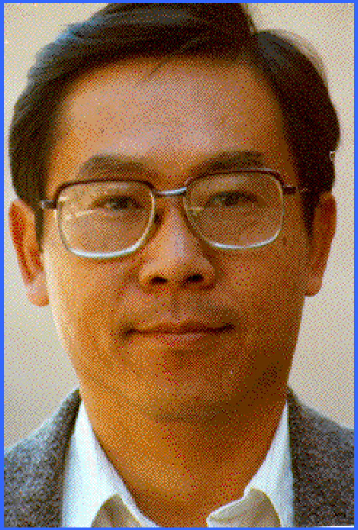
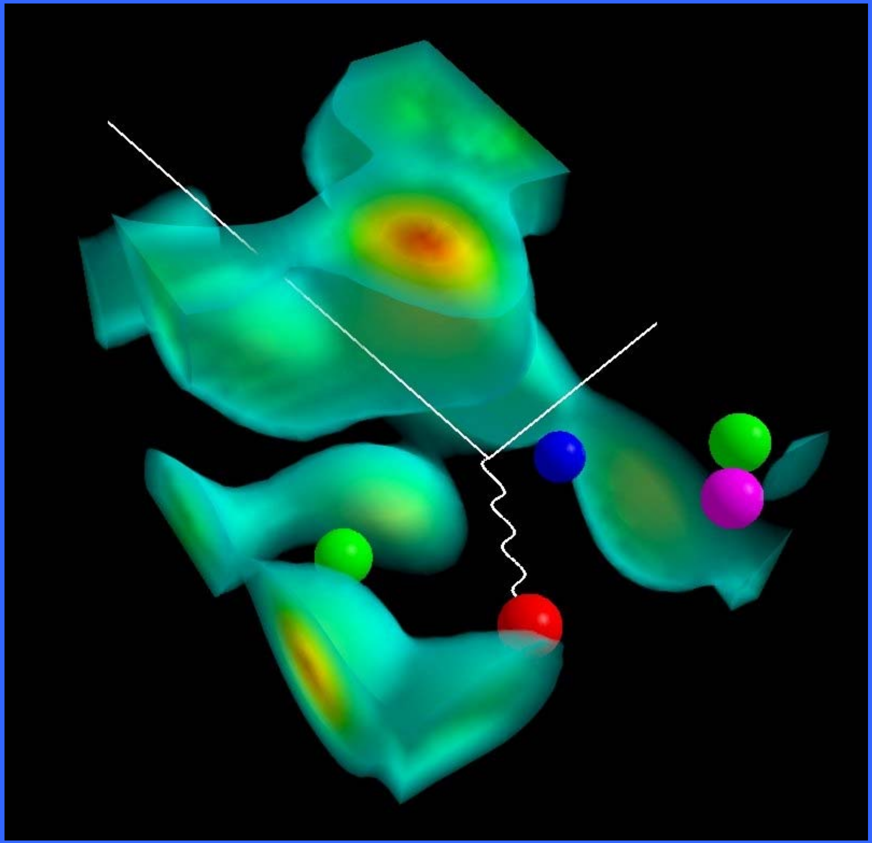
Insights into Hadron Structure through Lattice QCD





Anthony W. Thomas

Festschrift for Keh-Fei Liu
Lexington KY : April 19th 2007

Outline

- Quantum Chromodynamics

- Lattice QCD :
  there are problems
  ⇒ new opportunities!

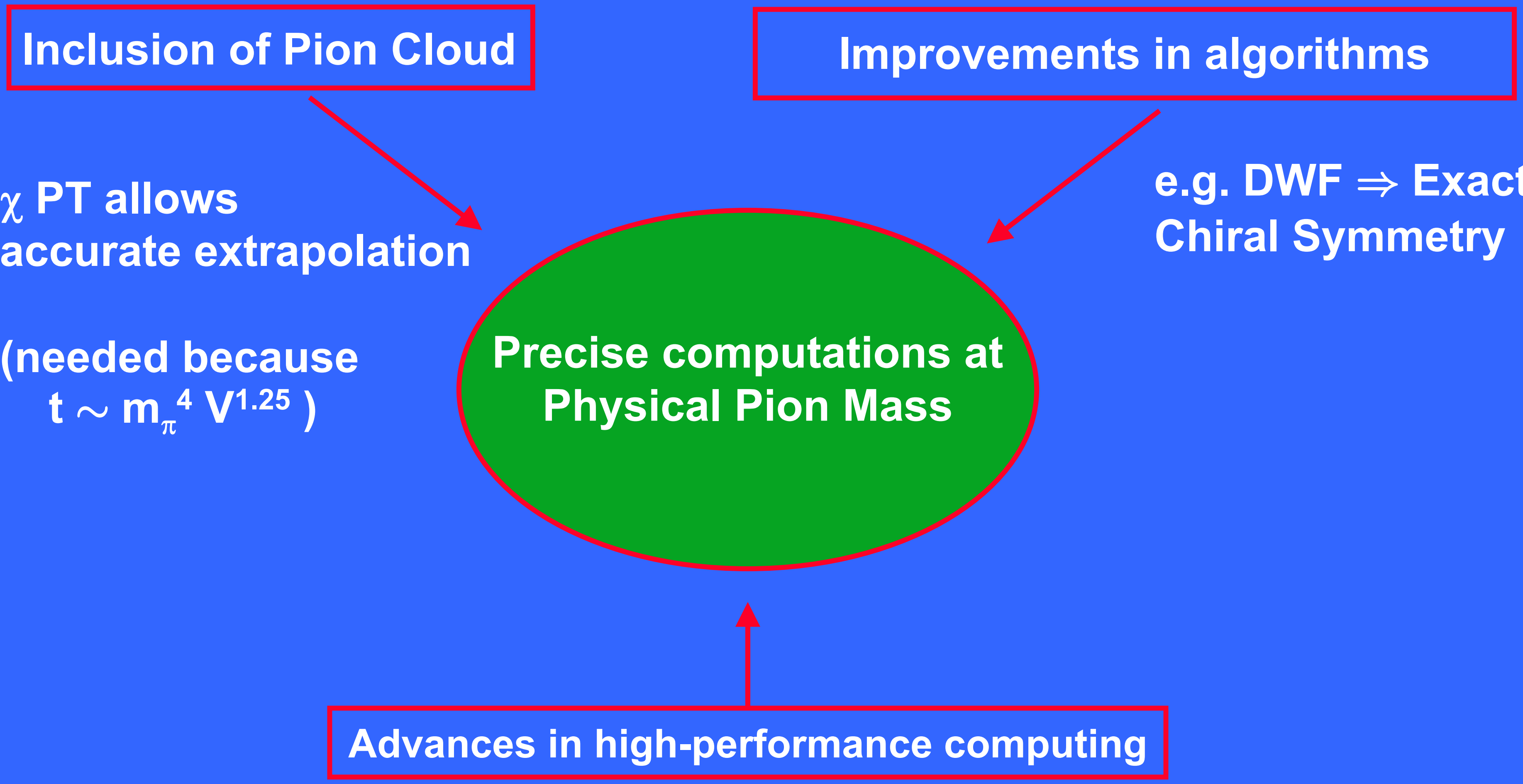
(and, by the way, some things CAN be calculated ACCURATELY)

- M_N , M_Δ , QQCD ⟺ QCD ⟺ pQQCD, m_ρ ;

  g_A, μ_B , $G_{E,M}^s$

- Something different to end!

Advances in Lattice QCD



Inclusion of Pion Cloud

Improvements in algorithms

χ PT allows accurate extrapolation

(needed because $t \sim m_\pi^4 V^{1.25}$)

Precise computations at Physical Pion Mass

e.g. DWF ⇒ Exact Chiral Symmetry

Advances in high-performance computing

from D. Richards

What are we aiming to do??

- Test whether χ `al EFT can describe the lattice data

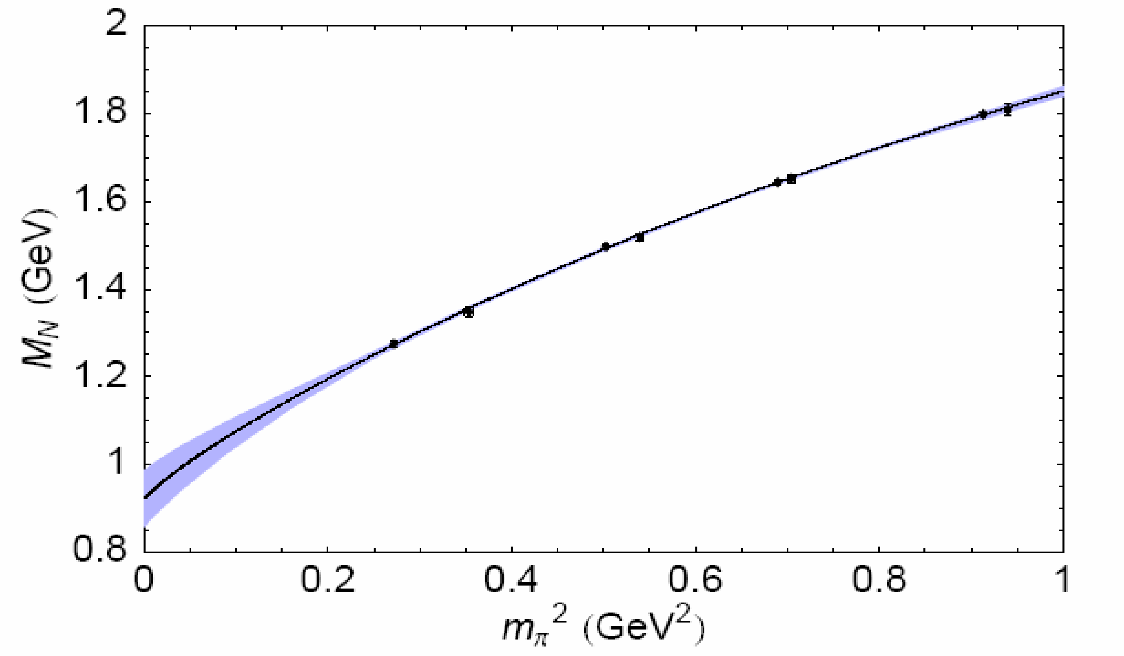
Hopeless for traditional dim-reg χ PT !

- Extrapolate the lattice data to the physical quark mass in order to compare with experimental data

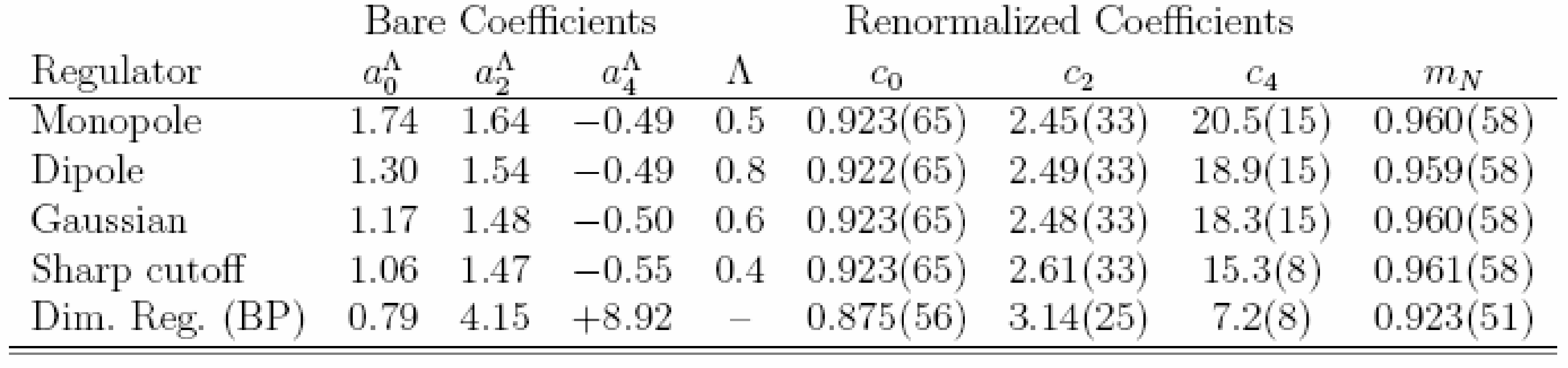
a) Anything would work…. (g_A)
b) Stop expansion when you get the "right" answer
c) Use FRR

- not quite sure…..

χ’al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon



FRR give same answer to <<1% systematic error!



	Bare Coefficients				Renormalized Coefficients			
Regulator	a_0^Λ	a_2^Λ	a_4^Λ	Λ	c_0	c_2	c_4	m_N
Monopole	1.74	1.64	−0.49	0.5	0.923(65)	2.45(33)	20.5(15)	0.960(58)
Dipole	1.30	1.54	−0.49	0.8	0.922(65)	2.49(33)	18.9(15)	0.959(58)
Gaussian	1.17	1.48	−0.50	0.6	0.923(65)	2.48(33)	18.3(15)	0.960(58)
Sharp cutoff	1.06	1.47	−0.55	0.4	0.923(65)	2.61(33)	15.3(8)	0.961(58)
Dim. Reg. (BP)	0.79	4.15	+8.92	–	0.875(56)	3.14(25)	7.2(8)	0.923(51)

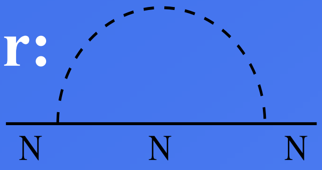
Leinweber et al., PRL 92 (2004) 242002

Extrapolation of Masses

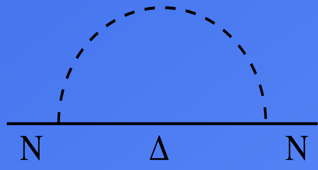
At "large m_π" preserve observed linear (constituent-quark-like) behaviour: $M_H \sim m_\pi^2$

As $m_\pi \sim 0$: ensure LNA & NLNA behaviour:

(BUT must die as $(\Lambda / m_\pi)^2$ for $m_\pi > \Lambda$)

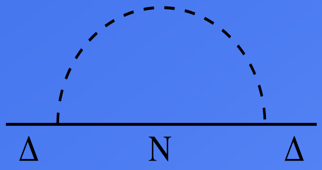


(a)

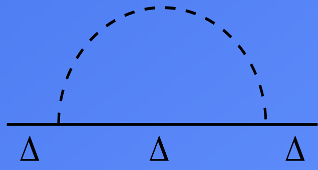


(b)

Hence use:



(c)

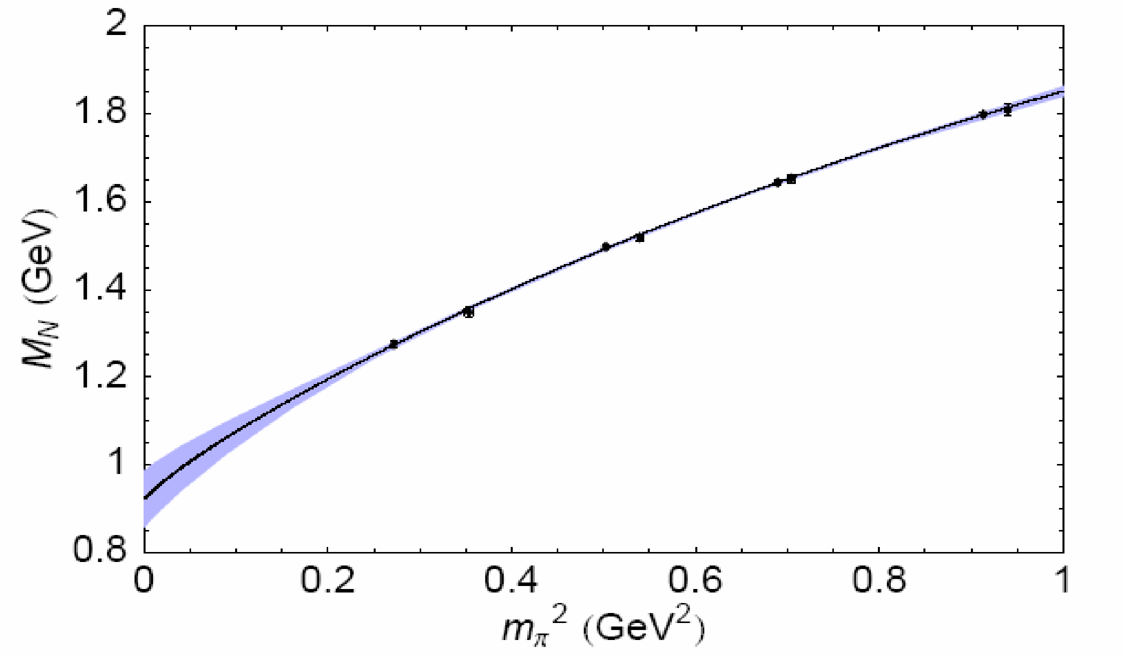


(d)

$$M_H = a_0 + a_2\, m_\pi^2 + a_4\, m_\pi^4 + \sigma_{LNA}(m_\pi, \Lambda) + \sigma_{NLNA}(m_\pi, \Lambda)$$

