Momenta are quantized
- ✓ Lattice Hamiltonian can have both resonance and decay channels states (scattering states)

- ✓ A → x+y, Spectra of m_A and $\sqrt{m_x^2 + p_n^2} + \sqrt{m_y^2 + p_n^2}, \; p_n = \frac{2\pi n}{La}$
- ✓ One needs to separate out resonance states from scattering states
- ✓ Need multiple volumes, stochastic propagators

Scattering state and its volume dependence

Normalization condition requires :

$$| n, \vec{p}, s > \infty \sqrt{\frac{1}{V}} \, | n, \vec{p}, s)$$

Lattice

Continuum

Two point function :

$$G(t) = \sum_{\vec{x}} < 0 \,|\, T(\chi(\vec{x}, t)\, \overline{\chi}(0)) \,|\, 0 >$$

$$= \sum_{\vec{x}} \sum_{n} \frac{\left|\langle 0 \,|\, \chi(0) \,|\, n \rangle\right|^2}{2 M_n V} e^{-M_n t}$$

$$= \sum_{n} W_n e^{-M_n t} \qquad \sum_{\vec{x}} \Rightarrow \mathrm{V}$$

$$W_n = \frac{\left|\langle 0 \,|\, \chi(0) \,|\, n \rangle\right|^2}{2 M_n}$$

For one particle bound state

Spectral weight (W) will NOT be explicitly dependent on lattice volume

Scattering state and its volume dependence

Normalization condition requires :

$$\underset{\uparrow\atop\text{Lattice}}{|\,n,\vec{p},s>} \infty \sqrt{\frac{1}{V}}\,\underset{\uparrow\atop\text{Continuum}}{|\,n,\vec{p},s)}$$

Two point function :

$$G(t)=\sum_{\vec{x}}<0\,|\,T(\chi_1(\vec{x},t)\chi_2(\vec{x},t)\overline{\chi}_1(0)\overline{\chi}_2(0))\,|\,0>$$

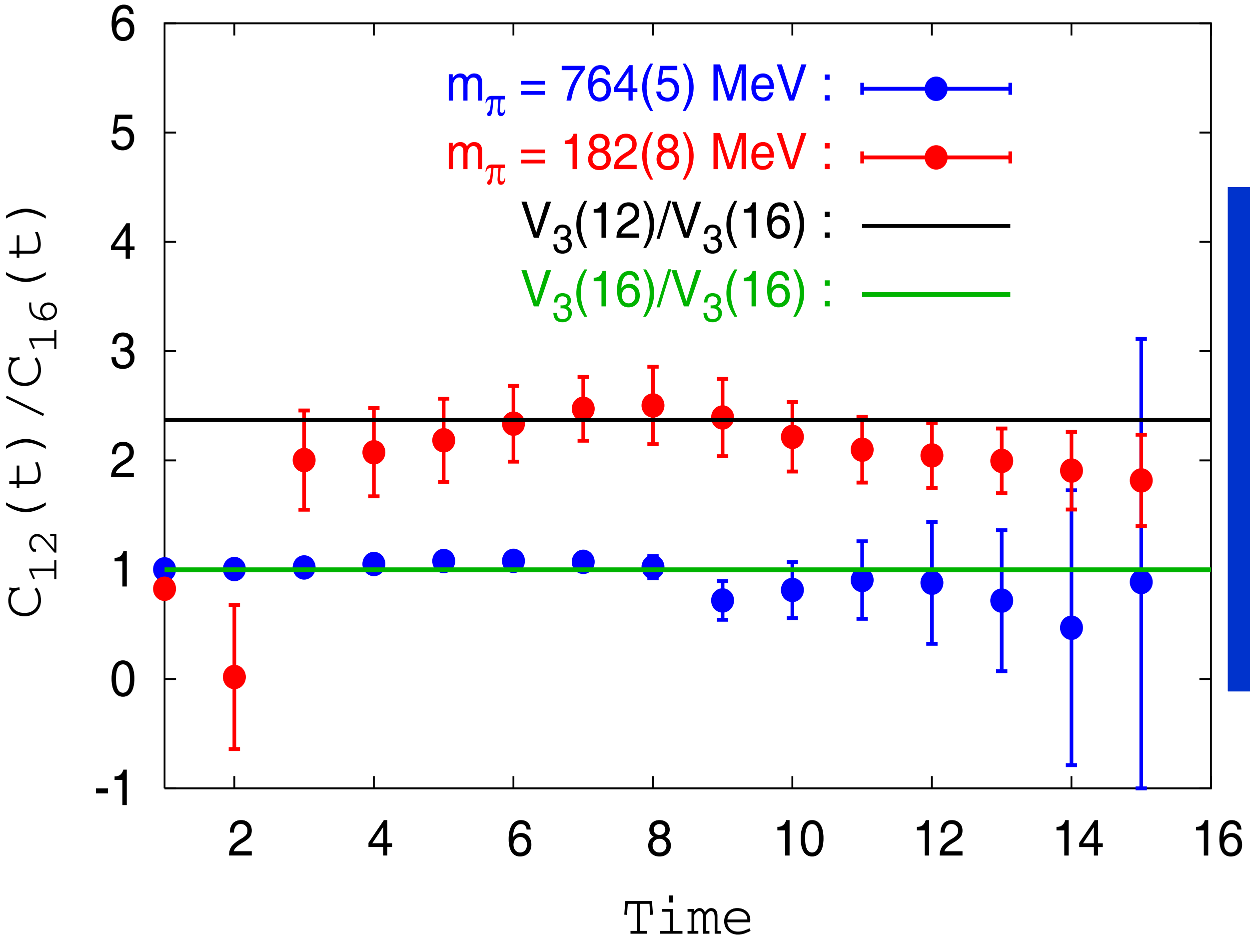
$$=\sum_{\vec{x}}\sum_{n_1,n_2}\frac{\left|\langle 0\,|\,\chi_1(0)\,|\,n_1\rangle\right|^2\left|\langle 0\,|\,\chi_2(0)\,|\,n_2\rangle\right|^2}{2M_{n_1}V\;\;2M_{n_2}V}\,e^{-\left(E_{n_1}+E_{n_1}\right)t}$$

$$\sum_{\vec{x}}\;\Rightarrow\;\mathrm{V}$$

$$=\sum_{n_1,n_2}\frac{W_{n_1}W_{n_2}}{\boxed{V}}\,e^{-\left(E_{n_1}+E_{n_1}\right)t}$$

For two particle scattering state

Spectral weight (W) WILL be explicitly dependent on lattice volume



$$\frac{G_{V1}(t)}{G_{V2}(t)} = \underset{t\to\infty}{\approx} \frac{W_{V1}\, e^{-m_0 t}}{W_{V2}\, e^{-m_0 t}}$$

$$\approx \frac{V_2}{V_1} \quad \text{two paticle}$$

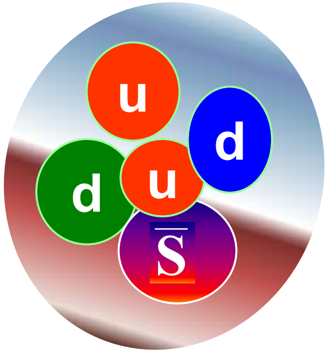
$$\approx 1 \quad \text{one particle}$$

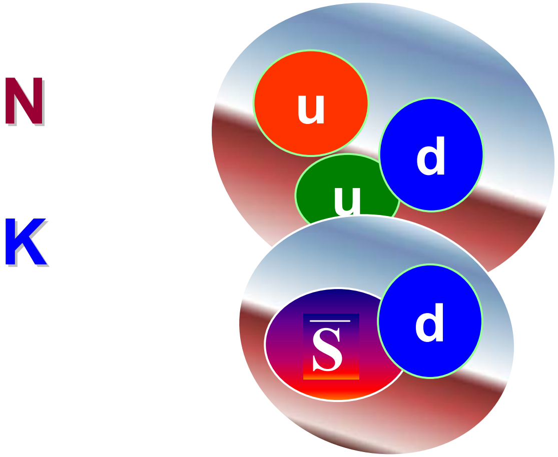
Ratio of scalar meson correlator at two volumes
and at two different quark mases

Problem of studying Θ^+ on the Lattice

Quark content : $uudd\bar{s}$

Two possible states





Θ^+ bound state
$m(\Theta^+) \sim 1540$ MeV

Two-particle NK scattering state

S-wave : $m_K + m_N \sim 1432$ MeV

P-wave : $\sqrt{m_K^2 + p^2} + \sqrt{m_N^2 + p^2}$

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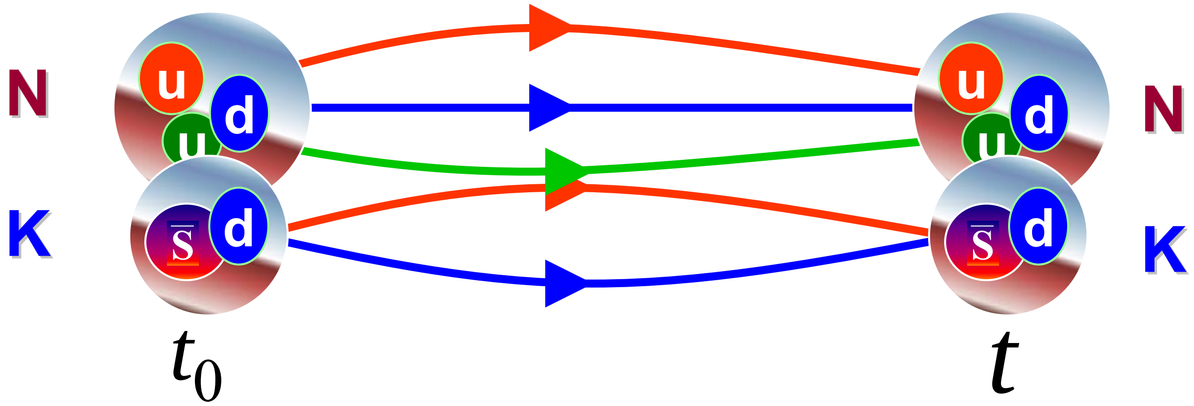
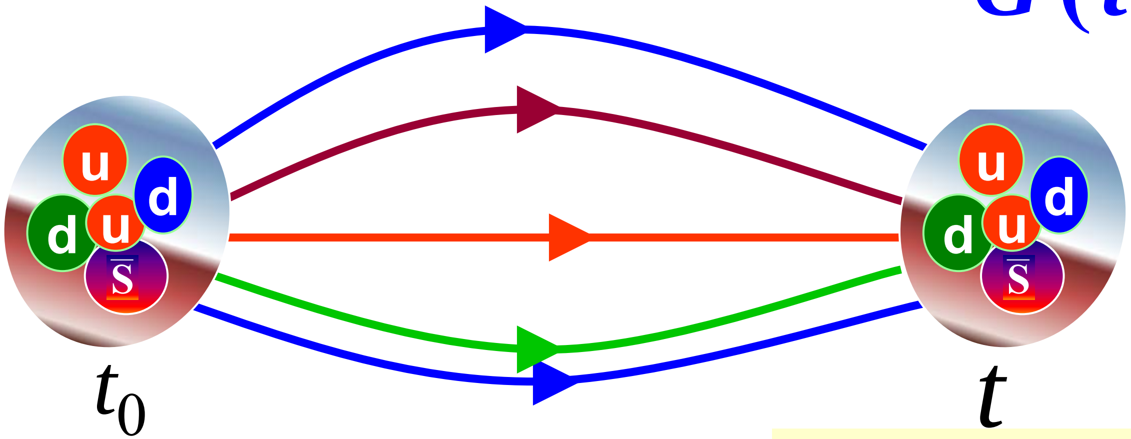


m_K+ m_N
~ 1432 MeV

$$G(t) = \sum_i W_i e^{-m_i t}$$



$m(\Theta^+)$ ~1540MeV

$C(t) \sim w_0 \exp(-m_{KN}\ t) + w_1 \exp(-m_\Theta\ t) + \ldots$

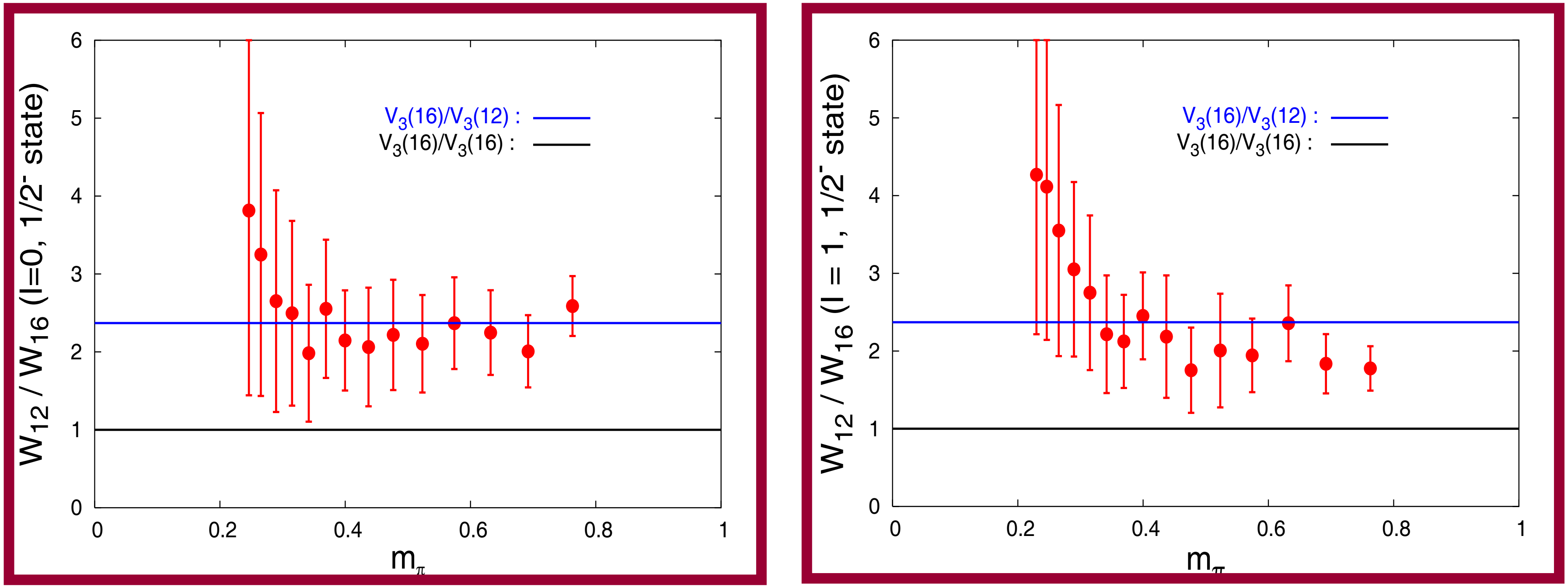
To separate out nearby states :