- Evaluate self-energies with form factor , "finite range regulator", FRR, with $\Lambda \sim$ 1/Size of Hadron

χ'al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon



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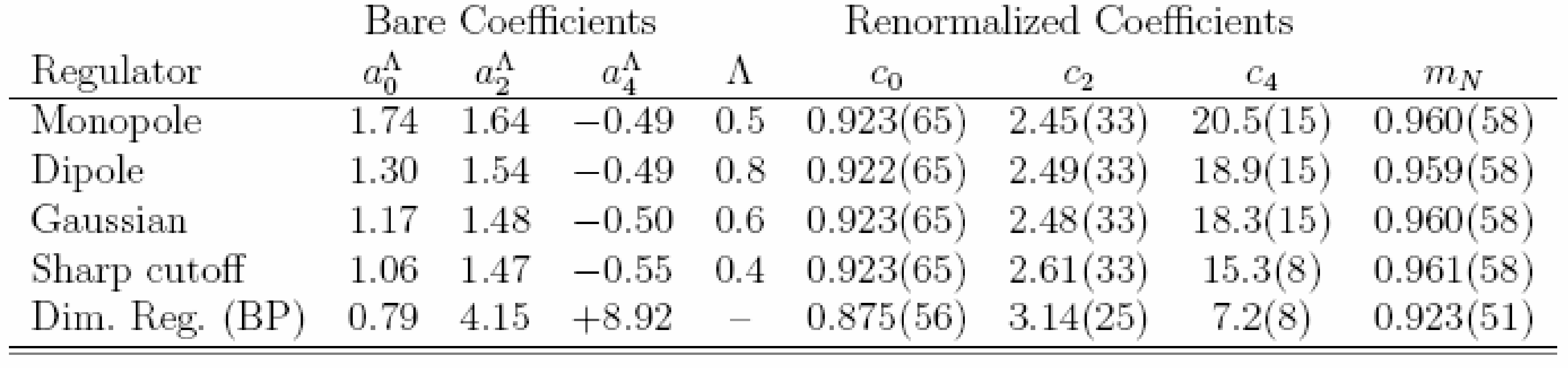

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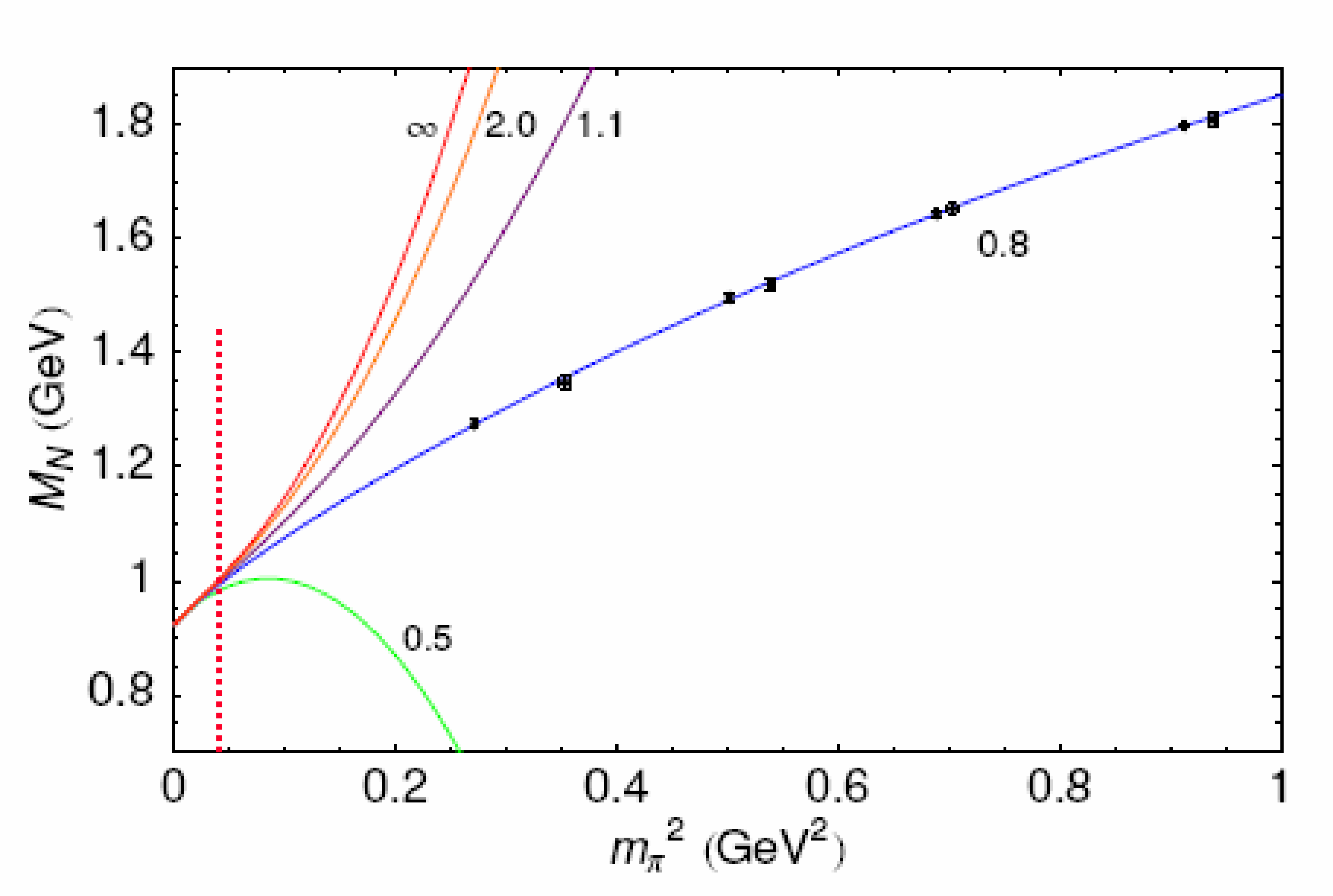
Leinweber et al., PRL 92 (2004) 242002

Power Counting Regime

Ensure coefficients c_0 , c_2 , c_4 all identical to 0.8 GeV fit



Leinweber, Thomas & Young, hep-lat/0501028

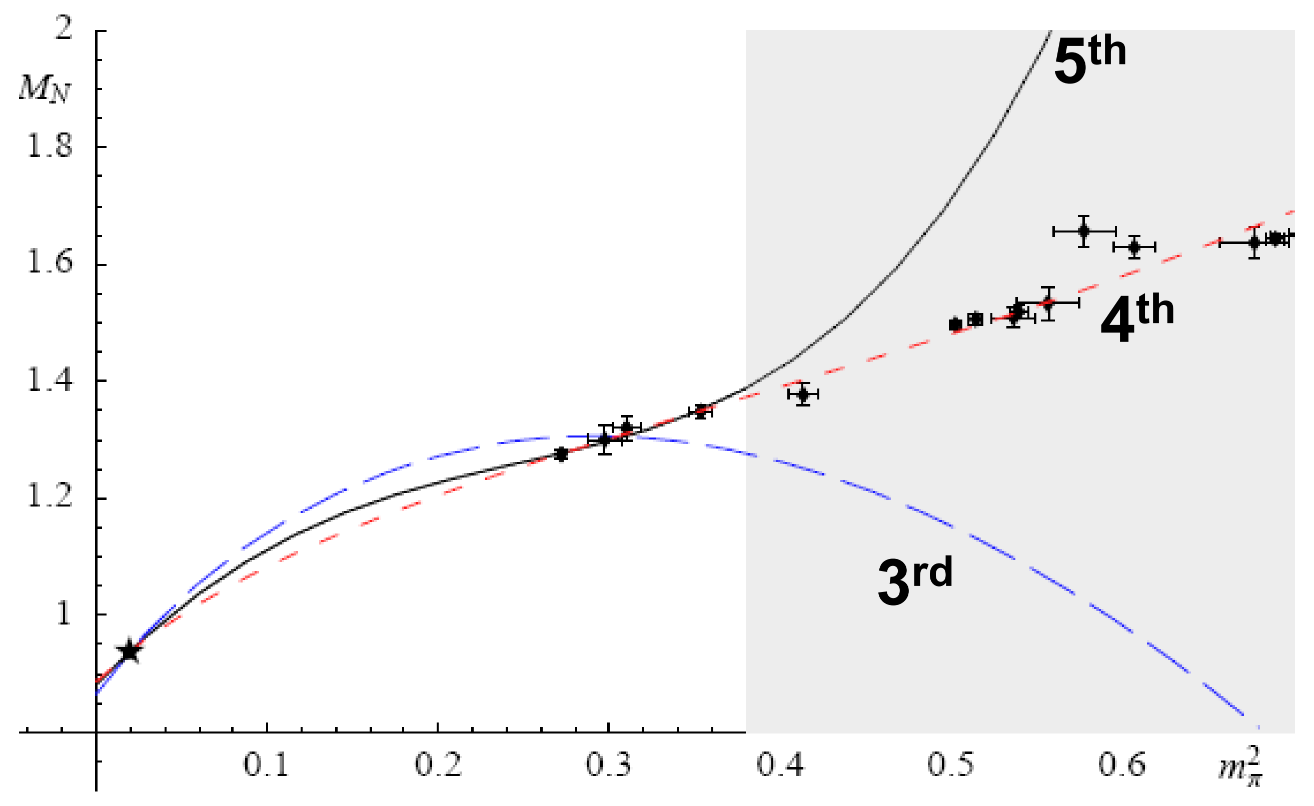
Convergence from LNA to NLNA is Rapid – Using Finite Range Regularization

Regulator	LNA	NLNA
Sharp	968	961
Monopole	964	960
Dipole	963	959
Gaussian	960	960
Dim Reg	784	884

M_N in MeV

McGovern & Birse: hep:lat/0608002

Fit data below 0.6 GeV by adjusting M_0 , e' and c_1 to fit M_N^{phys}



Low energy constant, e', quite different at 4th and 5th order…

McGovern & Birse (cont.)

- First to calculate two-loop, dim-reg χ PT
- Major correction is m_π dependence of $g_{\pi\,NN}$
  i.e. origin of GT discrepancy : $g_{\pi\,NN} \neq g_A/f_\pi$
- Leads to large Order (m_π^5) term
- Agree that convergence of formal chiral expansion is hopeless where current lattice data exists

$$M_N = 0.885 + 3.20 m_\pi^2 - 5.6 m_\pi^3 + 34\, m_\pi^4 - (50\text{-}110) m_\pi^5 \ldots$$

c.f. FRR fit required to include physical nucleon mass:

$$M_N = 0.897 + 2.83 m_\pi^2 - 5.6 m_\pi^3 + 22 m_\pi^4 - (44 \pm 18) m_\pi^5 \ldots$$

Leinweber et al., Lect. Notes in Phys. 663 (2005) 113

FRR works because…

- It preserves model independent LNA and NLNA behavior

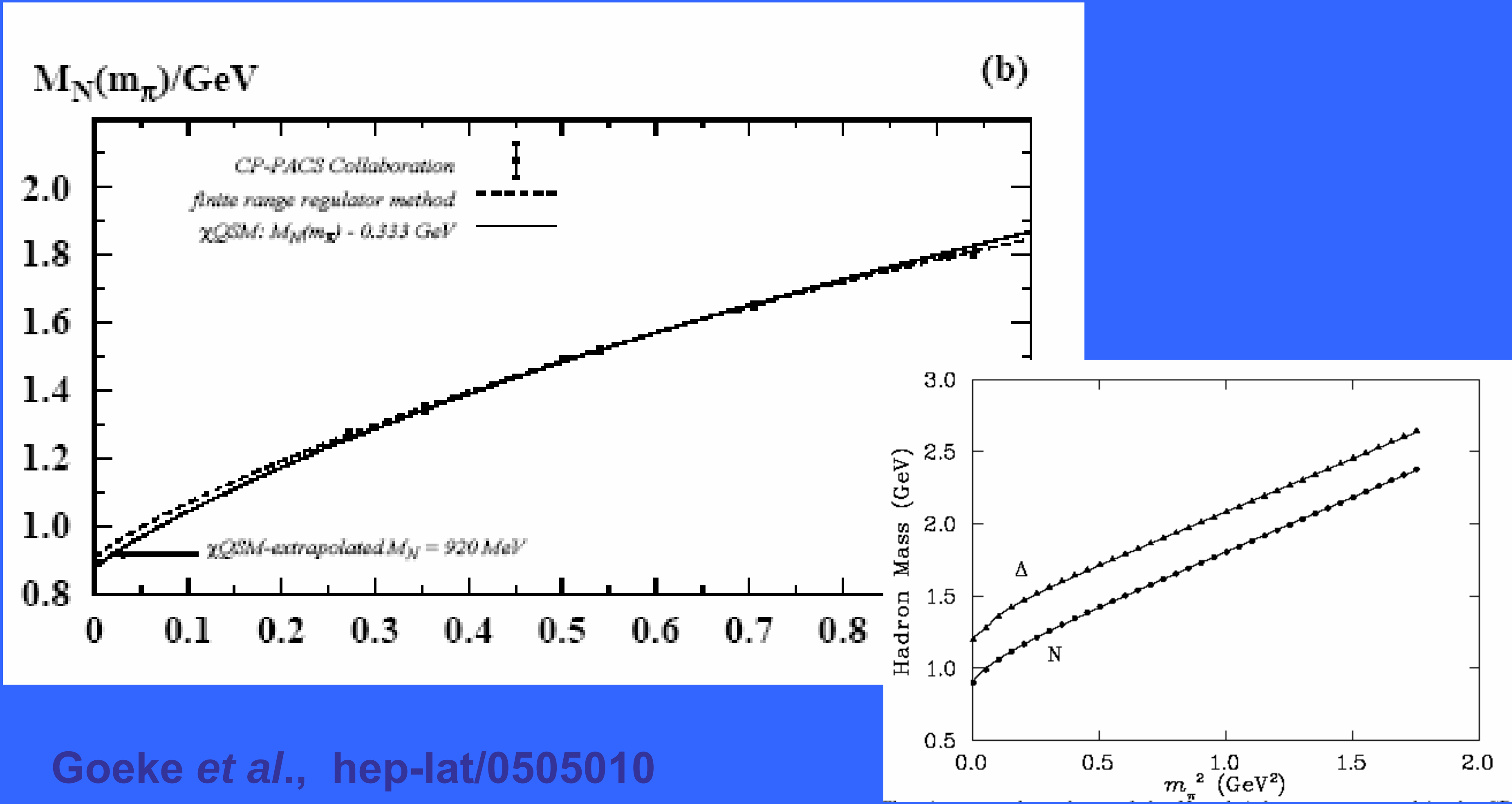
- Form factor naturally yields GT discrepancy of right sign and magnitude – and therefore correct m_π^5 term!
  - i.e. correct NNLNA behavior

- N.B. Usual EFT yields this term only at two loops

- For <u>sound physical reasons</u>, FRR suppresses meson loops once m_π exceeds about 0.4 GeV

- Yields convergent series expansion over mass region covered by lattice data

Comparison with χQSM and CBM

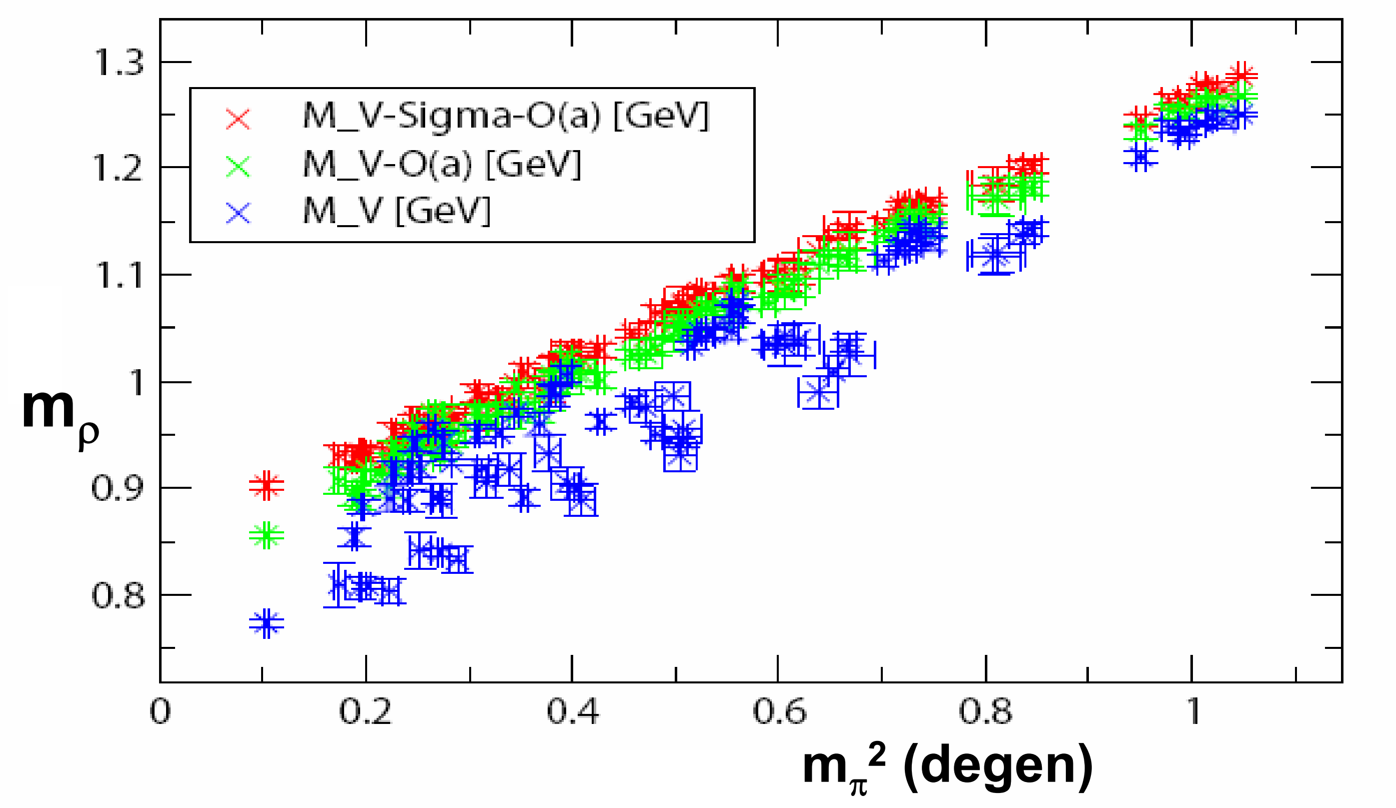


Goeke *et al.*, hep-lat/0505010

CBM: Leinweber et al., Phys.Rev.D61:074502,2000

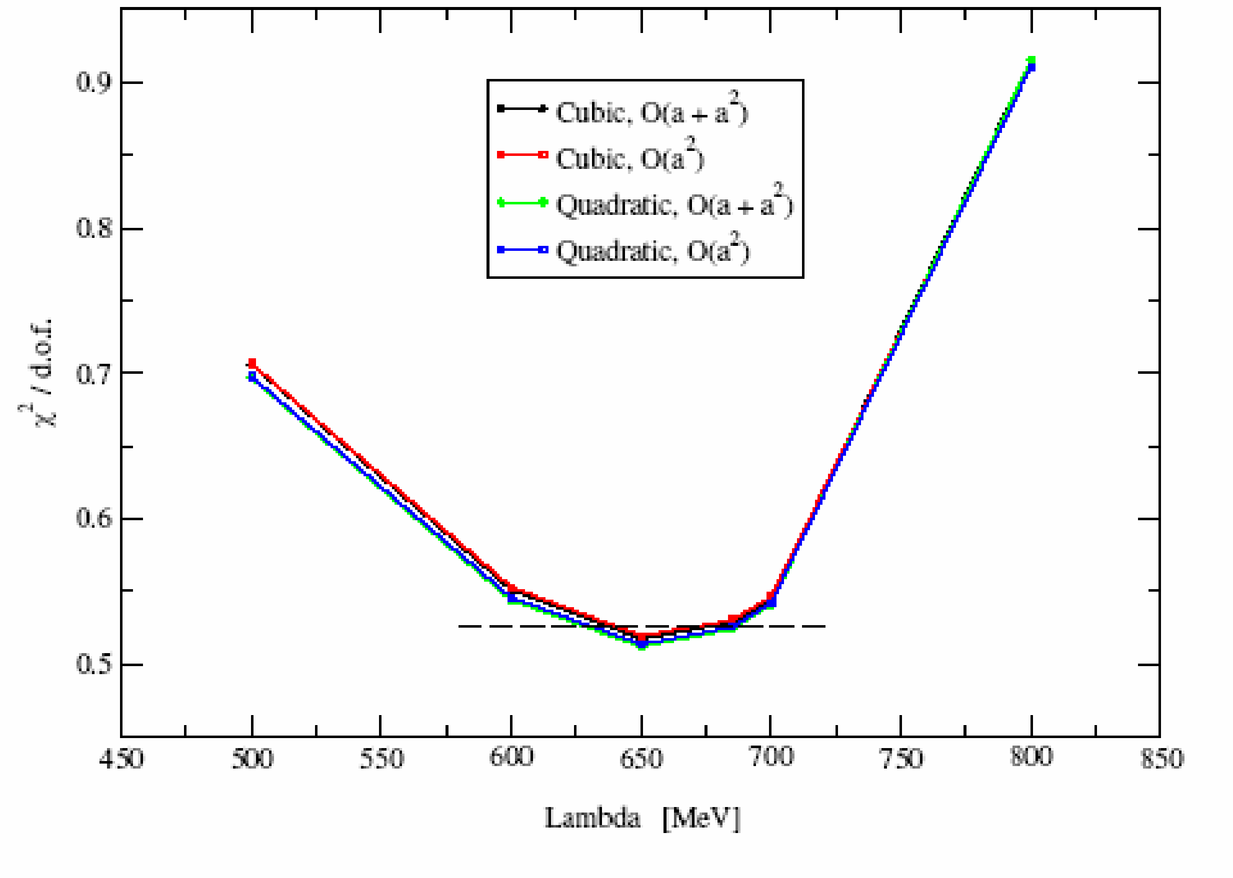
Analysis of pQQCD ρ data from CP PACS

i.e. $m_{val} \neq m_{sea}$



Fit with: $\sqrt{(M_V^{deg})^2 - \Sigma_{TOT}} = (a_0^{cont} + X_1 a + X_2 a^2) + a_2(M_{PS}^{deg})^2 + a_4(M_{PS}^{deg})^4 + a_6(M_{PS}^{deg})^6$

FRR Mass (in Σ_{TOT}) well determined by data



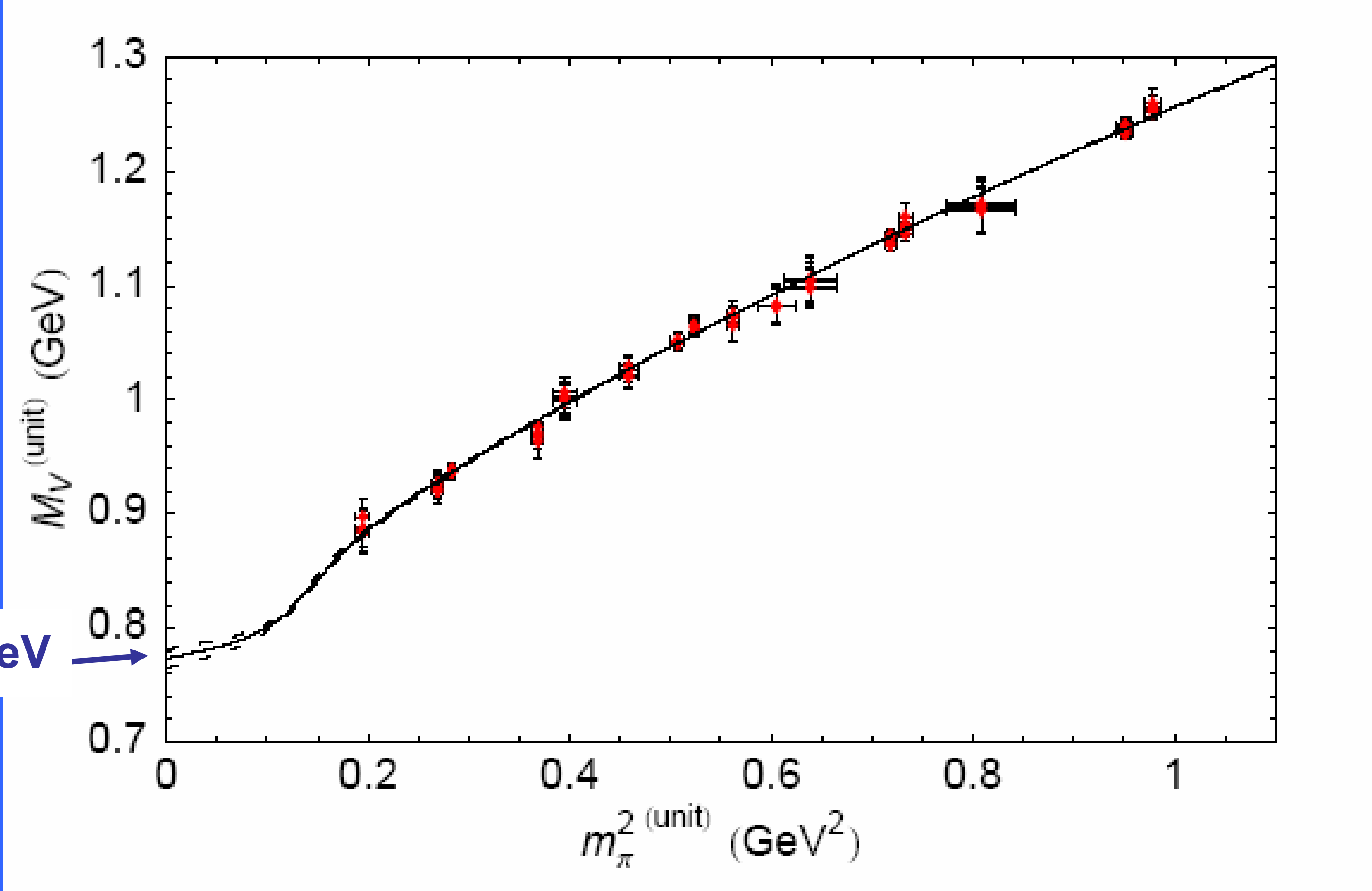
$$\sqrt{(M_V^{deg})^2 - \Sigma_{TOT}} = (a_0^{cont} + X_1 a + X_2 a^2) + a_2 (M_{PS}^{deg})^2 + a_4 (M_{PS}^{deg})^4 + a_6 (M_{PS}^{deg})^6$$

Infinite Volume Unitary Results

$a \to 0$ and $m_{sea} = m_{val}$

All 80 data points drop onto single, well defined curve !

777 ± 7 MeV



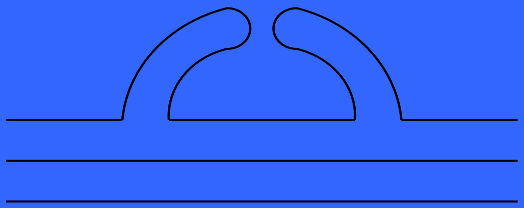
Allton, Young *et al.*, hep-lat/0504022

Baryon Masses in Quenched QCD

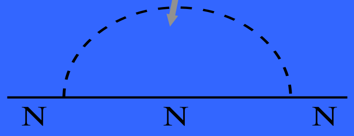
Chiral behaviour in QQCD quite different from full QCD

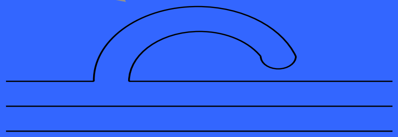
η' is an additional Goldstone Boson , so that:

$$m_N = m_0 + c_1\, m_\pi + c_2\, m_\pi^2 + c_3\, m_\pi^3 + c_4\, m_\pi^4 + m_\pi^4 \ln m_\pi + \ldots$$



Contribution from η' and π

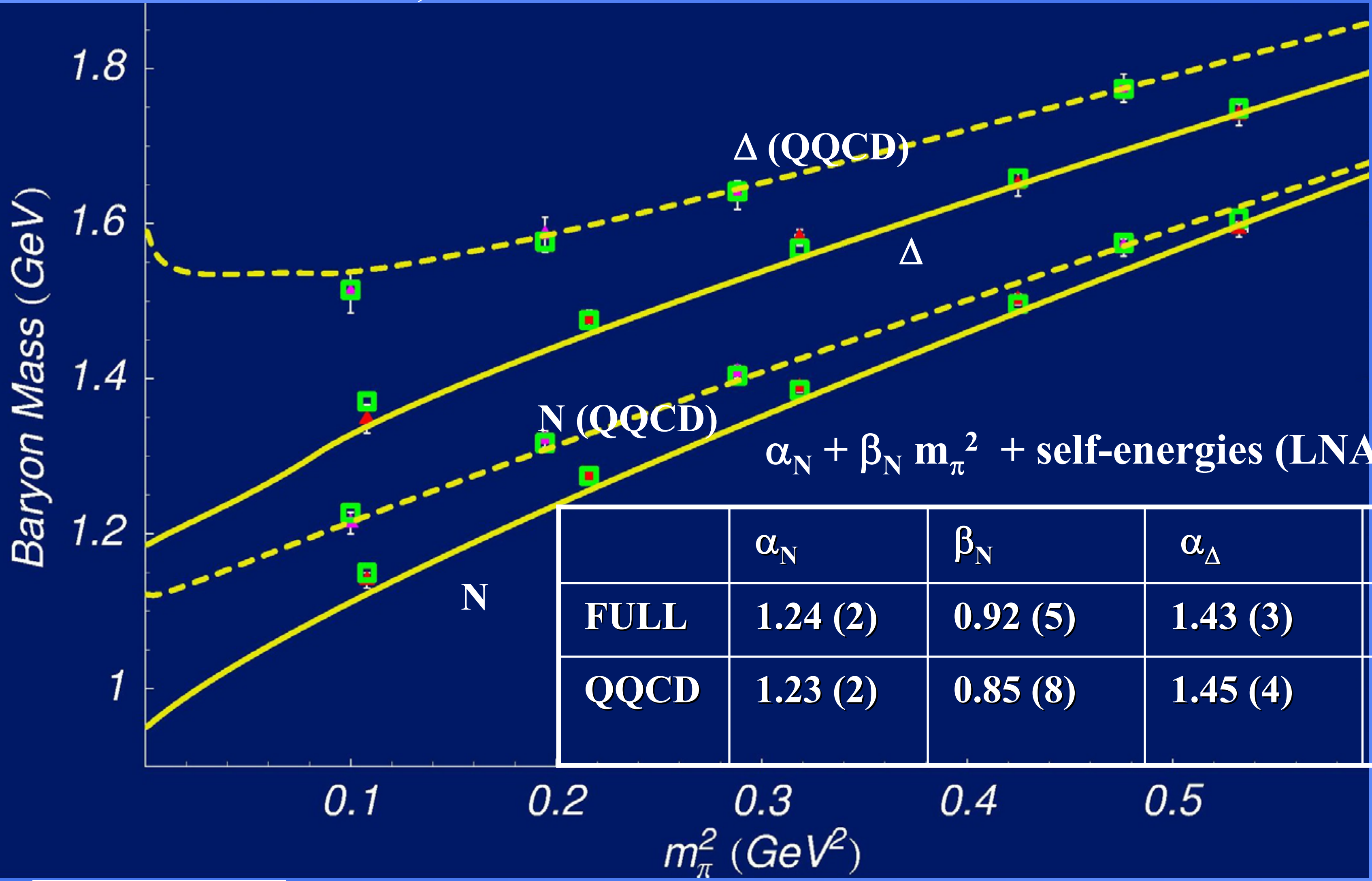




LNA term now ~ $m_q^{1/2}$

origin is η' double pole

- Lattice data (from **MILC Collaboration**) : red triangles
- Green boxes: fit evaluating σ's on <u>same finite grid as lattice</u>
- Lines are exact, continuum results