- Multi-exponential better fitting algorithm with high statistics
- Multi-operator cross correlator fitting with high statistics

Positive parity channel will have unphysical (ghost) states

Volume Dependence in 1/2⁻ channel

- For bound state, fitted weight will not show any volume dependence.
- For two particle scattering state, fitted weight will show inverse volume dependence



Phys.Rev.D70:074508,2004

Our observed ground state is S-wave scattering state

Spectrum Project

Octahedral group and lattice operators

Λ	J
G_1	$1/2 \oplus 7/2 \oplus 9/2 \oplus 11/2$ ...
G_2	$5/2 \oplus 7/2 \oplus 11/2 \oplus 13/2$ ...
H	$3/2 \oplus 5/2 \oplus 7/2 \oplus 9/2$ ...

Baryon

Λ	J
A_1	$0 \oplus 4 \oplus 6 \oplus 8$ ...
A_2	$3 \oplus 6 \oplus 7 \oplus 9$ ...
E	$2 \oplus 4 \oplus 5 \oplus 6$ ...
T_1	$1 \oplus 3 \oplus 4 \oplus 5$ ...
T_2	$2 \oplus 3 \oplus 4 \oplus 5$ ...

Meson

…R.C. Johnson, Phys. Lett.B 113, 147(1982)

Lattice operator construction

Three quark elemental operators :

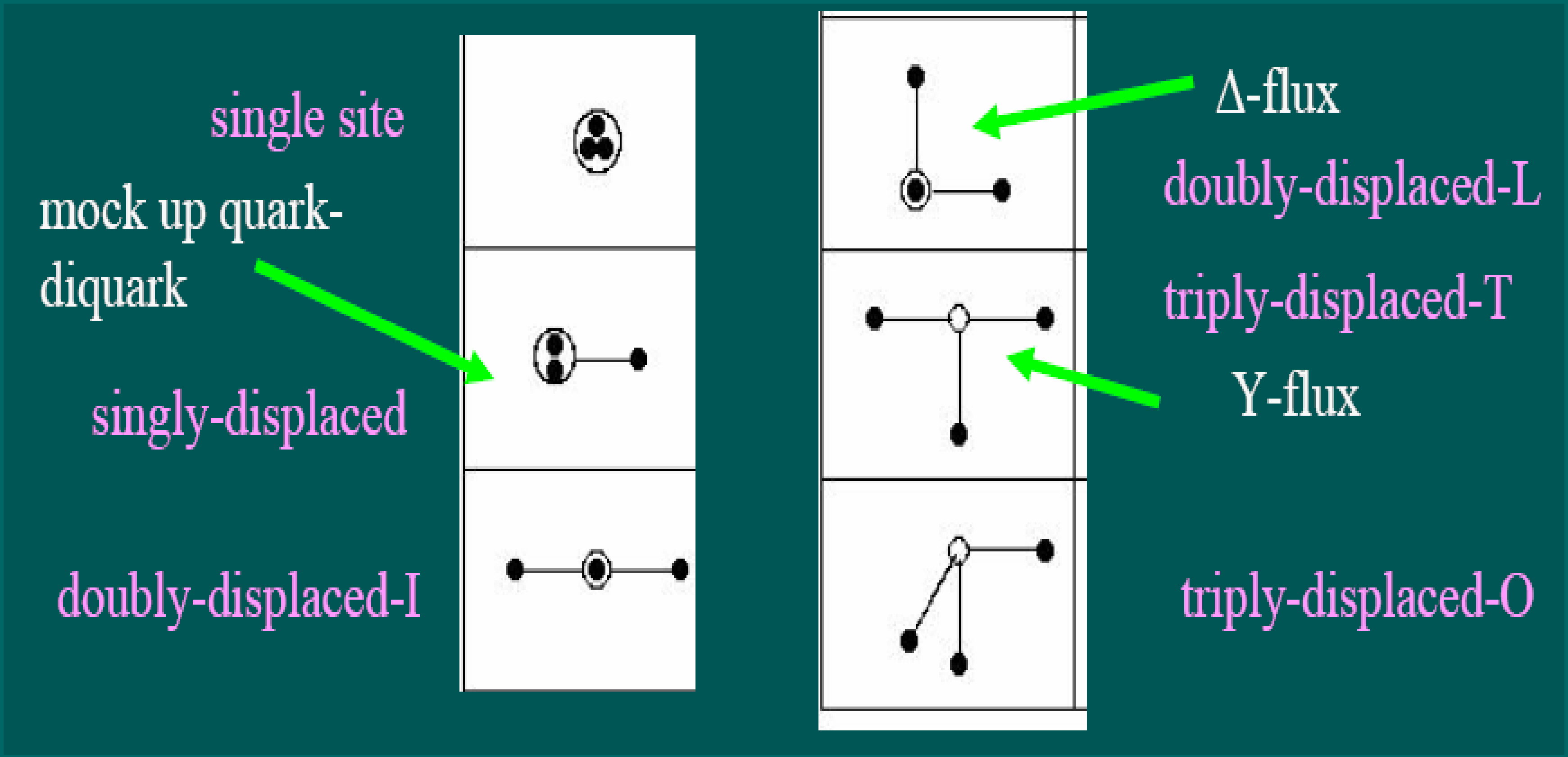
$$\Phi^{ABC}_{\alpha\beta\gamma,ijk}(t)=\sum_{\vec{x}}\varepsilon_{abc}\left(\tilde{D}_i^n\tilde{\psi}(\vec{x},t)\right)^A_{a\alpha}\left(\tilde{D}_i^n\tilde{\psi}(\vec{x},t)\right)^B_{b\beta}\left(\tilde{D}_i^n\tilde{\psi}(\vec{x},t)\right)^C_{c\gamma}$$

With covariant displacement

$$\tilde{D}_j^n(x,y)=\tilde{U}_j(x)\tilde{U}_j(x+\hat{j})\ldots\tilde{U}_j(x+(n-1)\hat{j})\delta_{y,x+n\hat{j}}\quad(j=\pm1,\pm2,\pm3)$$

Baryon	Operator
Δ^{++}	$\Phi^{uuu}_{\alpha\beta\gamma,ijk}$
Σ^{+}	$\Phi^{uus}_{\alpha\beta\gamma,ijk}$
N^{+}	$\Phi^{uud}_{\alpha\beta\gamma,ijk}-\Phi^{duu}_{\alpha\beta\gamma,ijk}$
Ξ^{0}	$\Phi^{ssu}_{\alpha\beta\gamma,ijk}$
Λ^{0}	$\Phi^{uds}_{\alpha\beta\gamma,ijk}-\Phi^{dus}_{\alpha\beta\gamma,ijk}$
Ω^{-}	$\Phi^{sss}_{\alpha\beta\gamma,ijk}$

…C. Morningstar



single site

mock up quark-diquark

singly-displaced

doubly-displaced-I

Δ-flux

doubly-displaced-L

triply-displaced-T

Y-flux

triply-displaced-O

Radial structure : displacements of different lengths
Orbital structure : displacements in different directions

…C. Morningstar

Variational Method …Luscher and Wolf, NPB 339, 220 (1990)

- Each operator $\phi_{\alpha}(t)$ can project to any quantum state

$$\phi_\alpha(t)|0\rangle = \sum_n |n\rangle\langle n|\phi_\alpha(t)|0\rangle$$

$$A_\alpha^n(t) = \langle n|\phi_\alpha(t)|0\rangle$$

- Need to find out variational coefficients $\{v_\alpha^{(m)}, \alpha = 1,2,\ldots, n\}$ such that the overlap to a state is maximum

$$\Phi^{(m)}(t)|0\rangle = \sum_\alpha^N v_\alpha^{(m)}\phi_\alpha(t)|0\rangle$$

$$= (1-\varepsilon_m)e^{-\hat{H}t}|m\rangle + \sum_{n\neq m}\varepsilon_n e^{-\hat{H}t}|n\rangle \quad \text{with } \varepsilon_n << 1$$

- In practice diagonalize the variational matrix

$$\begin{bmatrix} C_{11} & C_{12} & \ldots & C_{1n} \\ C_{21} & C_{22} & \ldots & C_{2n} \\ .. & .. & \ldots & .. \\ C_{n1} & C_{n2} & \ldots & C_{nn} \end{bmatrix}$$

$$C_{\alpha\beta}(t) = \langle 0|\phi_\alpha(t)\phi_\beta^+(0)|0\rangle \text{ by}$$

$$C(t_0)^{-1/2}C(t)C(t_0)^{-1/2} \text{ with eigenvalue } \lambda_\alpha(t,t_0),$$

$$\text{where, } \lim_{t\to\infty}\lambda_\alpha(t,t_0) = e^{-(t-t_0)E_\alpha}(1+e^{-t\Delta E_\alpha})$$

- Construct the optimal operator

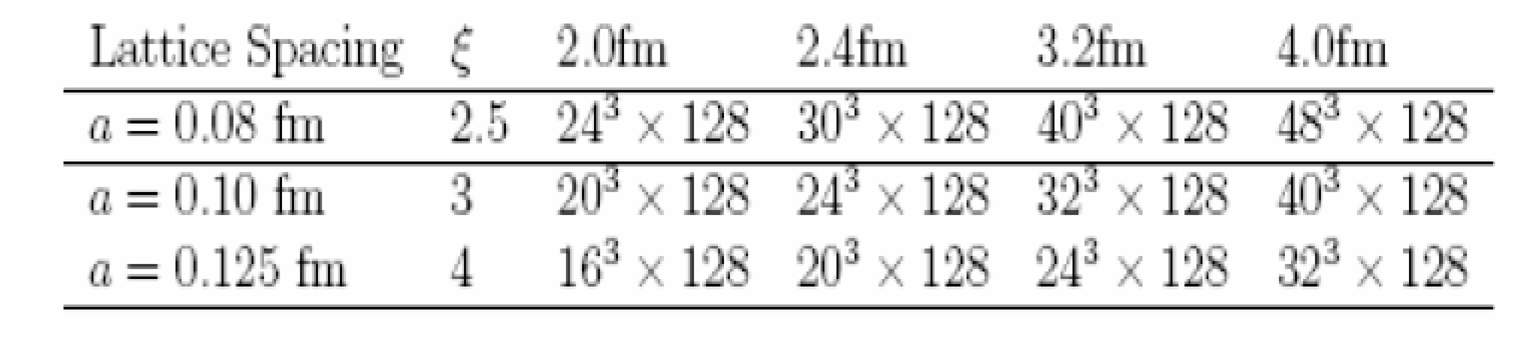
$$C^{(m)}(t) = \langle 0|\Phi^{(m)}(t)\Phi^{(m)}(0)|0\rangle$$

$$= \sum_{\alpha\beta} v_\alpha^{(m)}v_\beta^{(m)}C_{\alpha\beta}(t)$$

$$= (1-\varepsilon_m)^2 e^{-E_m t} + O(\varepsilon^2)$$

Anisotropic Clover Lattice

- Gauge Action : Wilson
- Fermion Action : Clover
- Anisotropy (finer temporal lattice spacing)
- Stout smearing