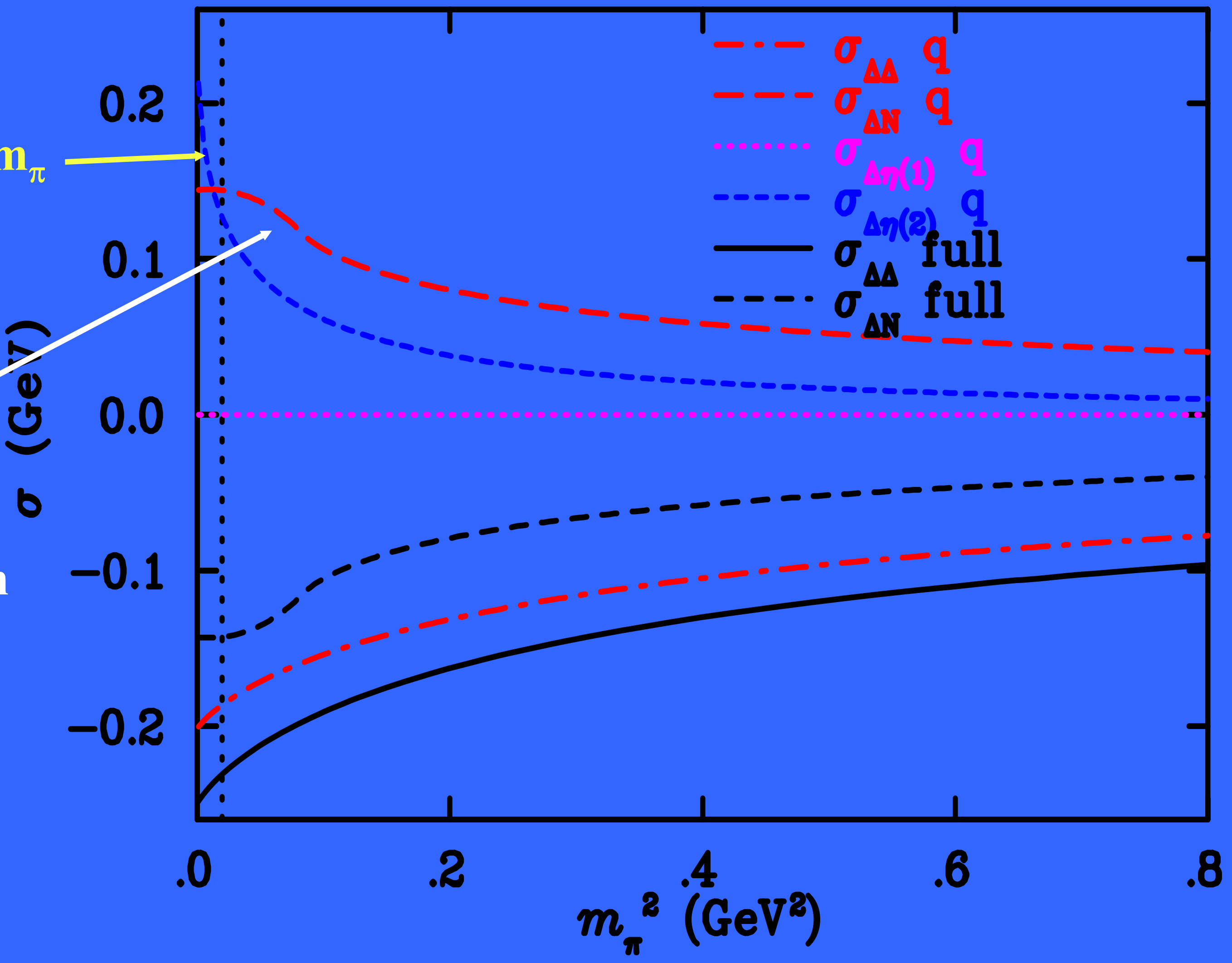


$\alpha_N + \beta_N m_\pi^2$ + self-energies (LNA+NLNA)

	α_N	β_N	α_Δ	β_Δ
FULL	**1.24 (2)**	**0.92 (5)**	**1.43 (3)**	**0.75 (8)**
QQCD	**1.23 (2)**	**0.85 (8)**	**1.45 (4)**	**0.71 (11)**

Young *et al.*, hep-lat/0111041; Phys. Rev. D66 (2002) 094507

Δ in QQCD

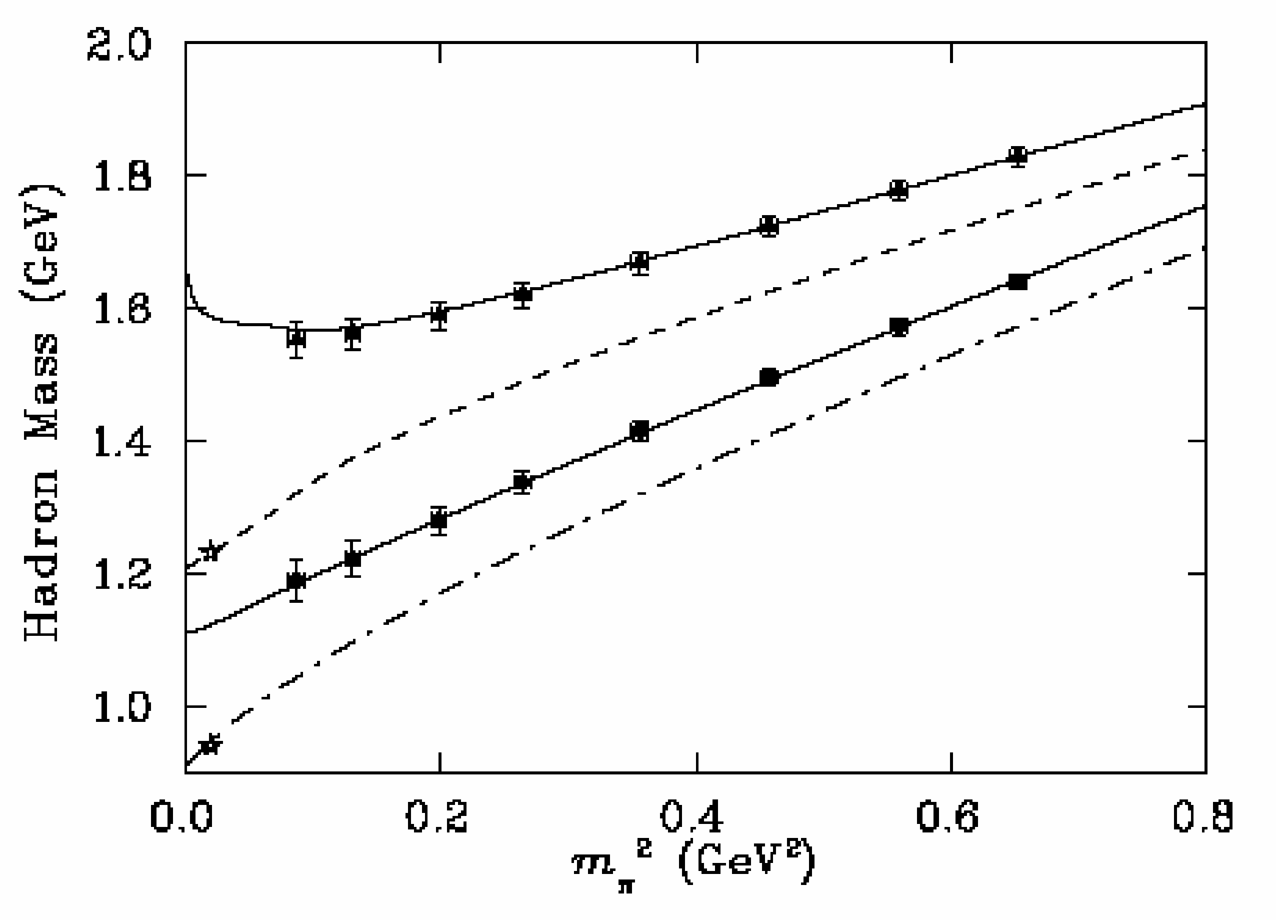


LNA term linear in m_π

Δ → N π contribution has opposite sign in QQCD (repulsive)

Overall σ_{QQCD} is repulsive !

Confirmation of Predicted Behavior of Δ



Zanotti et al., hep-lat/0407039

These results suggest following conjecture :

IF lattice scale is set using static quark potential (e.g. Sommer scale)

(insensitive to chiral physics)

Suppression of Goldstone loops for $m_\pi > \Lambda$ implies:

Analytic terms (e.g. $\alpha + \beta\, m_\pi^2 + \gamma\, m_\pi^4$)

representing "hadronic core" are the same in QQCD & QCD

Can then correct QQCD results by replacing LNA & NLNA behaviour in QQCD by corresponding terms in full QCD

Quenched QCD is then no longer an "uncontrolled approximation" !

Strangeness Widely Believed to Play a Major Role – Does It?

- As much as 100 to 300 MeV of proton mass:

$$M_N = \langle N(P)| -\frac{9\,\alpha_s}{4\,\pi}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u\bar{\psi}_u\psi_u + m_d\bar{\psi}_d\psi_d + m_s\bar{\psi}_s\psi_s |N(P)\rangle$$

y=0.2 ± 0.2 ?

$$\Delta M_N^{s-\mathrm{quarks}} = \frac{y\,m_s}{m_u + m_d}\,\sigma_N$$

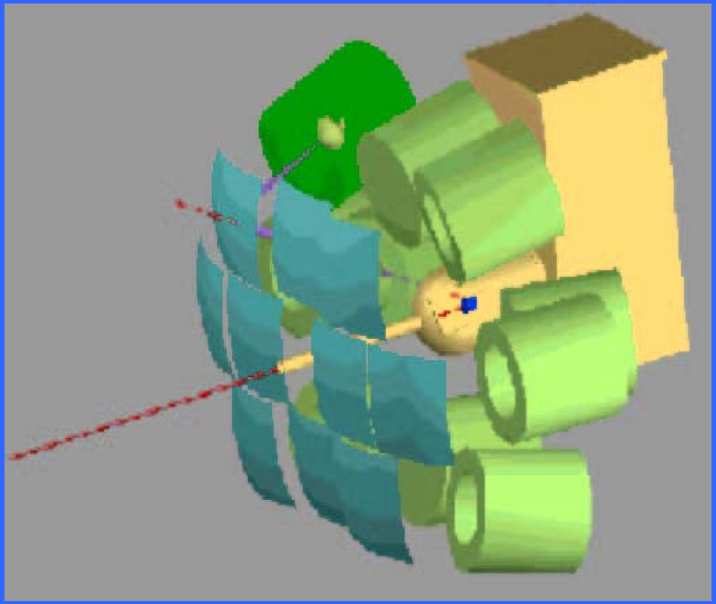
45 ± 8 MeV (or 70?)

Hence 110 ± 110 MeV (increasing to 180 for higher σ_N)

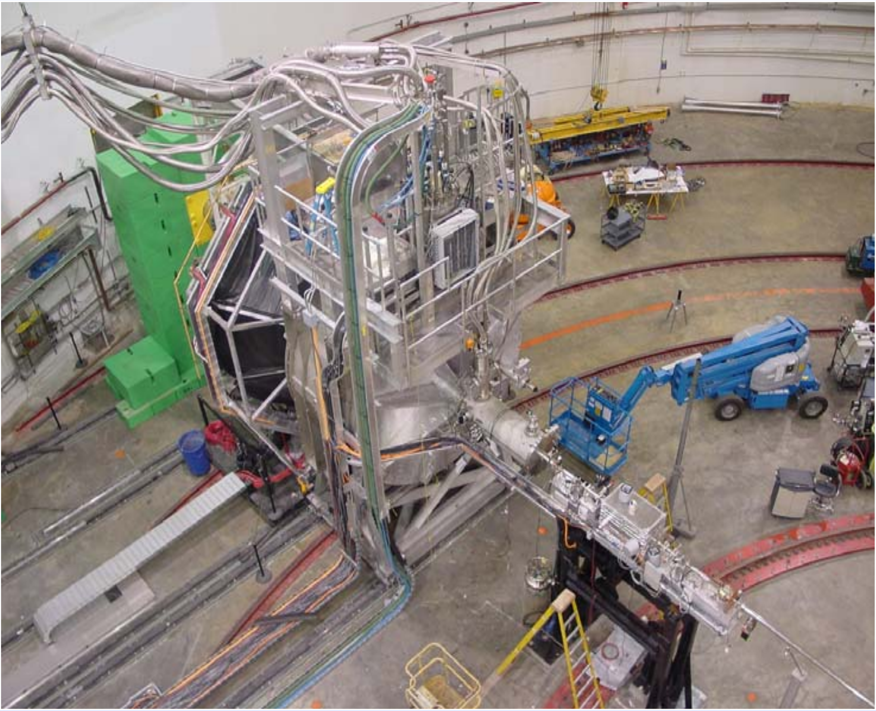
- Through proton spin crisis:
  As much as 10% of the spin of the proton

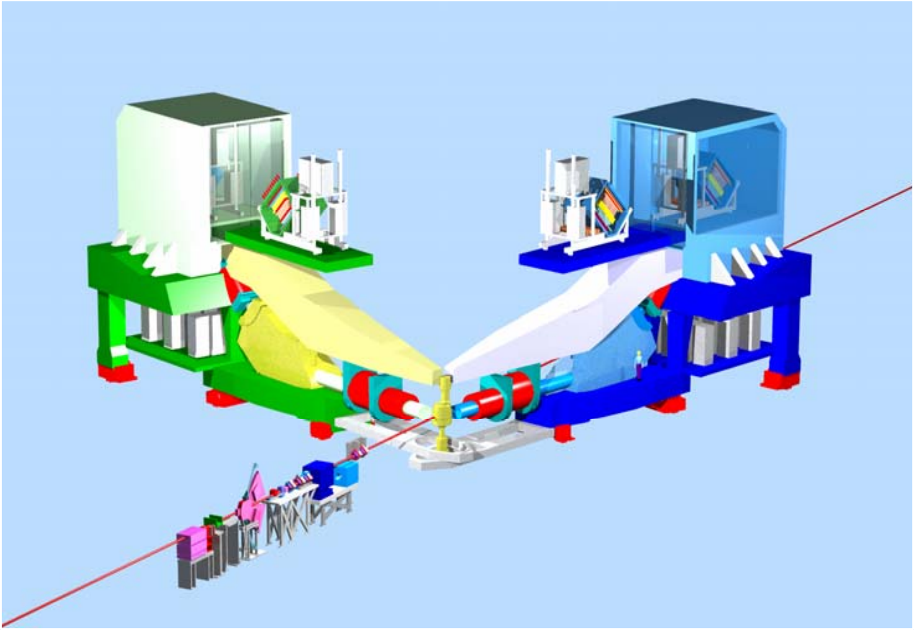
- HOW MUCH OF THE ELECTRIC and MAGNETIC FORM FACTORS ?