Lattice Spacing	ξ	2.0fm	2.4fm	3.2fm	4.0fm
$a = 0.08$ fm	2.5	$24^3 \times 128$	$30^3 \times 128$	$40^3 \times 128$	$48^3 \times 128$
$a = 0.10$ fm	3	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 128$	$40^3 \times 128$
$a = 0.125$ fm	4	$16^3 \times 128$	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 128$

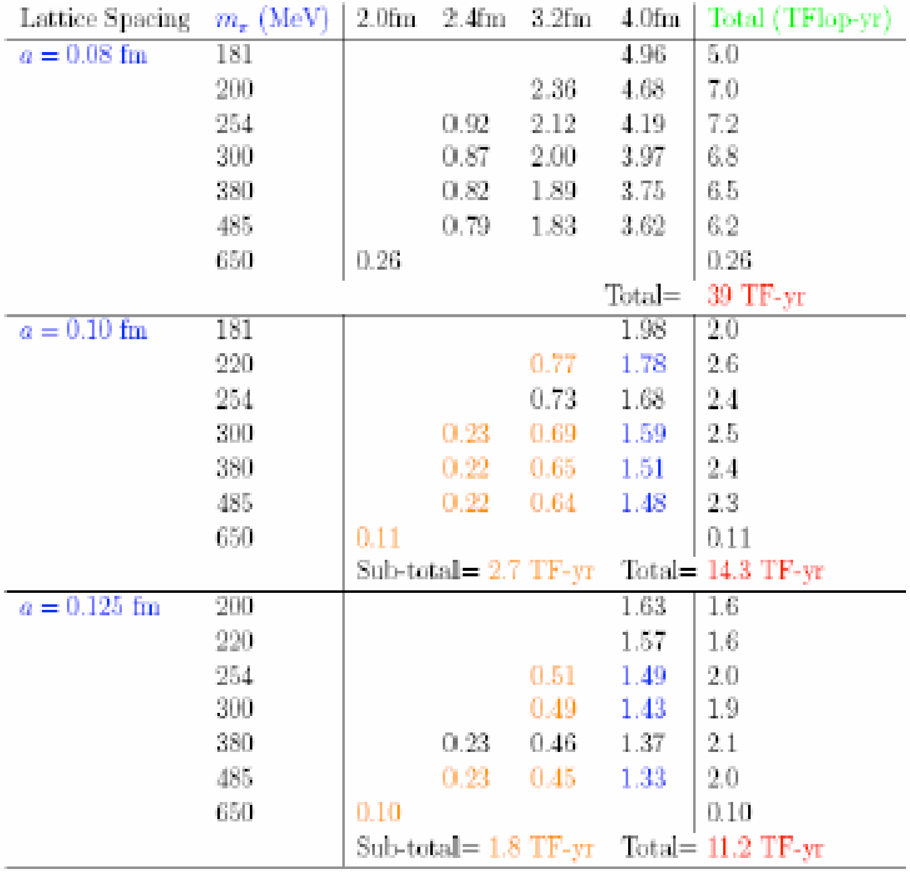
Expected lattice sizes

Anisotropic Clover: dynamical generation

Estimated cost of N_f=2+1 production (in TFlop-yrs) for 7K traj

$$\mathrm{Cost}_{\mathrm{traj}} = \xi^{1.25}\left(\frac{\mathrm{fm}}{a_s}\right)^6 \cdot \left[\left(\frac{L_s}{\mathrm{fm}}\right)^3\left(\frac{L_t}{\mathrm{fm}}\right)\right]^{5/4} \cdot [C_1 + C_2/m_l] .$$

- Phase I - initial production at a=0.1fm
  - Hybrid photo-couplings
  - cost = 2.7 TF-yr + 10% analysis
- Phase II - all of Phase I + 4.0fm
  - Baryon spectra
  - cost = 10 TF-yr + 50% analysis
- Phase III - add a=0.125fm
  - Spectra at light pion mass and 2 lattice spacing for cont. limit
  - cost = 16 TF-yr + 50% analysis
- Phase IV - add a=0.08fm
  - Full continuum limit
  - cost ~ 42 TF-yr + 50% analysis



Lattice Spacing	m_π (MeV)	2.0fm	2.4fm	3.2fm	4.0fm	Total (TFlop-yr)
$a = 0.08$ fm	181				4.96	5.0
	200			2.36	4.68	7.0
	254		0.92	2.12	4.19	7.2
	300		0.87	2.00	3.97	6.8
	380		0.82	1.89	3.75	6.5
	485		0.79	1.83	3.62	6.2
	650	0.26				0.26
					Total=	39 TF-yr
$a = 0.10$ fm	181				1.98	2.0
	220			0.77	1.78	2.6
	254			0.73	1.68	2.4
	300		0.23	0.69	1.59	2.5
	380		0.22	0.65	1.51	2.4
	485		0.22	0.64	1.48	2.3
	650	0.11				0.11
		Sub-total= 2.7 TF-yr			Total=	14.3 TF-yr
$a = 0.125$ fm	200				1.63	1.6
	220				1.57	1.6
	254			0.51	1.49	2.0
	300			0.49	1.43	1.9
	380		0.23	0.46	1.37	2.1
	485		0.23	0.45	1.33	2.0
	650	0.10				0.10
		Sub-total= 1.8 TF-yr			Total=	11.2 TF-yr

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R.G. Edwards, USQCD All Hands 2007

Recently Observed Hadrons

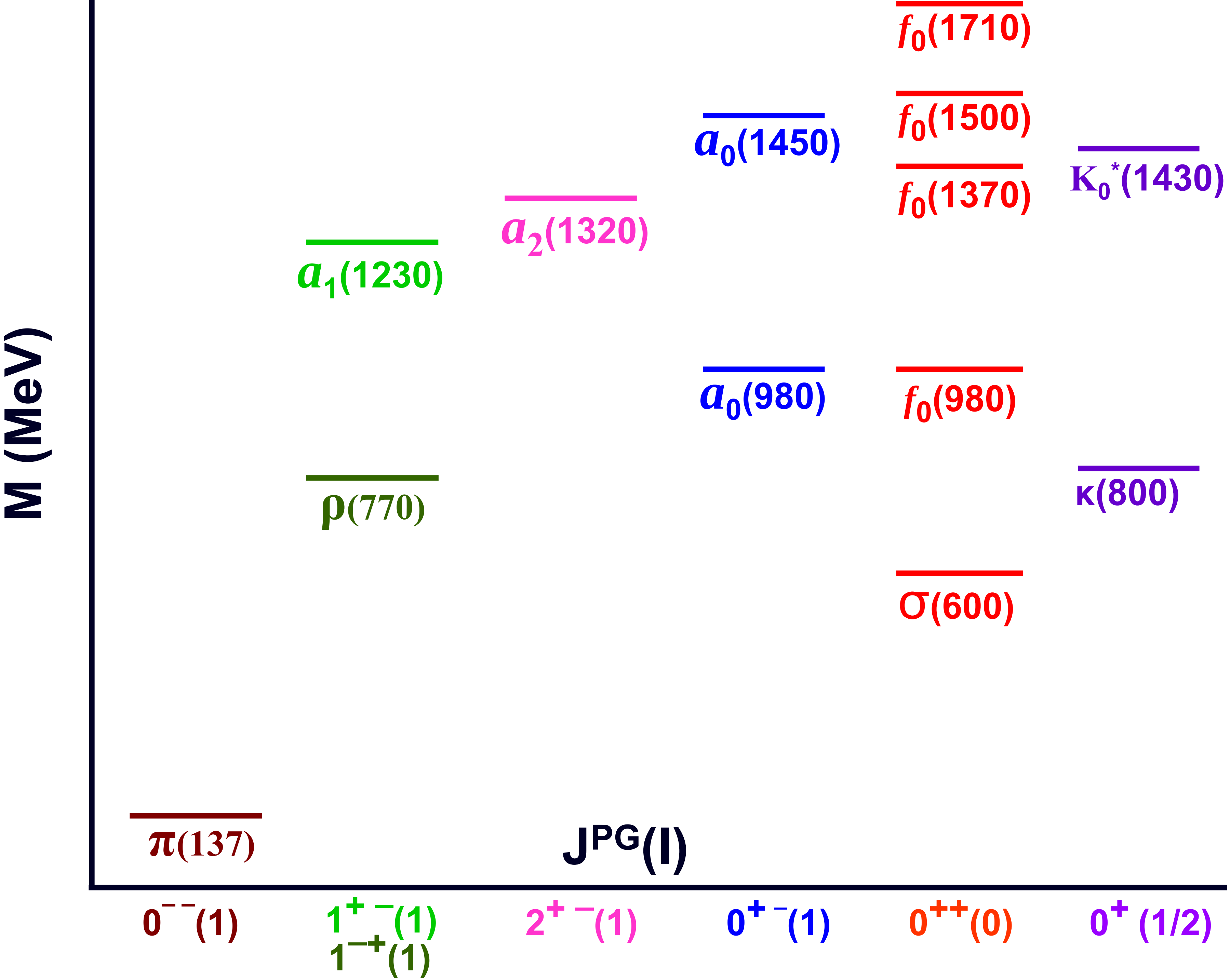
Hadrons	**Experiments**
D_{sJ} (2317) → $D_s\pi_0$	**PRL 90, 242001(2003) BABAR**
D_{sJ}(2460) → $D_s{}^*\pi_0$	**PRD68, 032002 (2003) CLEO**
X(3872) → $J/\psi\pi^+\pi^-$	**hep-ex/0308029, SELEX**
Y(3940), Y(4260)	**hep-ex/0507019, 0507033, 0506081**
$\Xi_{CC}{}^{++}$ (3460)	
$\Xi_{CC}{}^{+}$ (3520)	**PRL89,11 2001(2002) SELEX,**
$\Xi_{CC}{}^{++}$ (3780)	

… Mathur, Lewis, Woloshyn et. al. PRD66, 014502 (2002); PRD64, 094509 (2001)

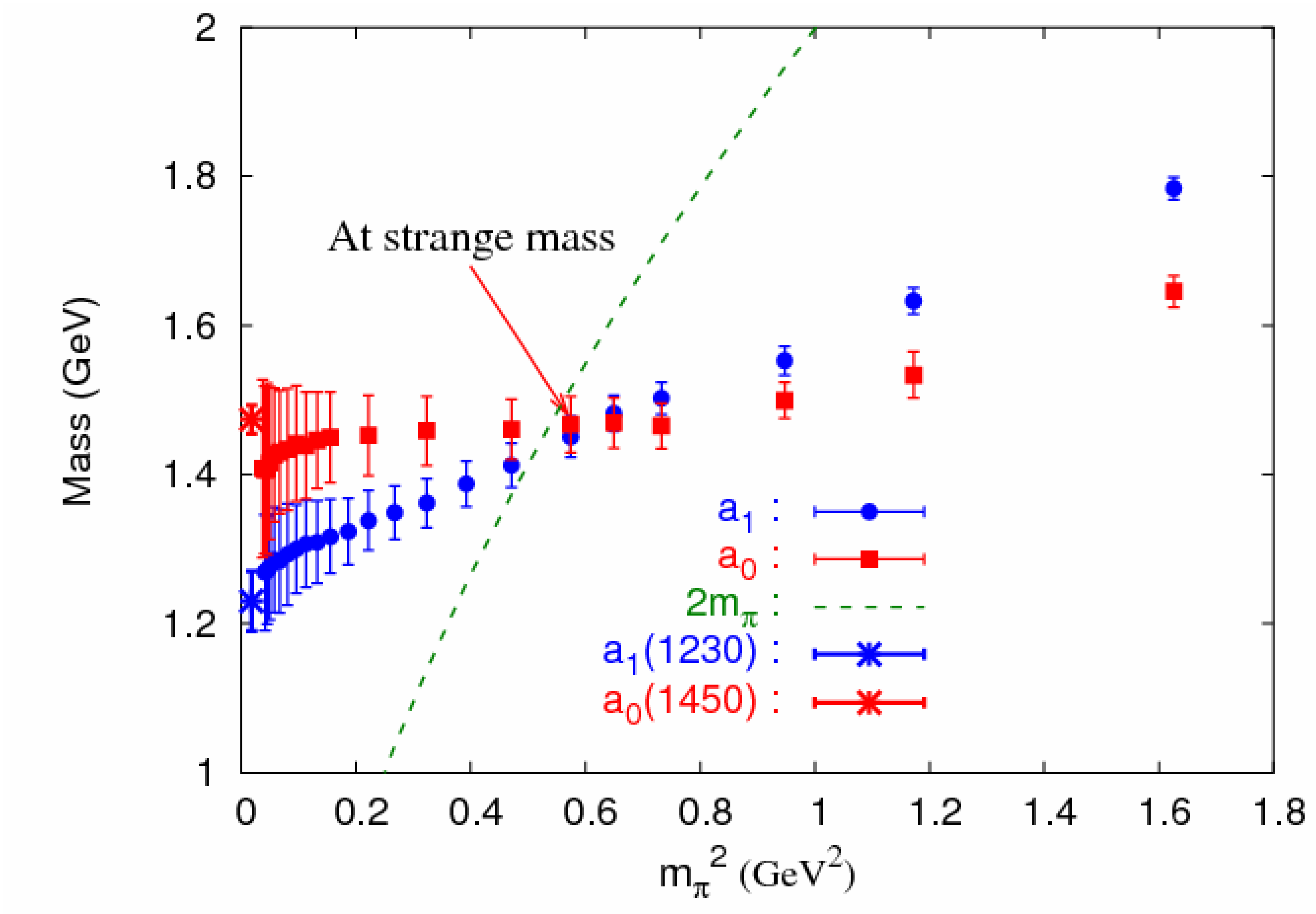
$\Xi_{CC}{}^{+}$: 3560(47)($^{27}{}_{25}$)