MIT-Bates & A4 at Mainz



G0 and HAPPEx at Jlab



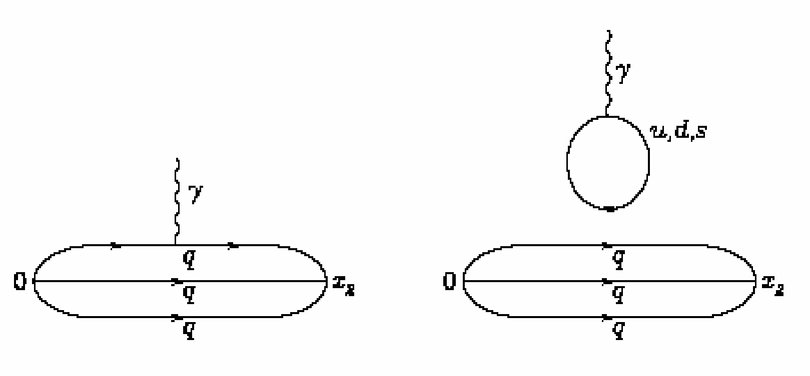


Direct calculation pioneered by Keh-Fei and collaborators BUT very difficult

Instead try indirect method…

Magnetic Moments within QCD

(Leinweber and Thomas, Phys Rev D62, 2000)



CS

$p = 2/3\ u^p - 1/3\ d^p + O_N$

$n = -1/3\ u^p + 2/3\ d^p + O_N$

$2p + n = u^p + 3\ O_N$

(and $p + 2n = d^p + 3\ O_N$)

$\Sigma^+ = 2/3\ u^\Sigma - 1/3\ s^\Sigma + O_\Sigma$

$\Sigma^- = -1/3\ u^\Sigma - 1/3\ s^\Sigma + O_\Sigma$

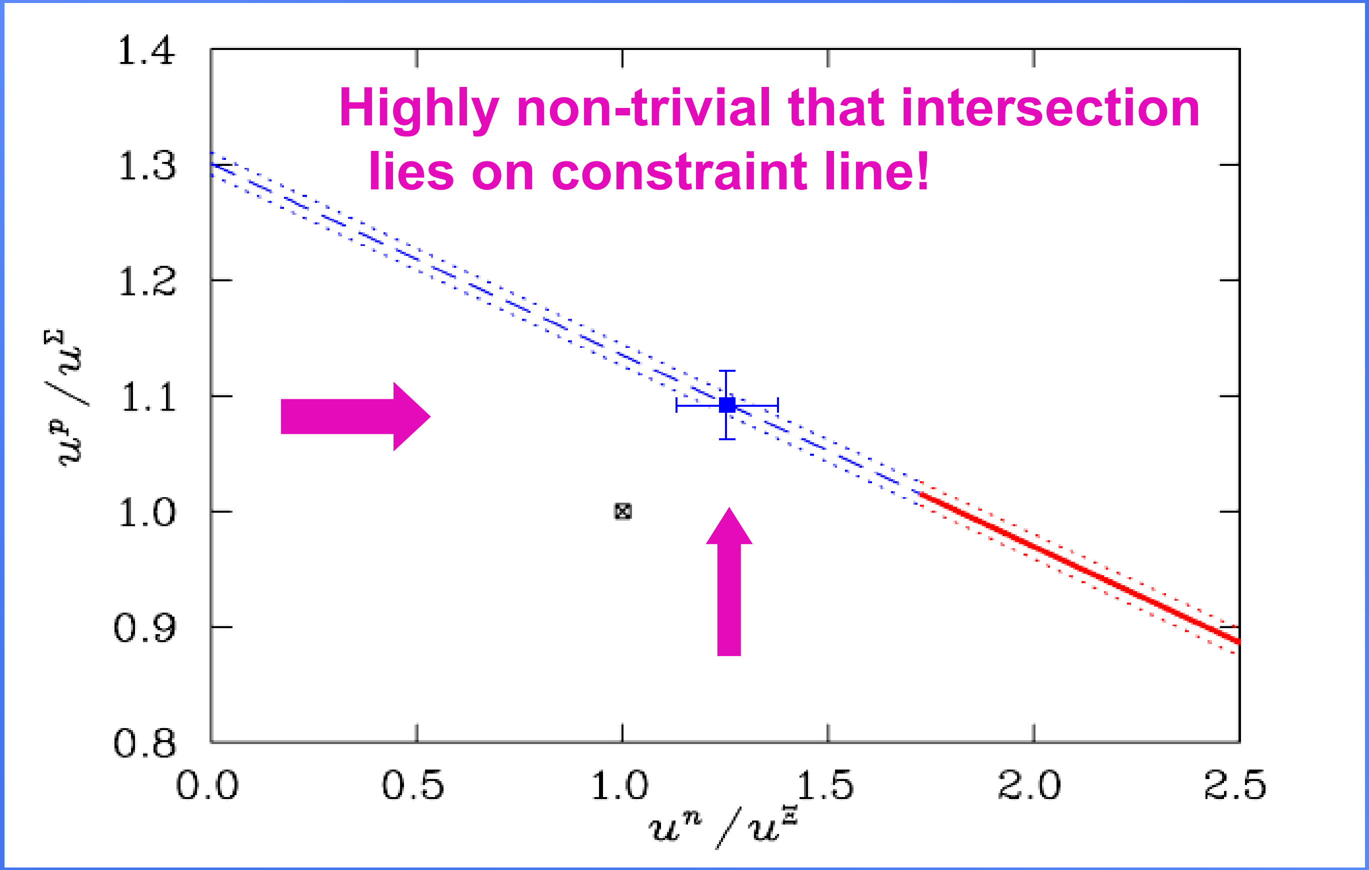
$\Sigma^+ - \Sigma^- = u^\Sigma$

HENCE: $O_N = 1/3\ [\ 2p + n - (\ u^p / u^\Sigma\ )\ (\Sigma^+ - \Sigma^-)\ ]$

Just these ratios from Lattice QCD

OR $O_N = 1/3\ [\ n + 2p - (\ u^n / u^\Xi\ )\ (\Xi^0 - \Xi^-)\ ]$

Accurate Final Result for G_M^s



Highly non-trivial that intersection lies on constraint line!

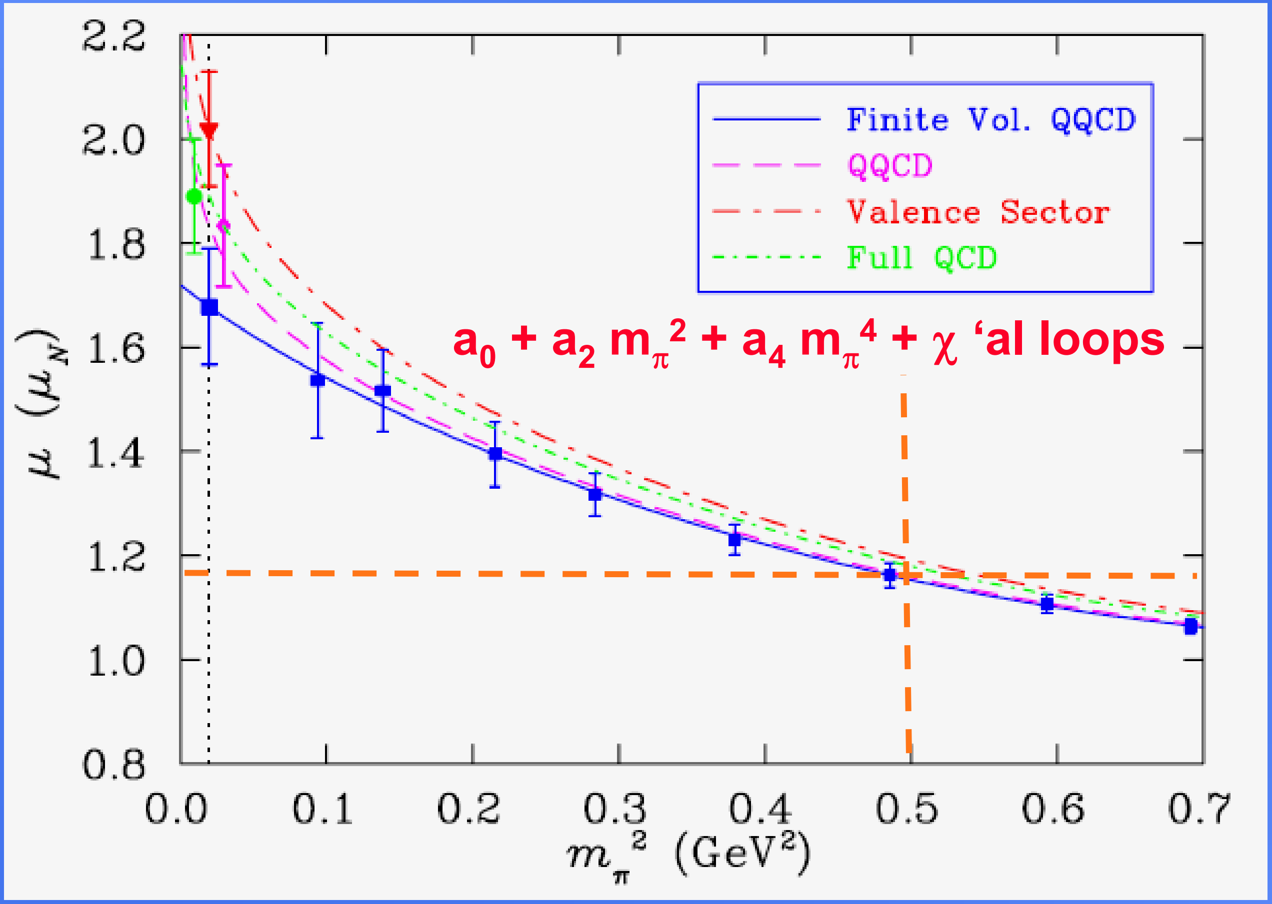
1.10±0.03

1.25±0.12

Yields : $G_M^s = -0.046 \pm 0.019\ \mu_N$

Leinweber et al., (PRL June '05) hep-lat/0406002

u^{p}_{valence} : QQCD Data Corrected for Full QCD Chiral Coefficients

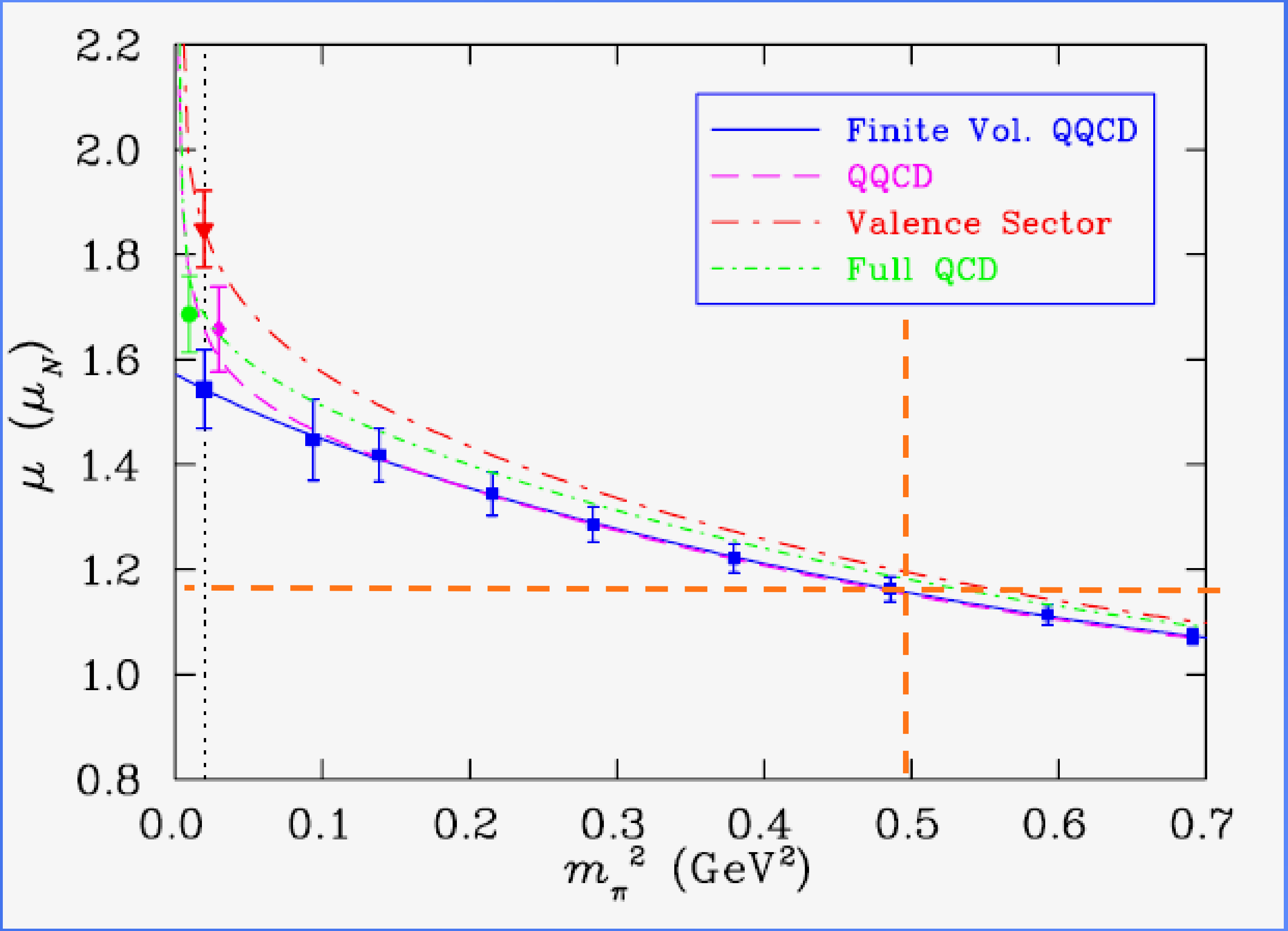


$a_0 + a_2\, m_\pi^2 + a_4\, m_\pi^4 + \chi$ ‘al loops

c.f. CQM
2/3 940/540
$\sim$ 1.18

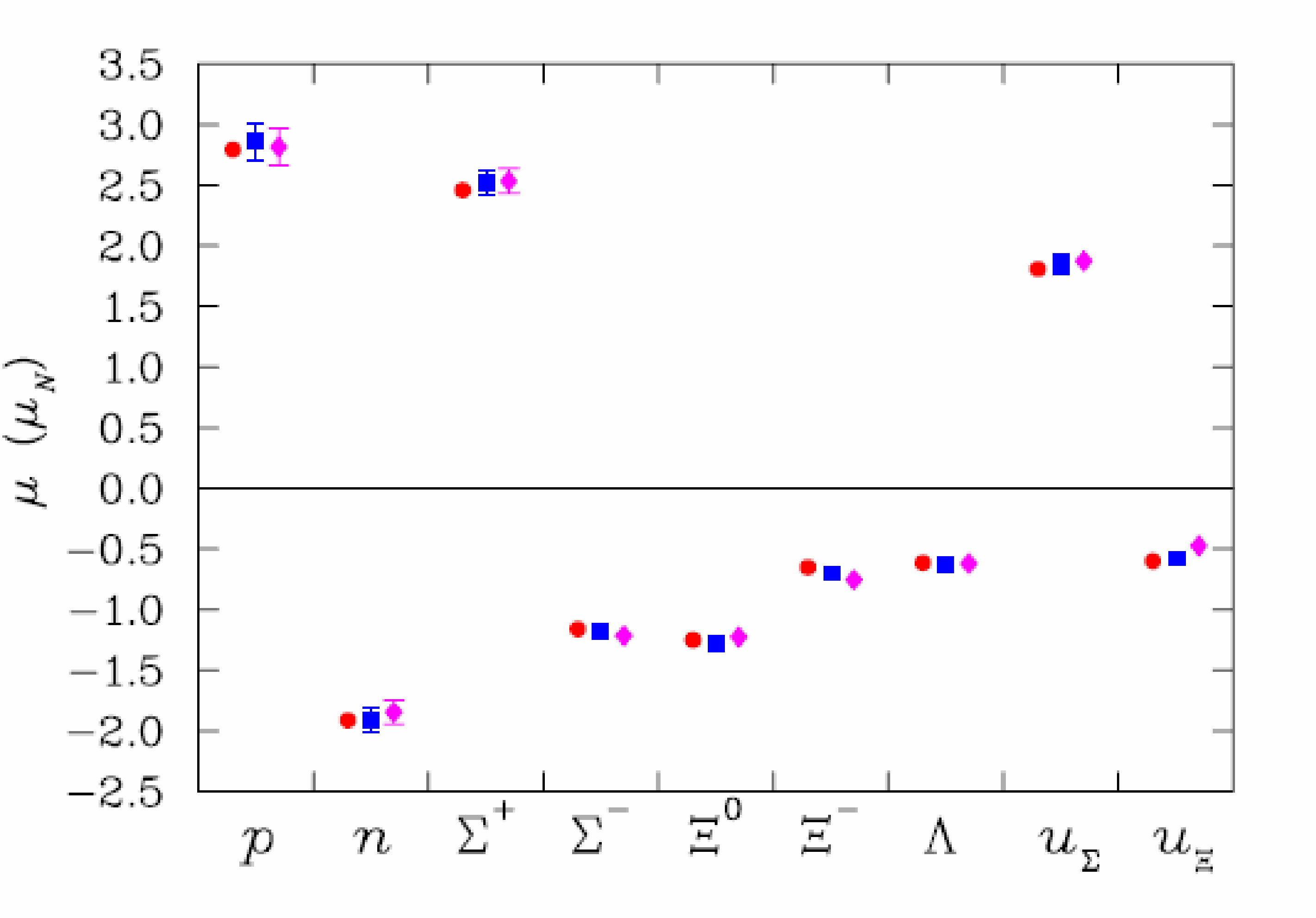
New lattice data from Zanotti et al. ; Chiral analysis Leinweber et al.