$\Omega_c^* - \Omega_c$	$65(13)(^{7}_{8})$
$\Sigma_c - \Lambda_c$	$128(28)(^{39}_{28})$
$\Xi_c' - \Xi_c$	$104(19)(^{20}_{23})$
Ξ_{cc}	$3562(47)(^{27}_{25})$
Ω_{cc}	$3681(44)(^{17}_{19})$
$\Xi_{cc}^* - \Xi_{cc}$	$63(14)(^{9}_{7})$
$\Omega_{cc}^* - \Omega_{cc}$	$56(8)(^{7}_{6})$
Λ_b	$5664(98)(^{33}_{46})$
Ξ_b	$5762(83)(^{29}_{38})$

MESONS


M (MeV)
f0(1710)
f0(1500)
a0(1450)
K0*(1430)
f0(1370)
a2(1320)
a1(1230)
a0(980)
f0(980)
κ(800)
ρ(770)
σ(600)
π(137)
JPG(I)
0−−(1)
1+−(1)
1−+(1)
2+−(1)
0+−(1)
0++(0)
0+ (1/2)

$\bar{\psi}\psi$ $I^G(J^{PC}) \equiv 1^-(0^{++}),\ 1^-(1^{++})$



Our results shows scalar mass around 1400-1500 MeV, suggesting

$a_0(1450)$ is a two quark state … hep-ph/0607110

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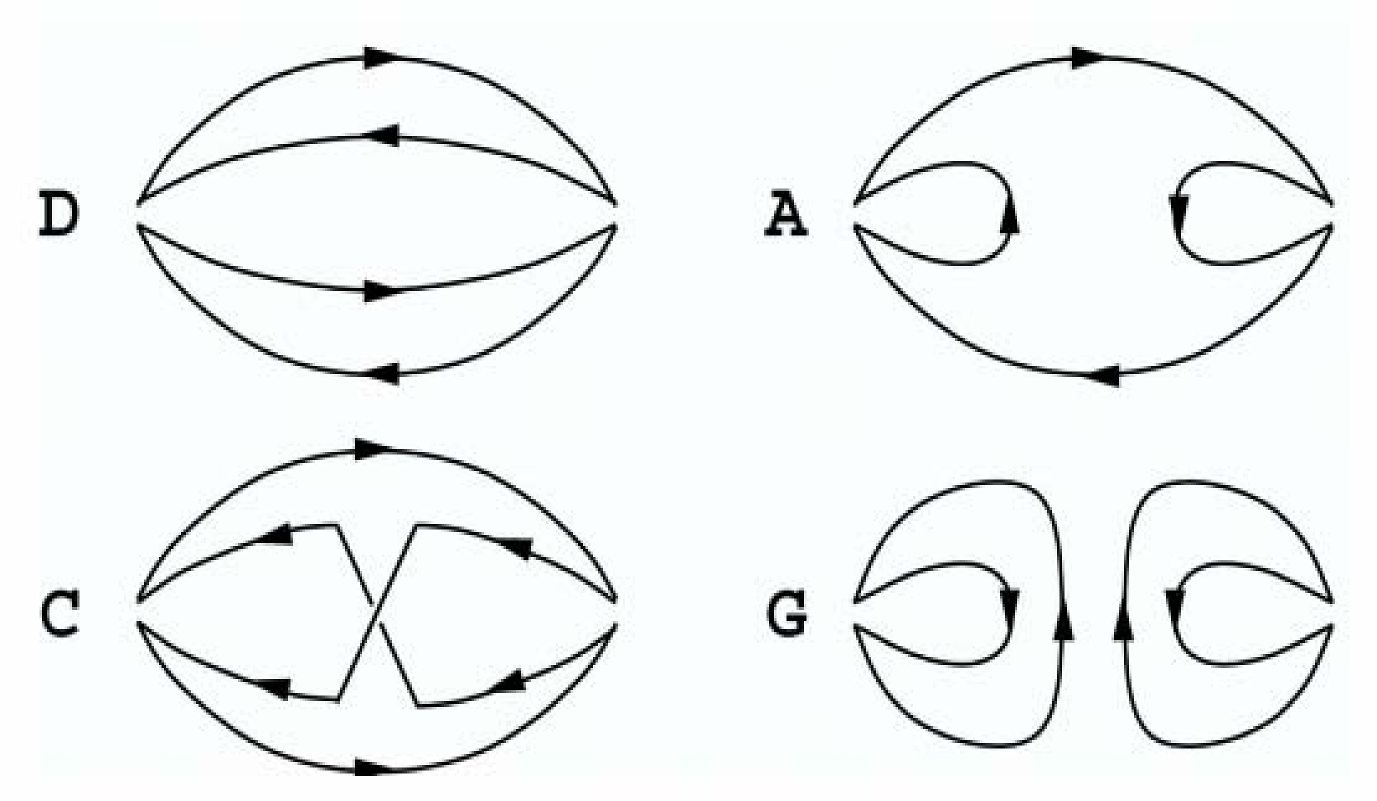
$\pi\pi$ four quark operator (I=0)

$$\chi = \frac{1}{\sqrt{3}}\left[\pi^+\pi^- + \pi^-\pi^+ + \pi^o\pi^o\right],$$

$$\pi^+ = \bar{u}\gamma_5 d$$

$$\pi^- = \bar{d}\gamma_5 u$$

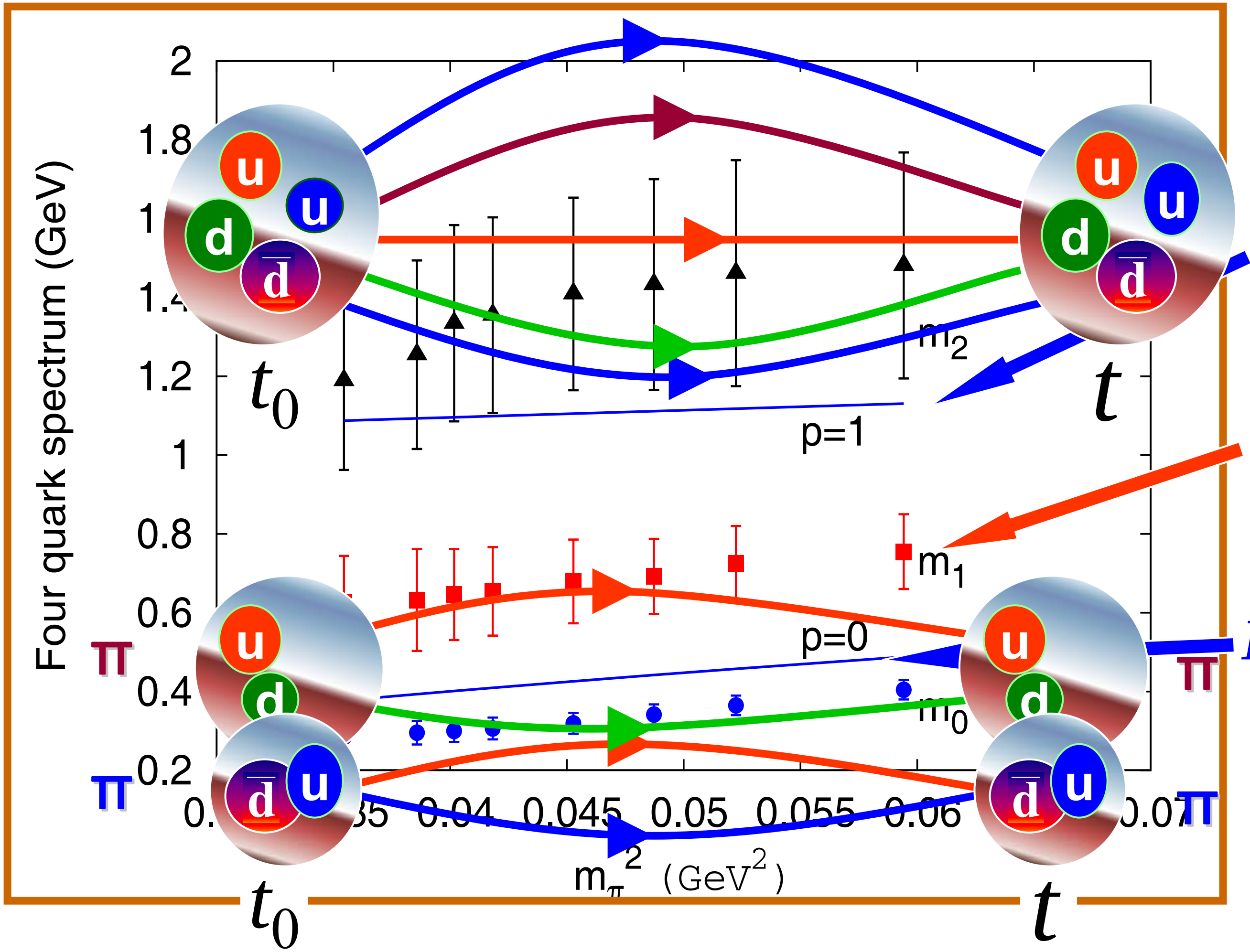
$$\pi^o = \frac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$$



$$\left\langle \chi(t)\chi^{+}(0)\right\rangle = 2\left[D(t)+\frac{1}{2}C(t)-3\left(A(t)-\frac{1}{2}G(t)\right)\right], \quad \mathbf{I=0}\ ,$$

$$= D(t)-C(t)\ , \quad \mathbf{I=2}$$

$\overline{\psi}\gamma_5\psi\ \overline{\psi}\gamma_5\psi\ [\pi\pi, \mathrm{I}^{\mathrm{G}}(\mathrm{J}^{\mathrm{PC}}) \equiv 0^{+}(0^{++})]$



Scattering states

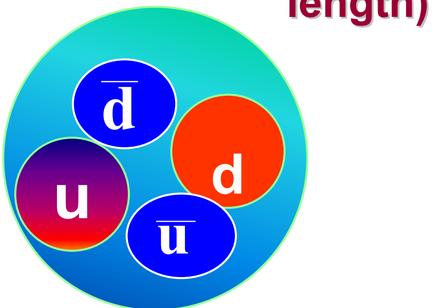
$E_\pi(p=1) + E_\pi(p=1)$

Possible BOUND state

σ(600)?

$E_\pi(p=0) + E_\pi(p=0)$

Scattering states
(Negative scattering
length)



Evidence for a tetraquark state?

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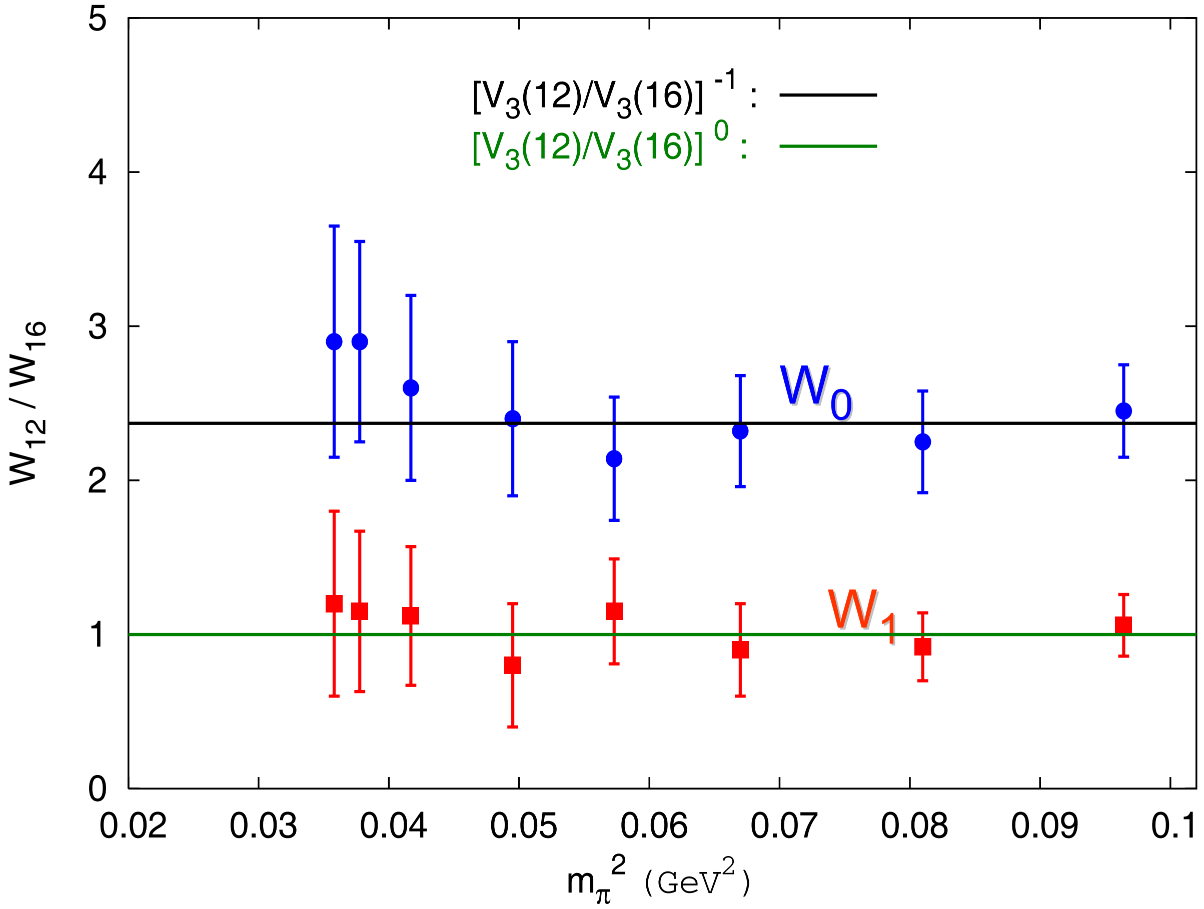
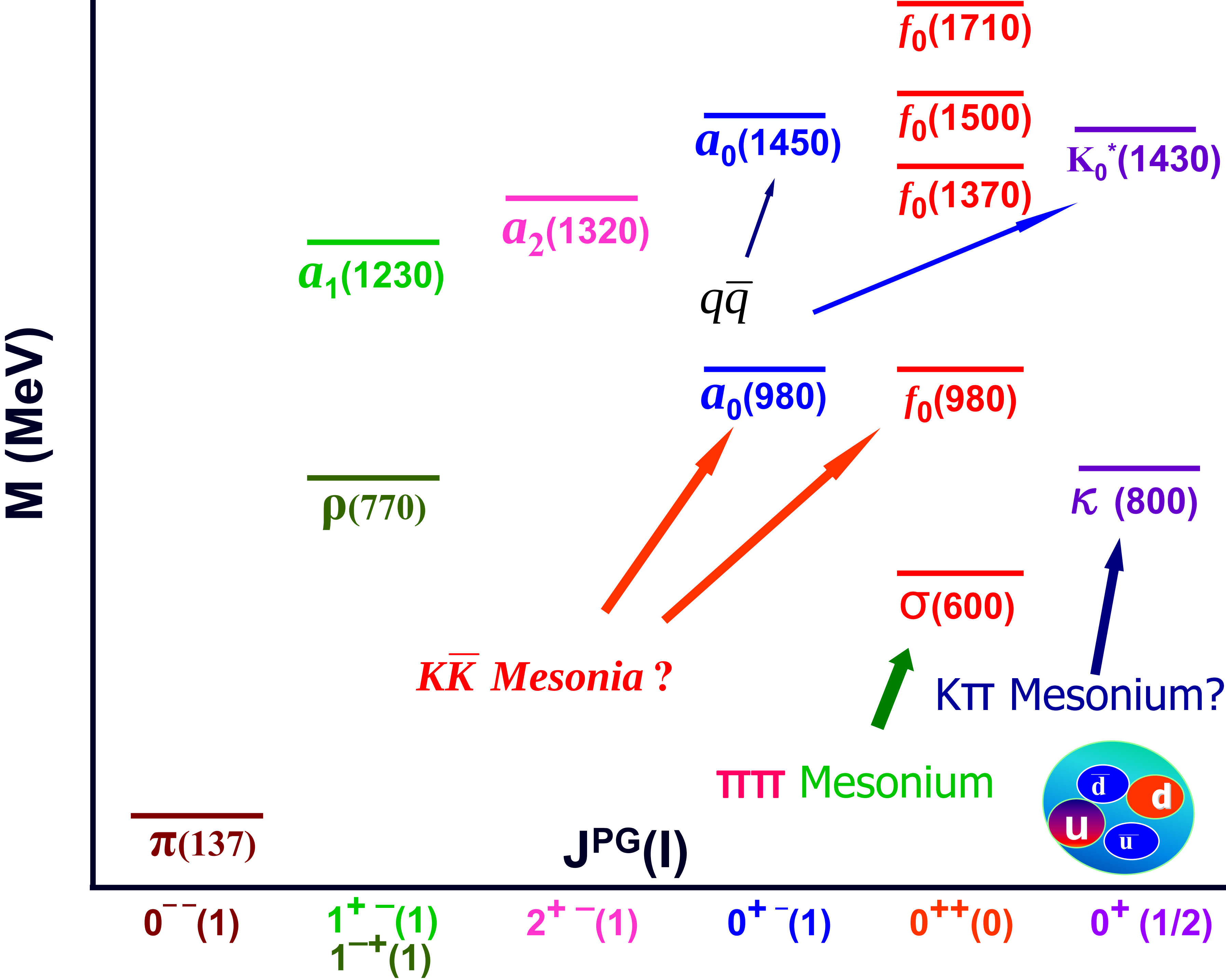
hep-ph/0607110

Volume dependence of spectral weights

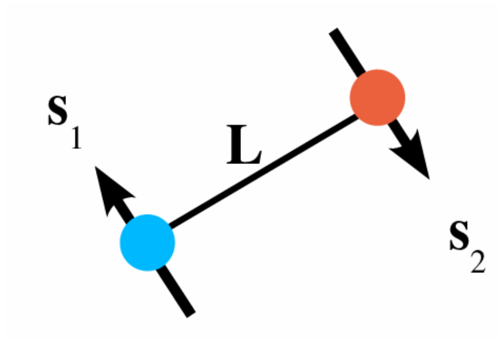


hep-ph/0607110

Volume independence suggests the observed state is an one particle state


M (MeV)
$f_0(1710)$
$f_0(1500)$
$a_0(1450)$
$K_0^*(1430)$
$f_0(1370)$
$a_2(1320)$
$a_1(1230)$
$q\bar{q}$
$a_0(980)$
$f_0(980)$
$\rho(770)$
κ (800)
$\sigma(600)$
$K\bar{K}$ Mesonia ?
Kπ Mesonium?
ππ Mesonium
$\bar{d}$
d
u
$\bar{u}$
$\pi(137)$
$J^{PG}(I)$
$0^{--}(1)$
$1^{+-}(1)$
$1^{-+}(1)$
$2^{+-}(1)$
$0^{+-}(1)$
$0^{++}(0)$
0^{+} (1/2)

Hybrids



S = 0, 1

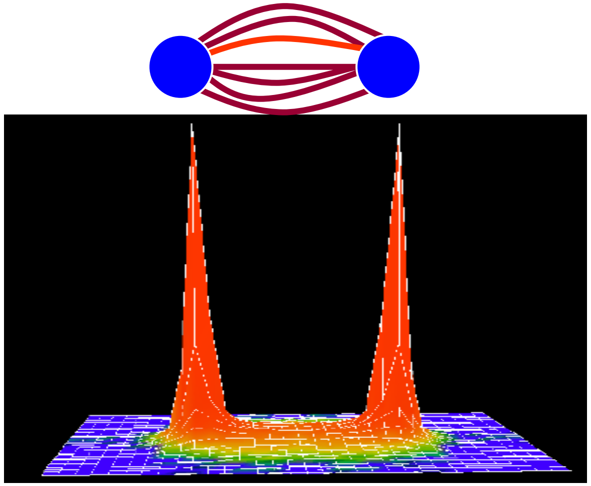
L = 0, 1, 2, 3…

J = L + S

$$\vec{J} = \vec{L} + \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

Allowed : $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}, \ldots.$

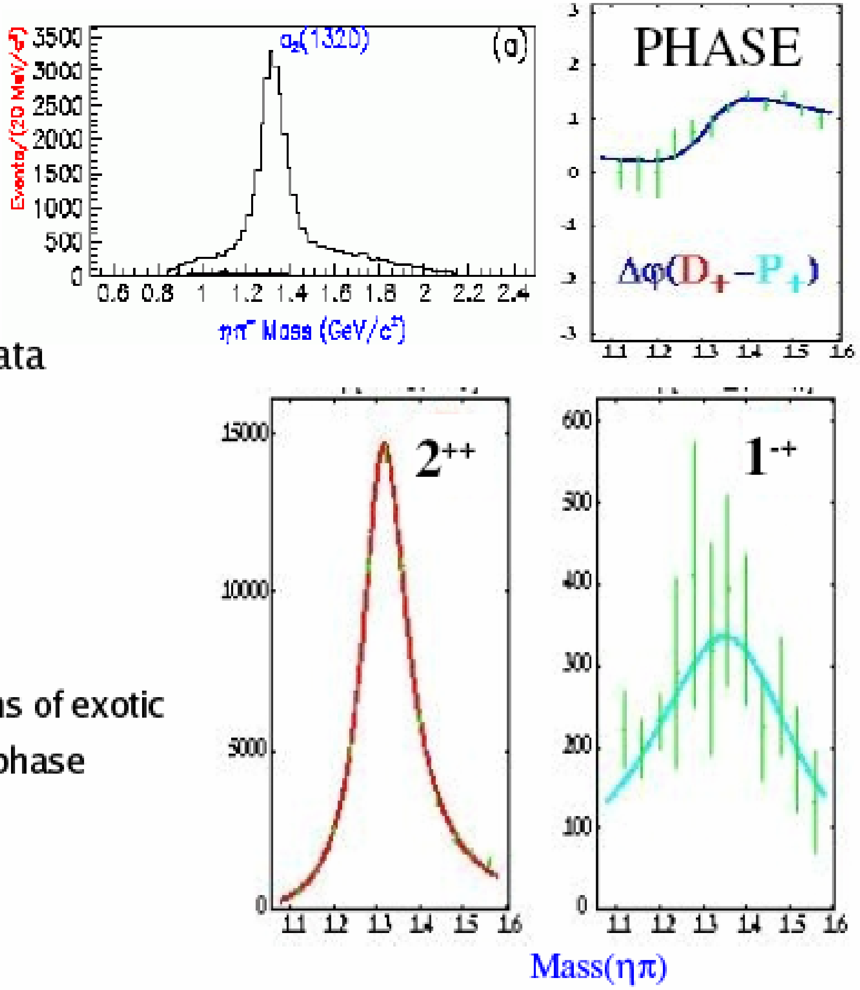
Forbidden (Exotics) : $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 1^{++}, 3^{-+}, 4^{+-}, \ldots.$



$J^{PC} = 1^{-+}$ Exotic Candidate

$\pi_1(1400) \rightarrow \eta\pi$

- BNL-E852 & Crystal Barrel
  - Observe structure in the cross section & rapid phase motion
  - Simple Breit-Wigner ansatz describes data
  - Both Claims of Exotic Nature
- VES & IU-E852$_{\eta\pi 0}$
  - Observe structure in the cross section & rapid phase motion
    - VES reported results earlier but no claims of exotic
    - IU-E852 $\eta\pi^0$ result differs slightly in the phase
  - Results compatible with BNL findings
  - VES Did not Claim Exotic
  - IU-E852 claims rescattering effect