$u^{\Sigma}_{\text{valence}}$



Universal Here!

Convergence LNA to NLNA Again Excellent (Effect of Decuplet)



State of the Art Magnetic Moments

	QQCD	Valence	Full QCD	Expt.
p	2.69 (16)	2.94 (15)	2.86 (15)	2.79
n	-1.72 (10)	-1.83 (10)	-1.91 (10)	-1.91
Σ^+	2.37 (11)	2.61 (10)	2.52 (10)	2.46 (10)
Σ^-	-0.95 (05)	-1.08 (05)	-1.17 (05)	-1.16 (03)
Λ	-0.57 (03)	-0.61 (03)	-0.63 (03)	-0.613 (4)
Ξ^0	-1.16 (04)	-1.26 (04)	-1.28 (04)	-1.25 (01)
Ξ^-	-0.65 (02)	-0.68 (02)	-0.70 (02)	-0.651 (03)
u^p	1.66 (08)	1.85 (07)	1.85 (07)	1.81 (06)
u^Ξ	-0.51 (04)	-0.58 (04)	-0.58 (04)	-0.60 (01)

G_E^s by similar technique

In this case only know Σ^- radius (and p and n)

hence use absolute values of u and d radii:

$2p + n = u^p + 3\,O_N$ $\qquad$ $p + 2n = d^p + 3\,O_N$

$\Rightarrow \langle r^2 \rangle_s = 0.000 \pm 0.006 \pm 0.007\ \mathrm{fm}^2\ ;\ 0.002 \pm 0.004 \pm 0.004\ \mathrm{fm}^2$

(c.f. using Σ^- : $-0.007 \pm 0.004 \pm 0.007 \pm 0.021\ \mathrm{fm}^2$)

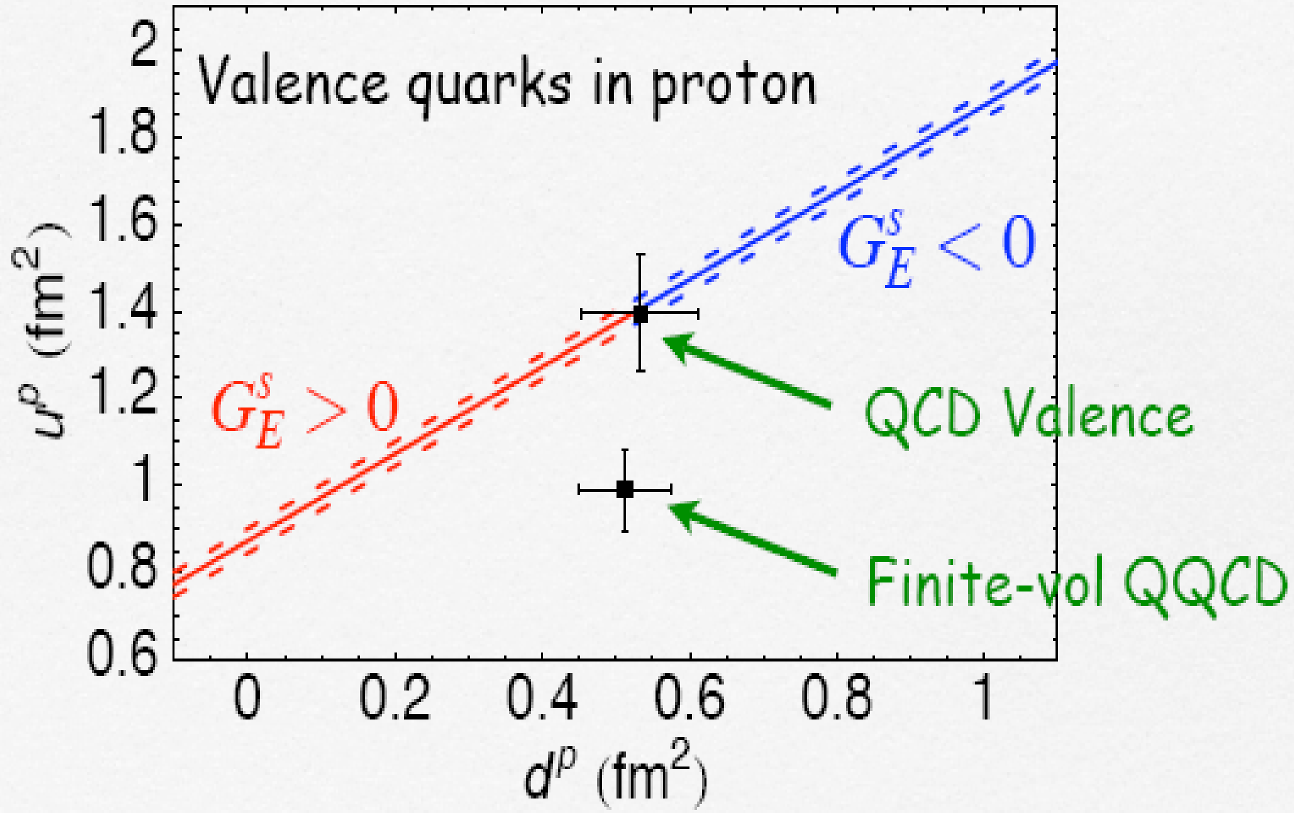
$$G_E^s(0.1\,\mathrm{GeV}^2) = +0.001 \pm 0.004 \pm 0.004$$

(up to order Q^4)

Note consistency and level of precision!

Leinweber, Young et al., hep-lat/0601025 (Jan 2006)

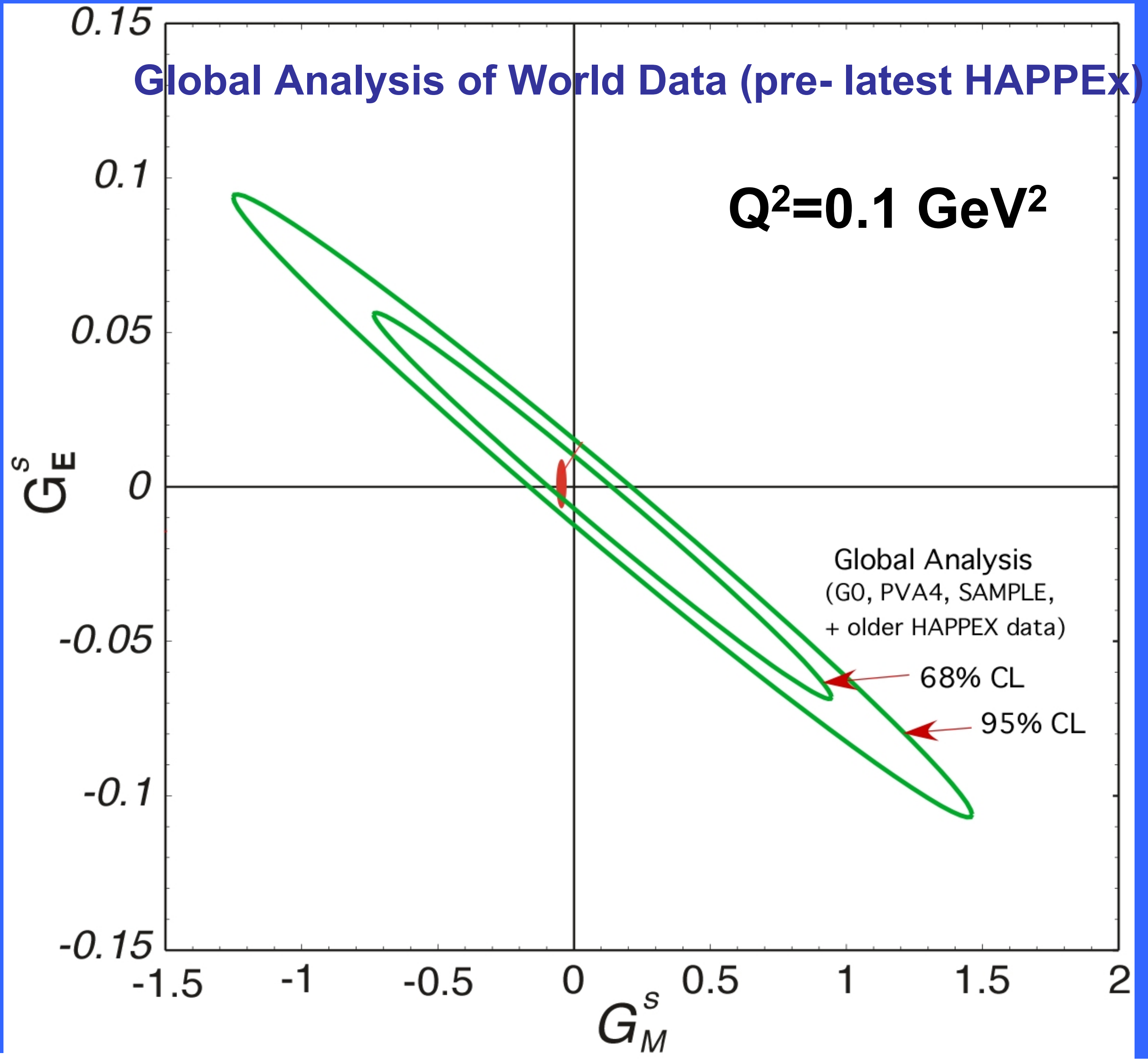
Model Independent Constraint Again Satisfied



$$G_E^s(Q^2 = 0.1) = +0.001 \pm 0.004 \pm 0.004$$

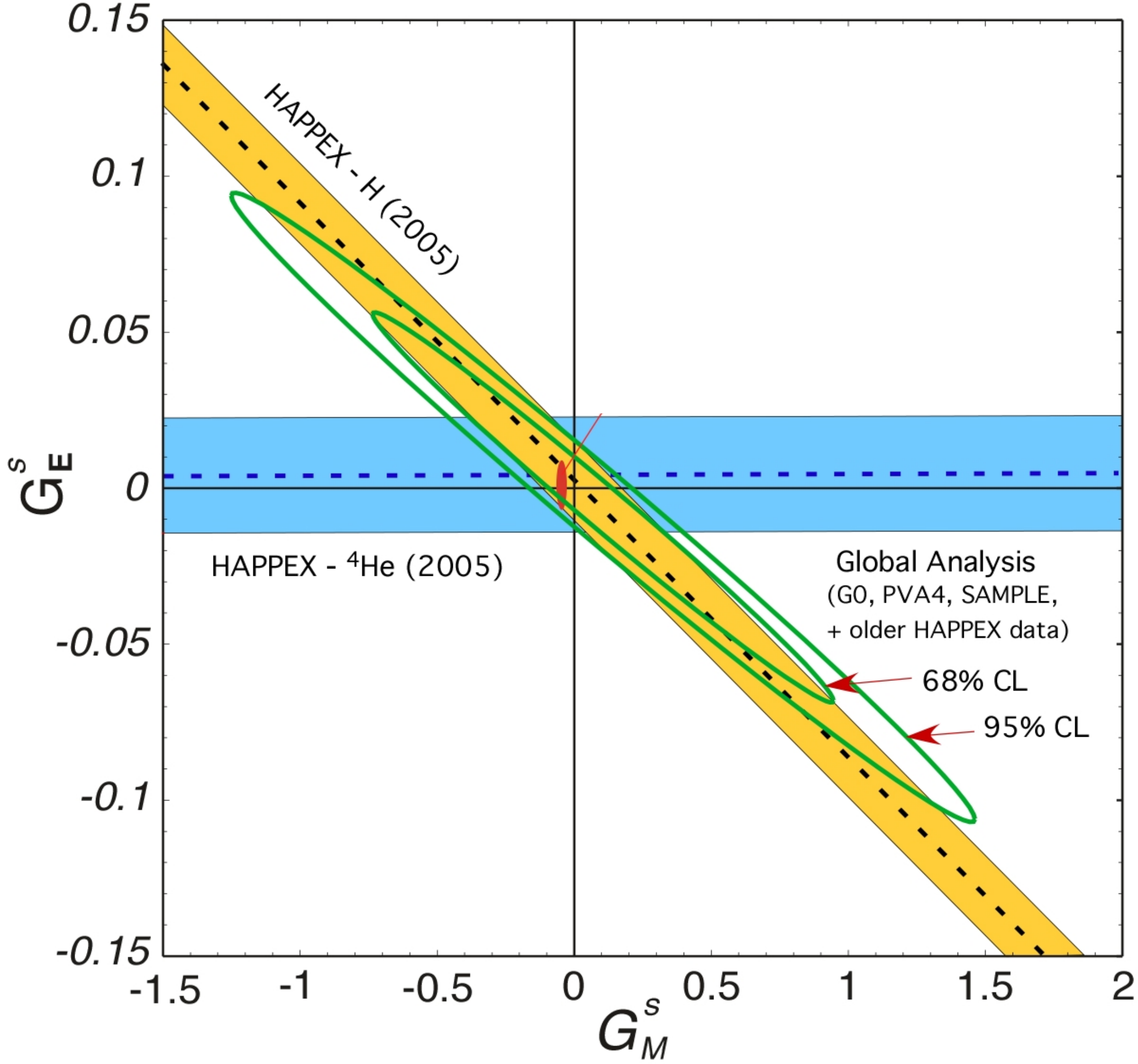
Leinweber, RDY *et al.* hep-lat/0601025

Young, Roche, Carlini, Thomas – nucl-ex/0604010 (PRL, Sept. 2006)

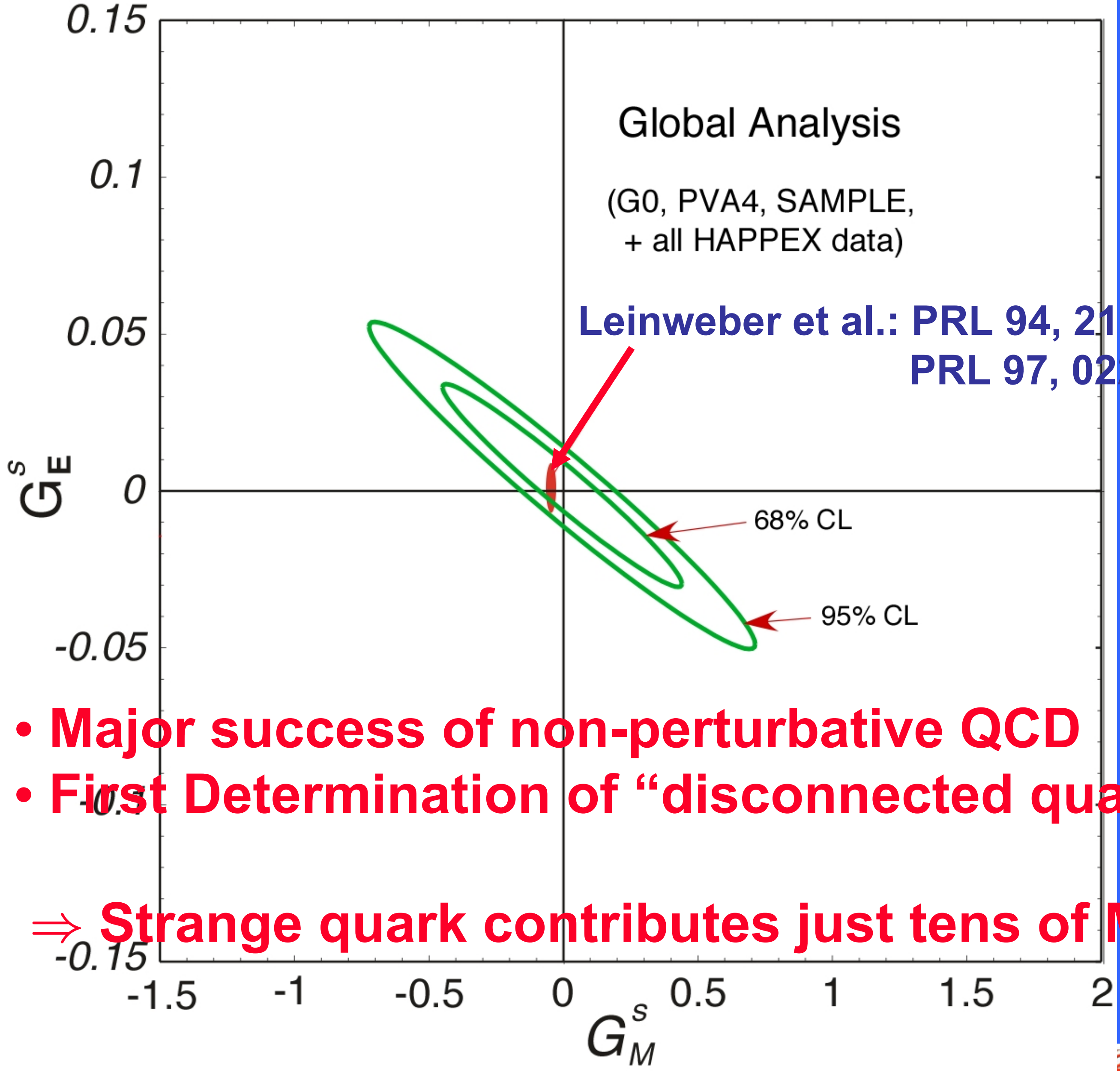

Global Analysis of World Data (pre- latest HAPPEx)

$Q^2=0.1\ \mathrm{GeV}^2$

Superimpose NEW HAPPEx Measurement (Dallas APS meeting, April 06)



Include new HAPPEx data : halves errors of previous world data !

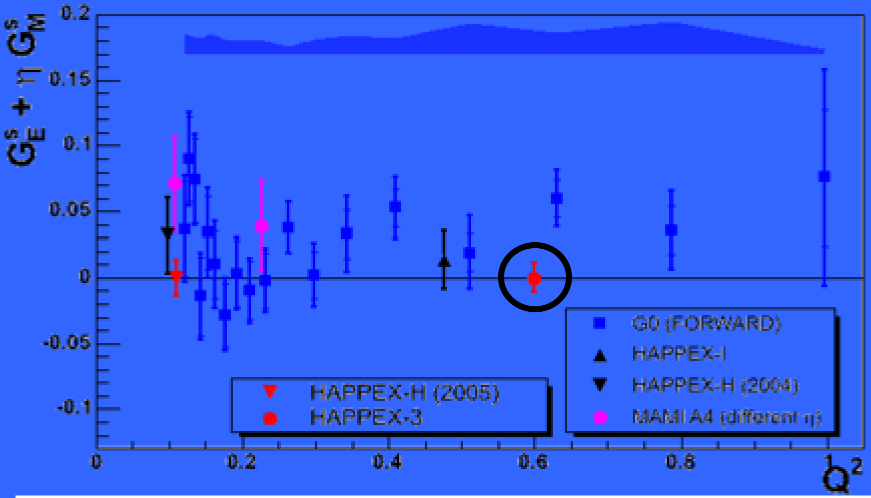


Leinweber et al.: PRL 94, 212001 (2005)
PRL 97, 022001 (2006)

- **Major success of non-perturbative QCD**
- **First Determination of "disconnected quark loop"**

⇒ Strange quark contributes just tens of MeV to M_N

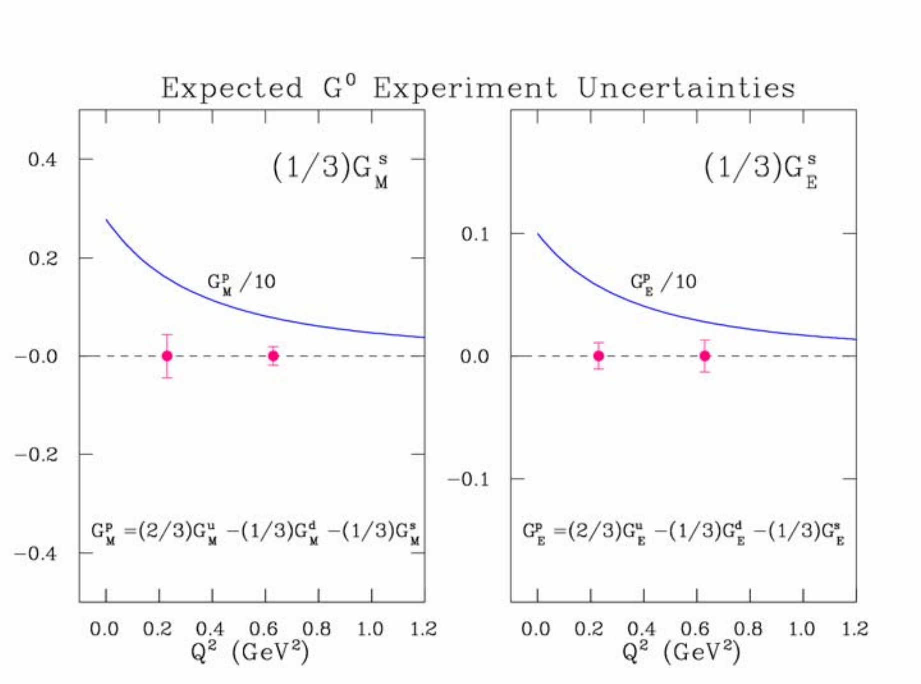
Strange Form Factor Measurements – Future Plans



HAPPEx: "HAPPEx3"
measure $G^s_E + 0.48G^s_M$ with high precision at Q^2~0.6 GeV^2

G^0: Turn experiment around

- **detect electrons at θ = 108°**
- **add Cerenkov for pion rejection**
- **measure at Q^2 = .23 and .63 GeV^2**
- **LH_2 and LD_2 targets**

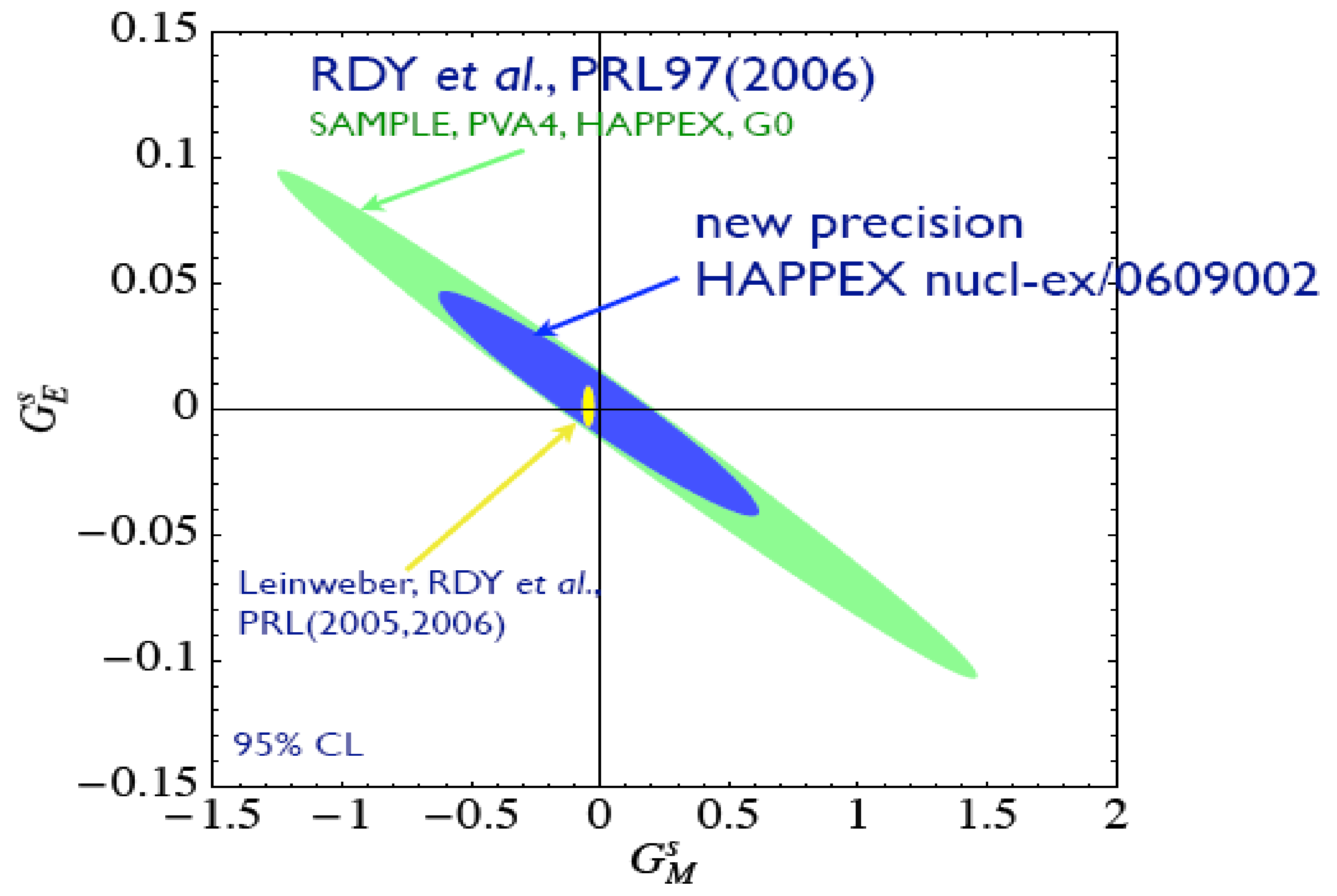


Mainz A4: Turn experiment around

- **detect electrons at θ = 145°**
- **Measure at Q^2 = .23 and .47 GeV^2**
- **LH_2 and LD_2 targets**

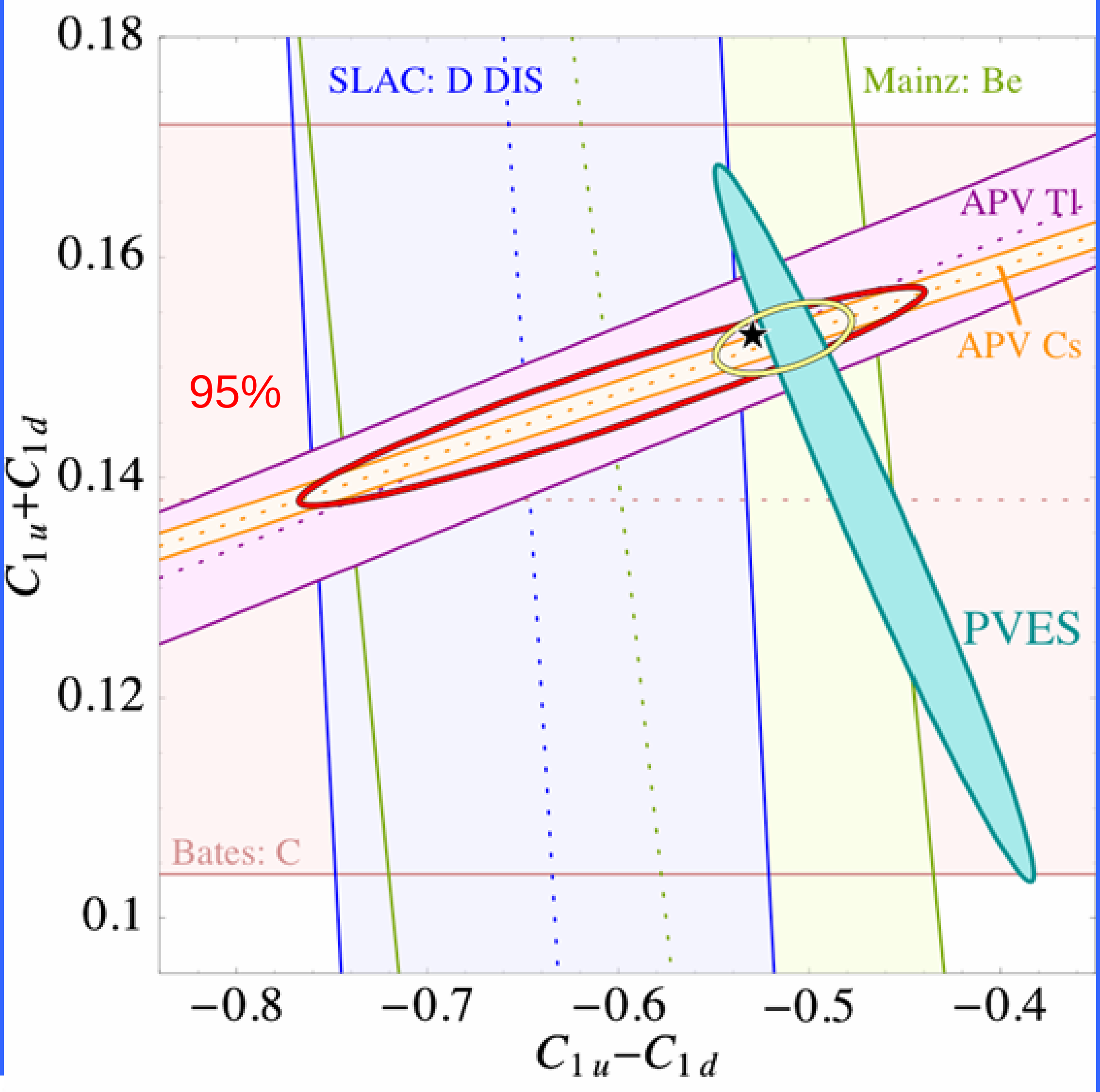
from Mark Pitt

Previously: Saw Precision of PVES for Strange Form Factors



Can we achieve meaningful accuracy in testing Standard Model now?

New update on C_{1q} couplings – Dec 2006



(Young, Carlini, Roche and AWT)

Dramatic improvement in knowledge of weak couplings!