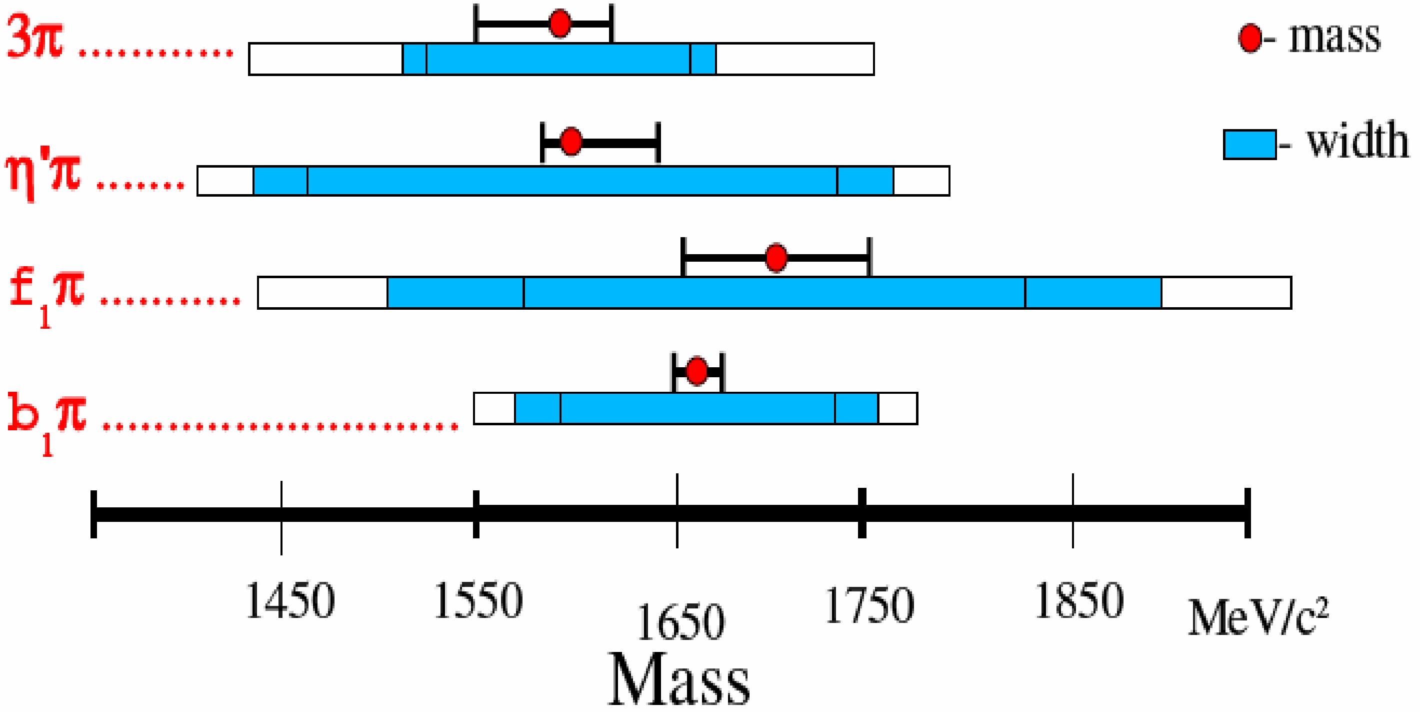


All groups agree on findings but disagree on interpretation

Jefferson Lab

Paul Eugenio …Lattice 2006

$\pi_1(1600)$ Consistency



Not Outrageous, but not great agreement

$$\pi^- p \rightarrow p X^-$$

Results from all 4 channels suggest Pomeron production

Paul Eugenio …Lattice 2006

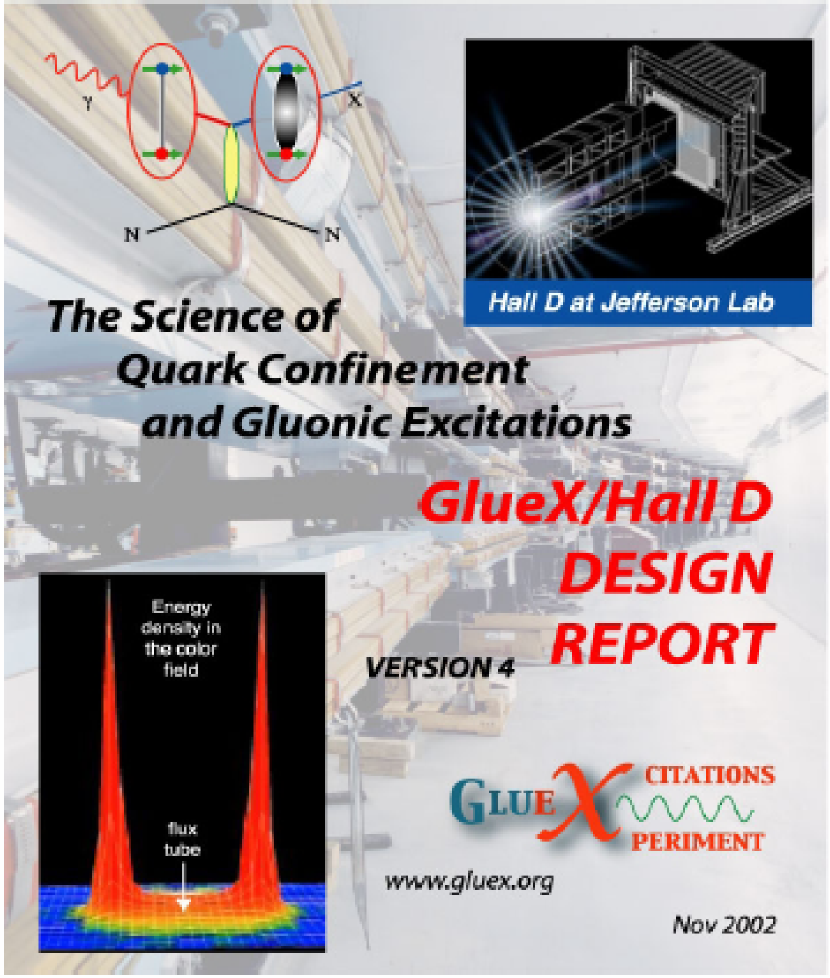
GlueX Collaboration

~100 Physicists   7 Countries   25 Institutions

Australia

Canada

Greece



Hall D at Jefferson Lab

The Science of Quark Confinement and Gluonic Excitations

GlueX/Hall D DESIGN REPORT

VERSION 4

GLUE X CITATIONS PERIMENT

www.gluex.org

Nov 2002

USA

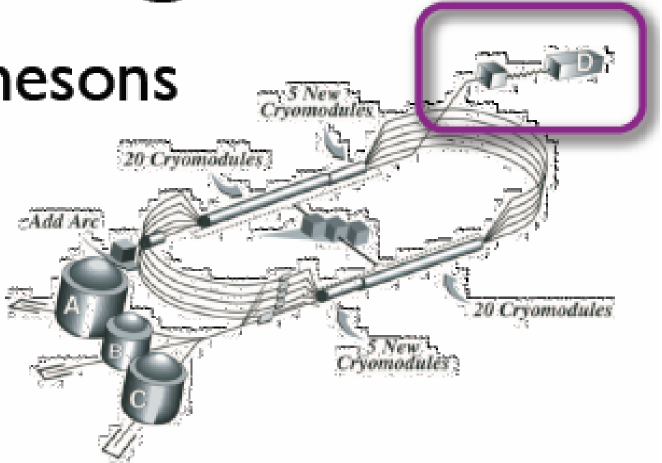
Russia

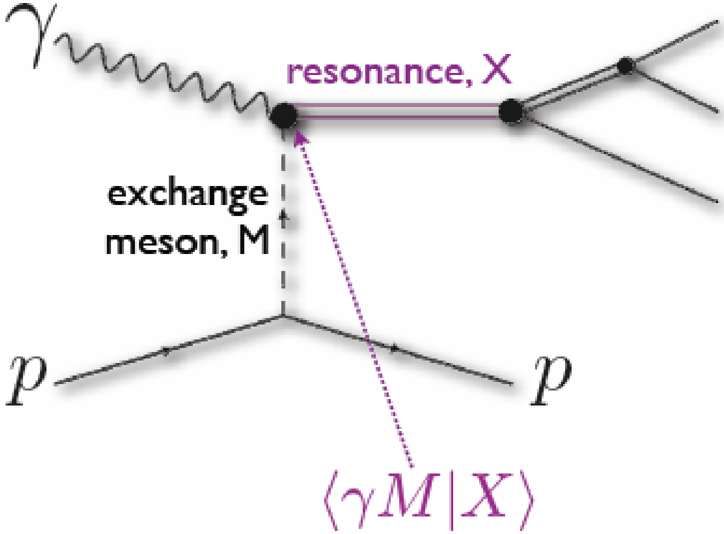
Scotland

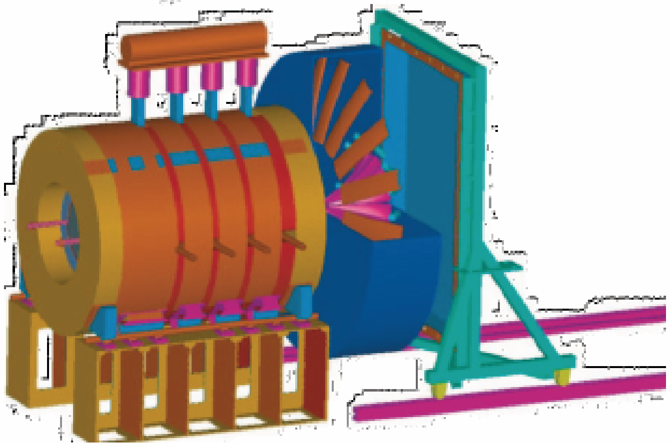
Mexico

JLab, GlueX and photocouplings

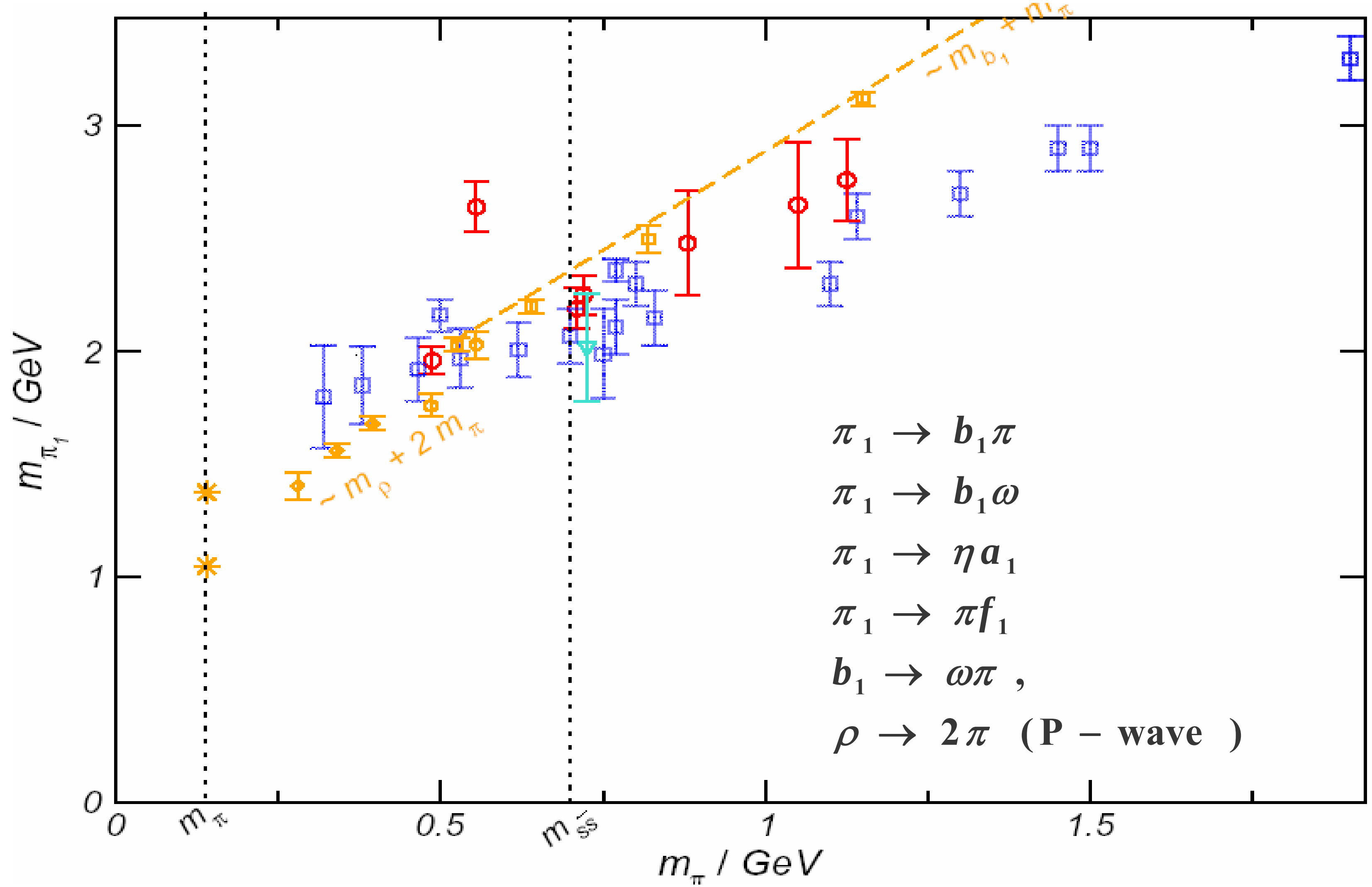
- GlueX plans to photoproduce mesons
- especially exotic J^{PC} mesons







Charmonium is our test bed



$\pi_1 \rightarrow b_1\pi$

$\pi_1 \rightarrow b_1\omega$

$\pi_1 \rightarrow \eta a_1$

$\pi_1 \rightarrow \pi f_1$

$b_1 \rightarrow \omega\pi$,

$\rho \rightarrow 2\pi$ (P – wave)

Multi-hadron states : lightest

1^{-+} Meson

$\overline{\Psi}\gamma_4\overleftrightarrow{D}_i\Psi$ Use D instead of $F_{\mu\nu}$

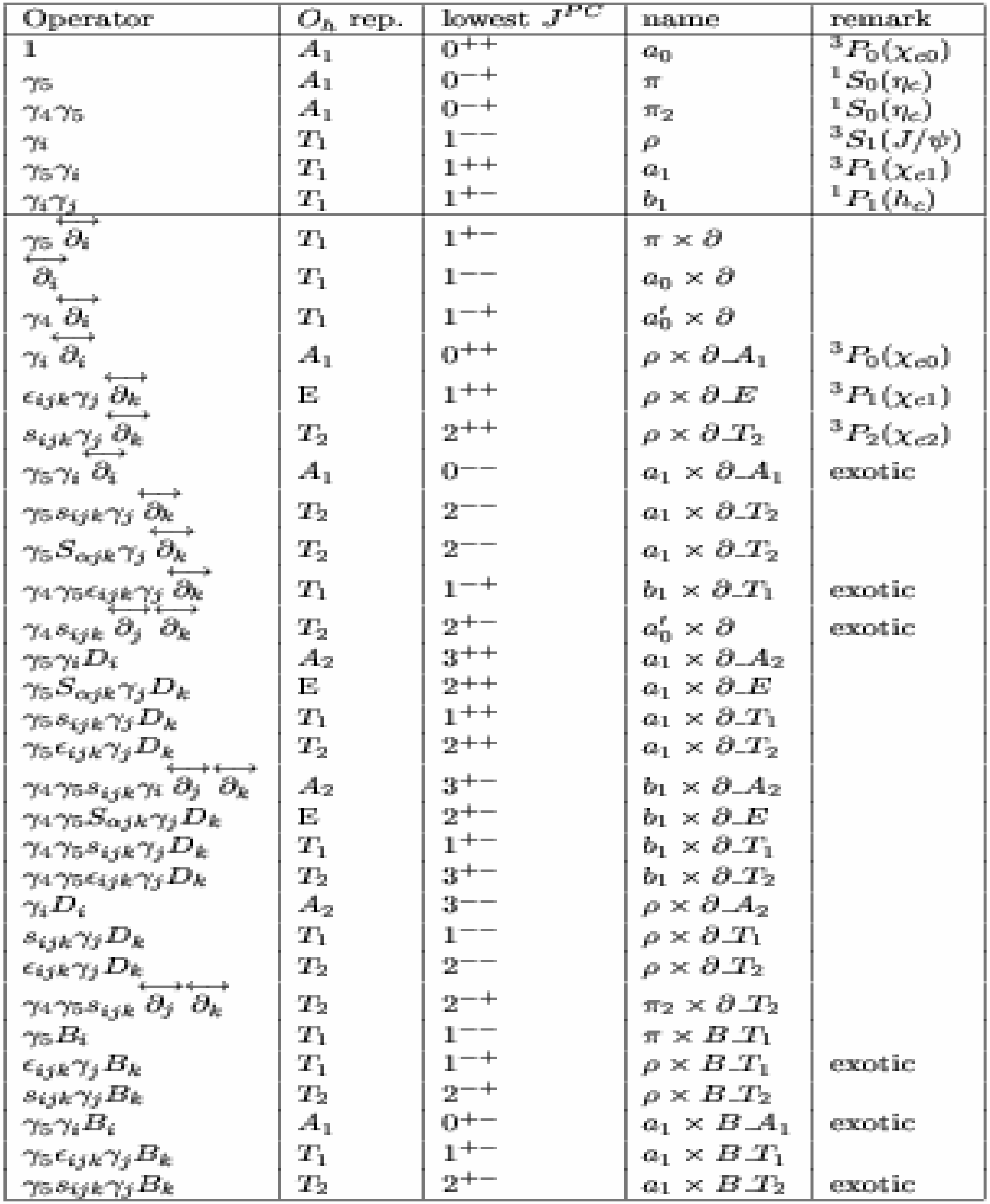
$$P: \quad \Psi(\vec{x}) \rightarrow \gamma_4\Psi(-\vec{x})$$

$$C: \quad \Psi \rightarrow C\ \overline{\widetilde{\Psi}}$$

Note : Need $\overleftrightarrow{D}$ **and not** $\vec{D}$

Required charge conjugation is not possible for non-zero momentum with single side derivative

Bing An Li…Acta Physica Sinica Vol 24, 1 (1975)
K. F. Liu and N. Mathur (Lattice 05)



Operator	O_h rep.	lowest J^{PC}	name	remark
1	A_1	0^{++}	a_0	$^3P_0(\chi_{c0})$
γ_5	A_1	0^{-+}	π	$^1S_0(\eta_c)$
$\gamma_4\gamma_5$	A_1	0^{-+}	π_2	$^1S_0(\eta_c)$
γ_i	T_1	1^{--}	ρ	$^3S_1(J/\psi)$
$\gamma_5\gamma_i$	T_1	1^{++}	a_1	$^3P_1(\chi_{c1})$
$\gamma_i\gamma_j$	T_1	1^{+-}	b_1	$^1P_1(h_c)$
$\gamma_5\ \overleftrightarrow{\partial}_i$	T_1	1^{+-}	$\pi\times\partial$	
$\overleftrightarrow{\partial}_i$	T_1	1^{--}	$a_0\times\partial$	
$\gamma_4\ \overleftrightarrow{\partial}_i$	T_1	1^{-+}	$a_0'\times\partial$	
$\gamma_i\ \overleftrightarrow{\partial}_i$	A_1	0^{++}	$\rho\times\partial\_A_1$	$^3P_0(\chi_{c0})$
$\epsilon_{ijk}\gamma_j\ \overleftrightarrow{\partial}_k$	E	1^{++}	$\rho\times\partial\_E$	$^3P_1(\chi_{c1})$
$s_{ijk}\gamma_j\ \overleftrightarrow{\partial}_k$	T_2	2^{++}	$\rho\times\partial\_T_2$	$^3P_2(\chi_{c2})$
$\gamma_5\gamma_i\ \overleftrightarrow{\partial}_i$	A_1	0^{--}	$a_1\times\partial\_A_1$	exotic
$\gamma_5 s_{ijk}\gamma_j\ \overleftrightarrow{\partial}_k$	T_2	2^{--}	$a_1\times\partial\_T_2$	
$\gamma_5 S_{\alpha jk}\gamma_j\ \overleftrightarrow{\partial}_k$	T_2	2^{--}	$a_1\times\partial\_T_2$	
$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j\ \overleftrightarrow{\partial}_k$	T_1	1^{-+}	$b_1\times\partial\_T_1$	exotic
$\gamma_4 s_{ijk}\ \overleftrightarrow{\partial}_j\ \overleftrightarrow{\partial}_k$	T_2	2^{+-}	$a_0'\times\partial$	exotic
$\gamma_5\gamma_i D_i$	A_2	3^{++}	$a_1\times\partial\_A_2$	
$\gamma_5 S_{\alpha jk}\gamma_j D_k$	E	2^{++}	$a_1\times\partial\_E$	
$\gamma_5 s_{ijk}\gamma_j D_k$	T_1	1^{++}	$a_1\times\partial\_T_1$	
$\gamma_5\epsilon_{ijk}\gamma_j D_k$	T_2	2^{++}	$a_1\times\partial\_T_2$	
$\gamma_4\gamma_5 s_{ijk}\gamma_i\ \overleftrightarrow{\partial}_j\ \overleftrightarrow{\partial}_k$	A_2	3^{+-}	$b_1\times\partial\_A_2$	
$\gamma_4\gamma_5 S_{\alpha jk}\gamma_j D_k$	E	2^{+-}	$b_1\times\partial\_E$	
$\gamma_4\gamma_5 s_{ijk}\gamma_j D_k$	T_1	1^{+-}	$b_1\times\partial\_T_1$	
$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j D_k$	T_2	3^{+-}	$b_1\times\partial\_T_2$	
$\gamma_i D_i$	A_2	3^{--}	$\rho\times\partial\_A_2$	
$s_{ijk}\gamma_j D_k$	T_1	1^{--}	$\rho\times\partial\_T_1$	
$\epsilon_{ijk}\gamma_j D_k$	T_2	2^{--}	$\rho\times\partial\_T_2$	
$\gamma_4\gamma_5 s_{ijk}\ \overleftrightarrow{\partial}_j\ \overleftrightarrow{\partial}_k$	T_2	2^{-+}	$\pi_2\times\partial\_T_2$	
$\gamma_5 B_i$	T_1	1^{--}	$\pi\times B\_T_1$	
$\epsilon_{ijk}\gamma_j B_k$	T_1	1^{-+}	$\rho\times B\_T_1$	exotic
$s_{ijk}\gamma_j B_k$	T_2	2^{-+}	$\rho\times B\_T_2$	
$\gamma_5\gamma_i B_i$	A_1	0^{+-}	$a_1\times B\_A_1$	exotic
$\gamma_5\epsilon_{ijk}\gamma_j B_k$	T_1	1^{+-}	$a_1\times B\_T_1$	
$\gamma_5 s_{ijk}\gamma_j B_k$	T_2	2^{+-}	$a_1\times B\_T_2$	exotic

$$\overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu\,,$$

$$D_i = s_{ijk}\overleftrightarrow{\partial}_j\overleftrightarrow{\partial}_k\,,\quad \textbf{symmetric}$$

$$B_i = \varepsilon_{ijk}\overleftrightarrow{\partial}_j\overleftrightarrow{\partial}_k\,,\quad \textbf{anti-symmetric}$$

$$s_{ijk} = \left|\varepsilon_{ijk}\right|$$

$$S_{\alpha jk} = 0(j \neq k), S_{111} = 1\,,$$

$$S_{122} = -1, S_{222} = 1, S_{233} = -1$$

a la Manke and Liao,
hep-lat/0210030

Charmonium—Laboratory to test hybrid technology

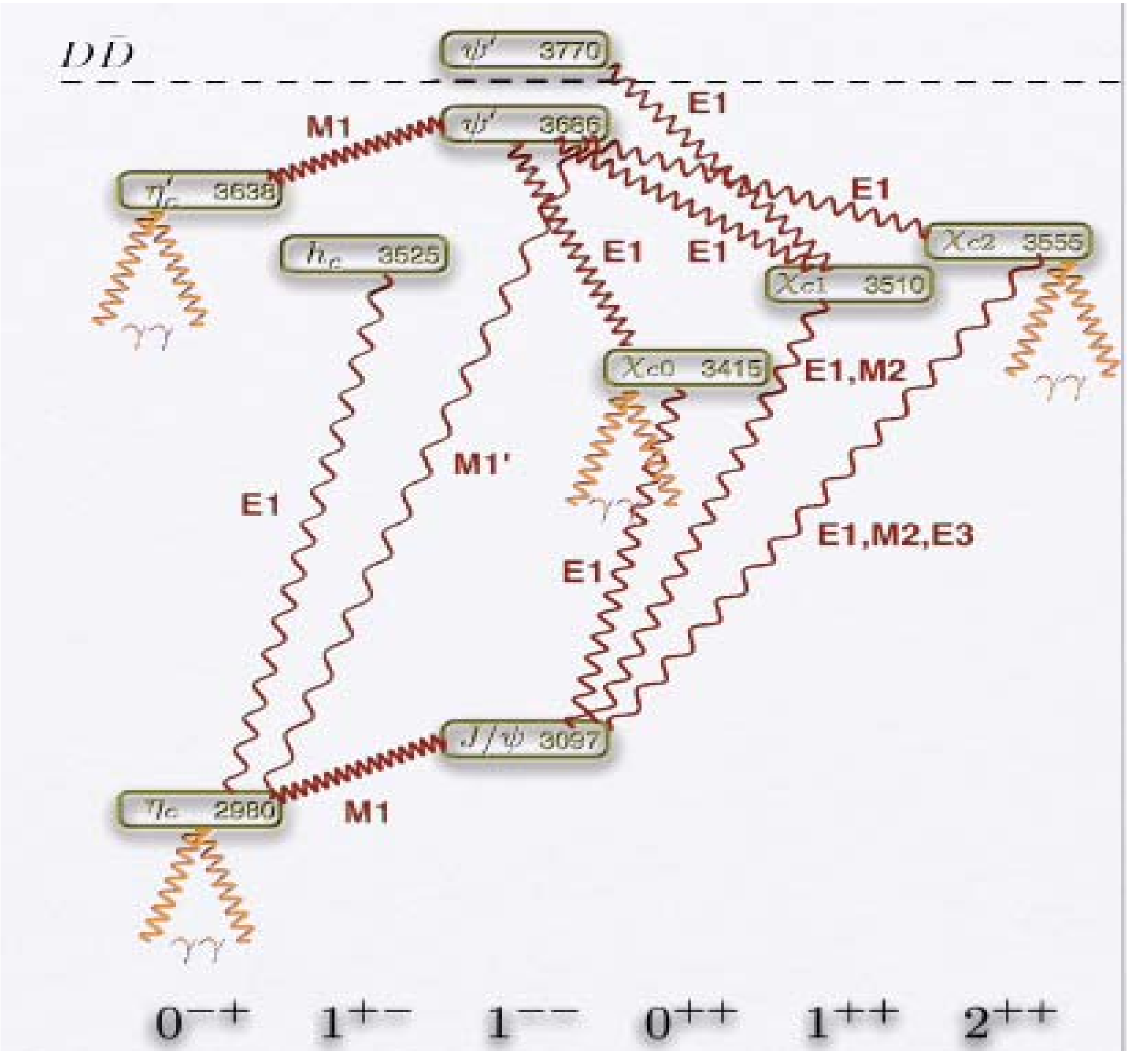
- Light quark hybrids and higher spin mesons are problematic so far

✓ Charmonium needs less constraints

-- Chiral extrapolation
-- Quenching

✓ Experimental data exists to test photo-couplings

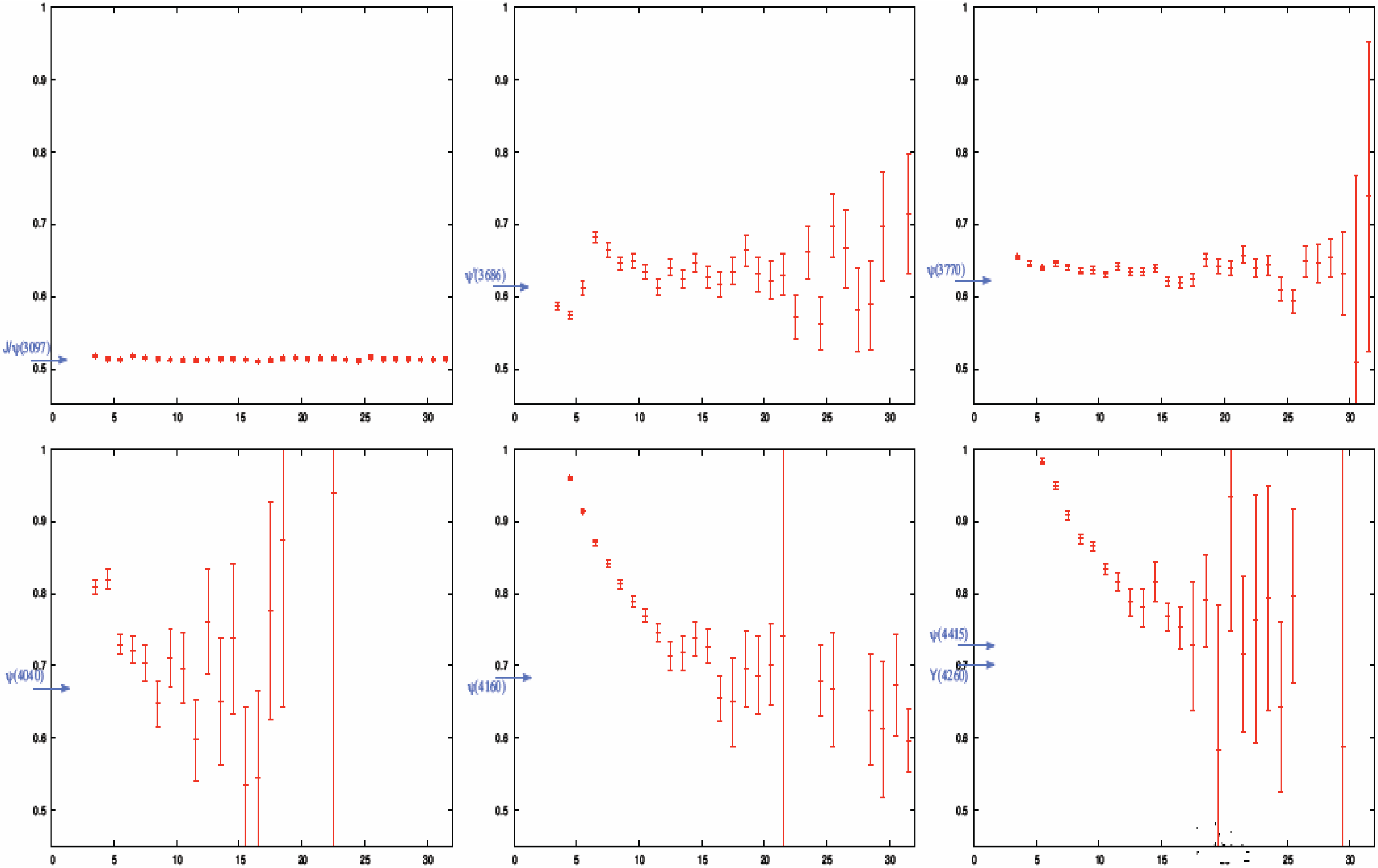
Radiative transition in Charmonium



Phys.Rev.D73, 074507 (2006),

Would be a good testing ground before going to light quark (GlueX observables)

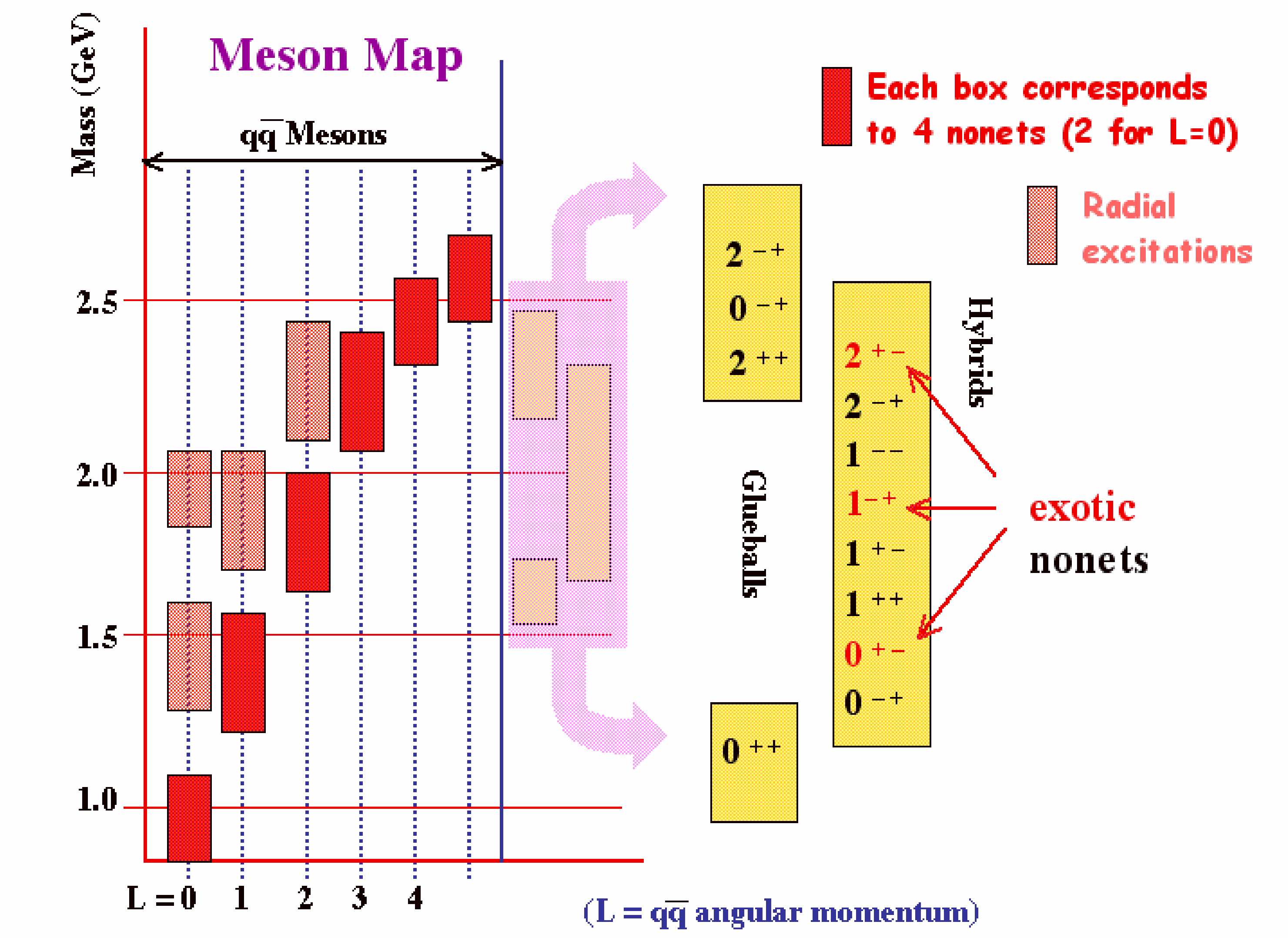
1^{--}

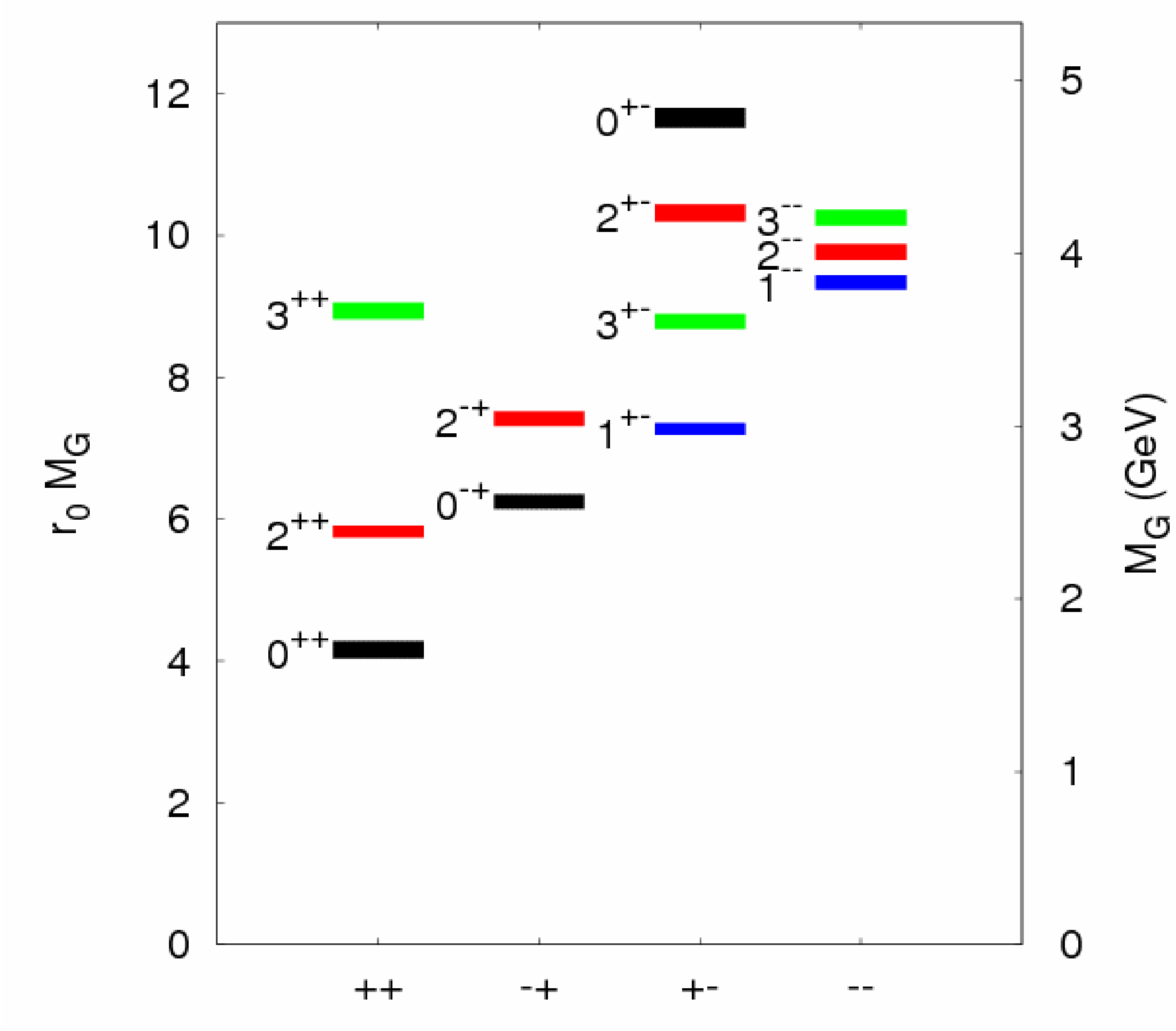


With Jo Dudek, R Edwards and D. Richards

Jefferson Lab

Glueballs and hybrid mesons





Jefferson Lab

Y. Chen et al. Phys. Rev. D73, 014516 (2006)

Conclusion

Lattice QCD is entering an era where it can make significant contributions in nuclear and particle physics.

- **Particle Masses :** Understanding the Structure and Interaction of Hadrons.
  - **Quenched lattice calculations can reproduce masses for many ground state hadrons within 10% of experimental numbers. Qualitatively spectrum ordering may well be understood by quench calculations.**
  - However, excited state masses are still not accessible comprehensively. Data analysis becomes increasingly difficult as we go towards chiral limit due to the appearance of unphysical ghost states. In low quark mass region one should consider these states in fitting function.
  - **For full QCD one needs to consider multiquark decay channels along with resonance states. Multivolume and posssibly stochastic propagators are necessary to carry out a reliable study**
  - **A very comprehensive program is ongoing by Spectrum group by using group theoretical multi-operator variational method in order to extract resonance states both for baryons and mesons including hybrid states.**

- **Multiquark (>3) and hybrid states :**
  - Multiquark and hybrid states may exist in nature and lattice QCD can contribute significantly in this area.
  - One need to be careful to distinguish a bound state from a scattering state by volume dependence or other methods.