Factor of 5 increase in precision of Standard Model test

Model-independent limits on New Physics

$$\mathcal{L}_{\mathrm{SM}}^{\mathrm{PV}} = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\mu\gamma_5 e \sum_q C_{1q}^{\mathrm{SM}}\bar{q}\gamma^\mu q$$

Erler et al., PR D68 (2003)

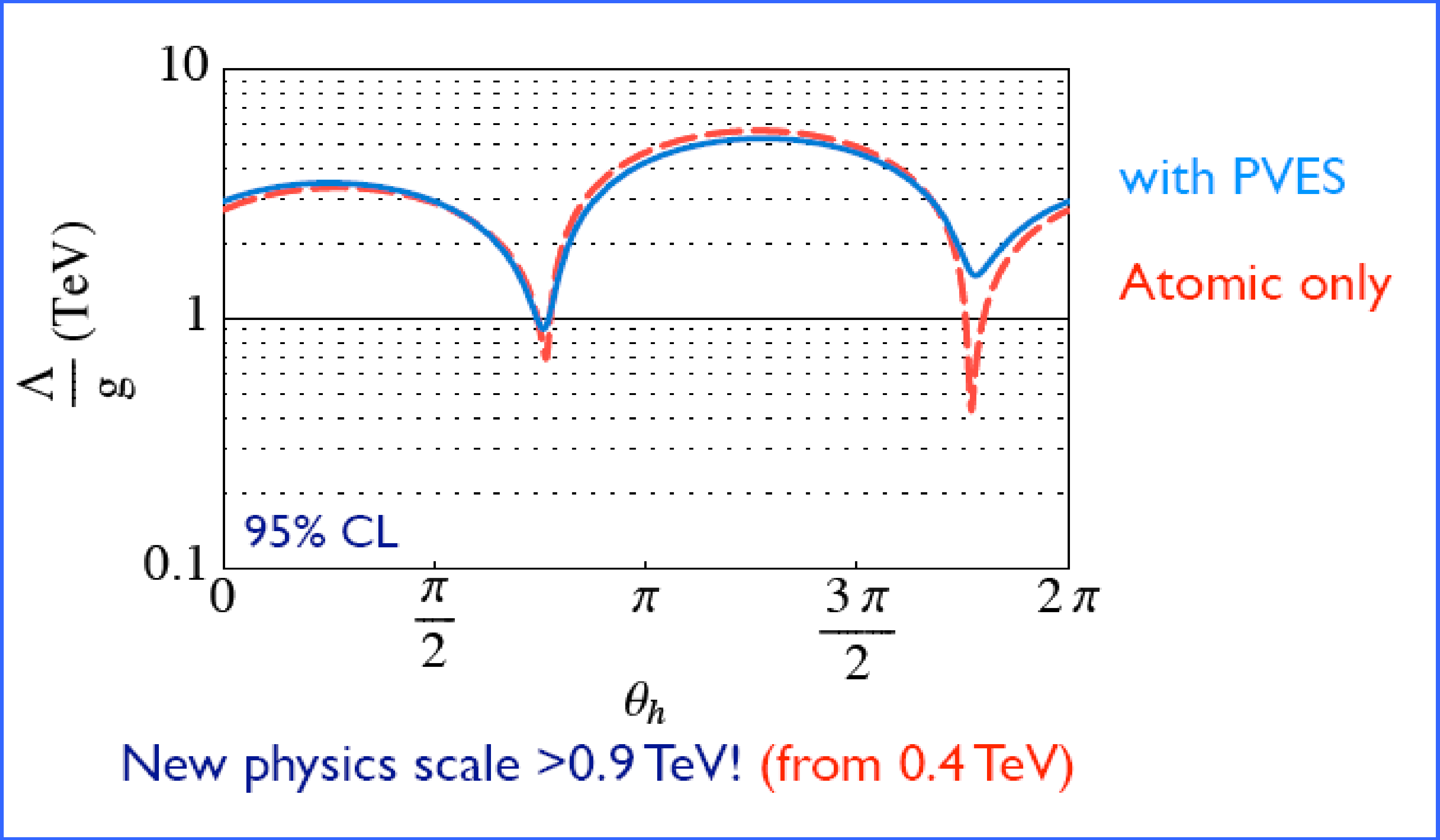
$$\mathcal{L}_{\mathrm{NP}}^{\mathrm{PV}} = \frac{g^2}{4\Lambda^2}\bar{e}\gamma_\mu\gamma_5 e \sum_q h_V^q\bar{q}\gamma^\mu q$$

Full isospin coverage for limits on new physics!

$$h_V^u = \cos\theta_h \qquad h_V^d = \sin\theta_h$$

Data sets limits on $\dfrac{g^2}{\Lambda^2}$

Lower bound on scale of New Physics

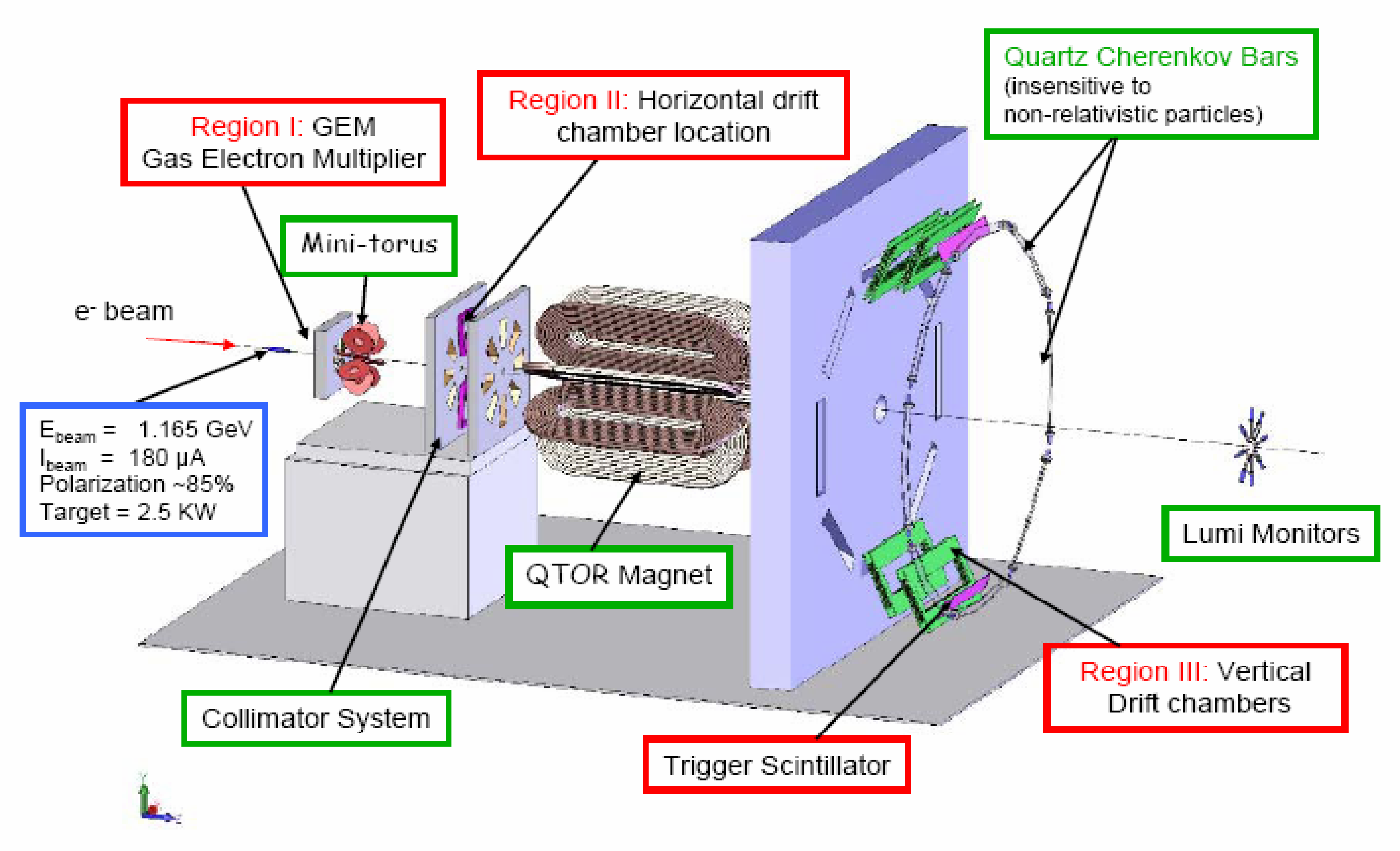


New physics scale >0.9 TeV! (from 0.4 TeV)

Young et al. (Dec 2006)

New physics scale >0.9 TeV! (up from 0.4 TeV)

Q_{weak} Apparatus



Region I: GEM
Gas Electron Multiplier

Region II: Horizontal drift chamber location

Quartz Cherenkov Bars
(insensitive to non-relativistic particles)

Mini-torus

e^- beam

E_{beam} = 1.165 GeV
I_{beam} = 180 µA
Polarization ~85%
Target = 2.5 KW

QTOR Magnet

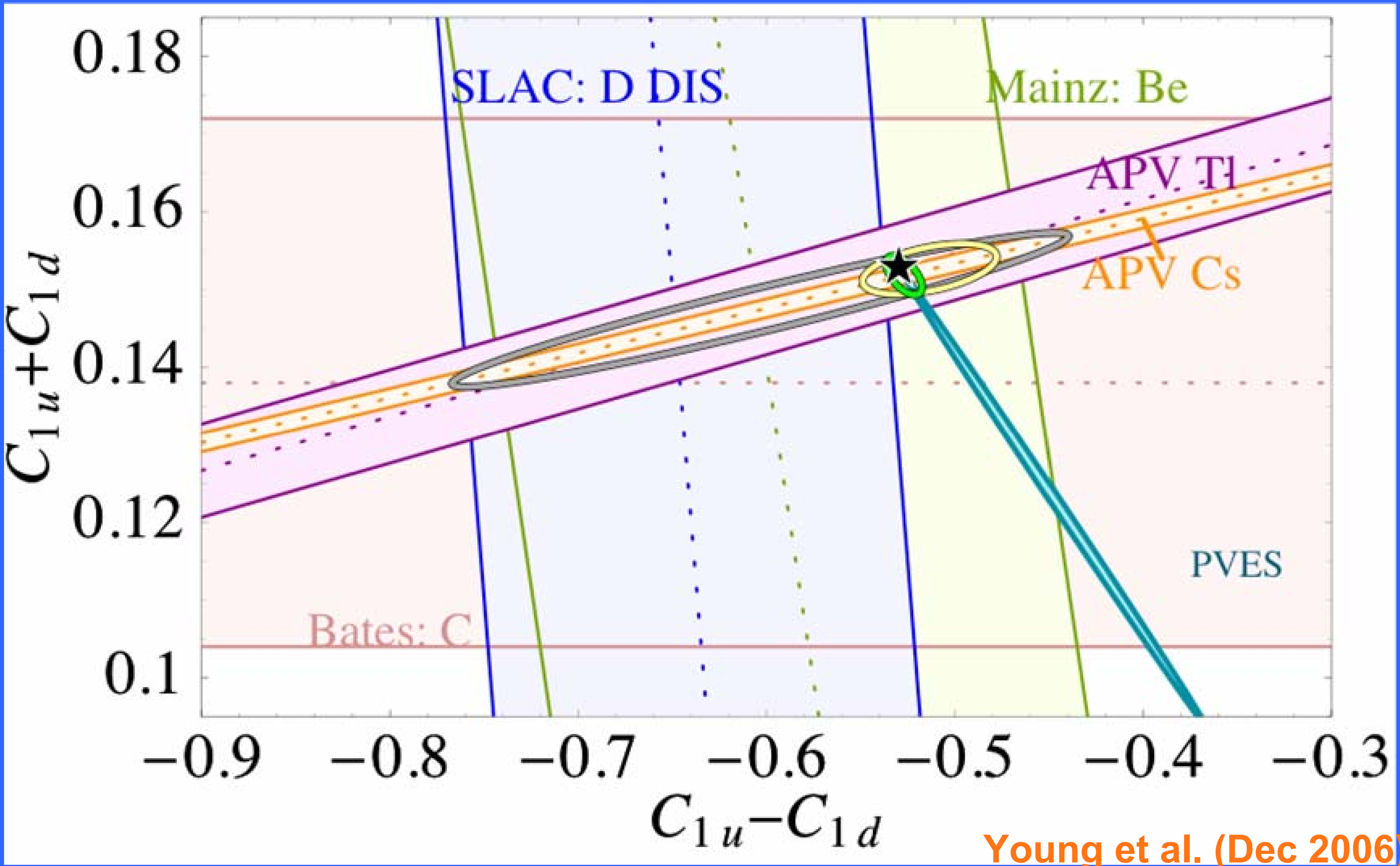
Lumi Monitors

Collimator System

Trigger Scintillator

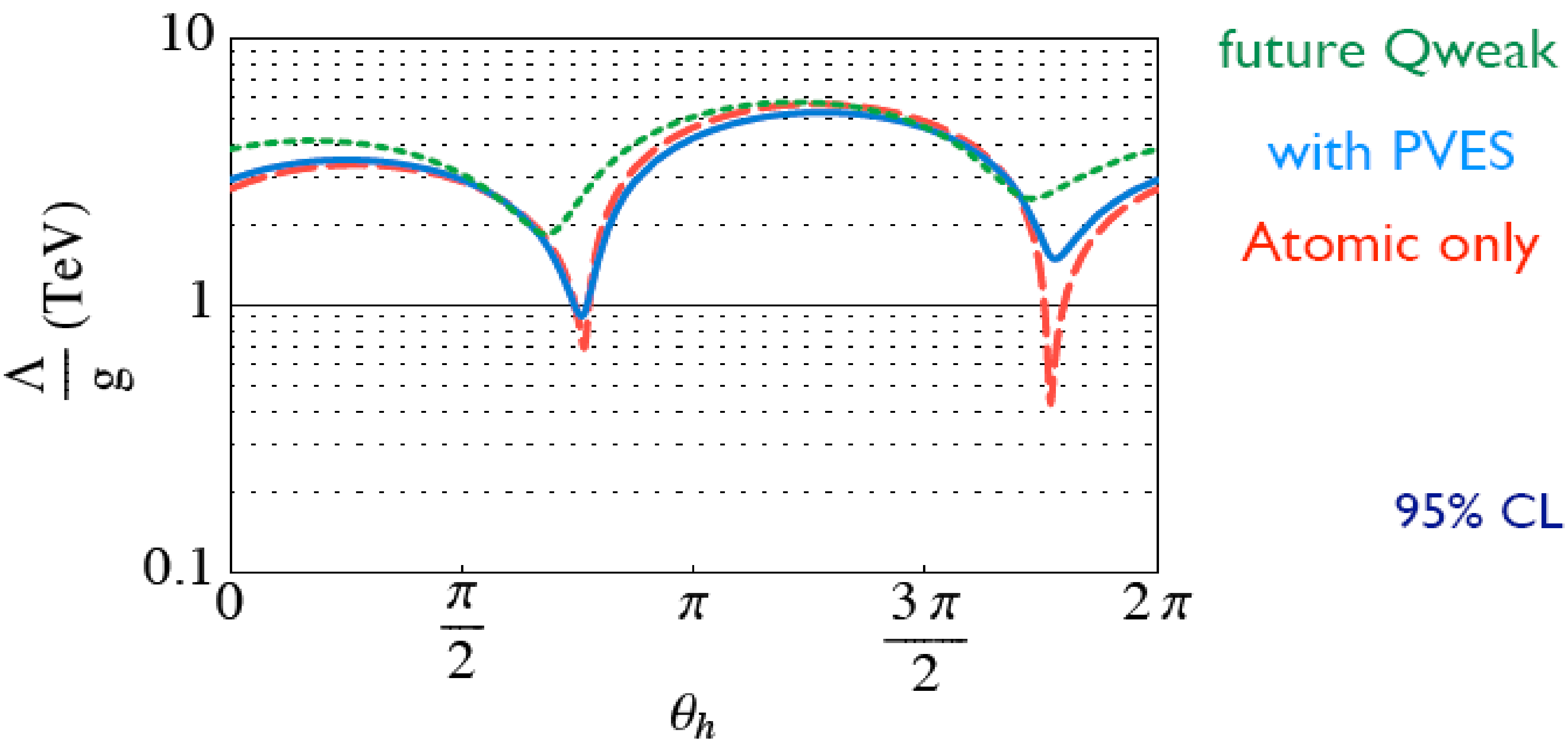
Region III: Vertical Drift chambers

Possible Impact of Qweak



Young et al. (Dec 2006)

New Physics Limits
(if result consistent with Standard Model)

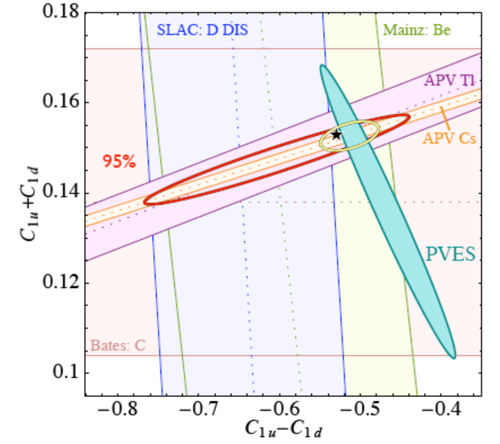


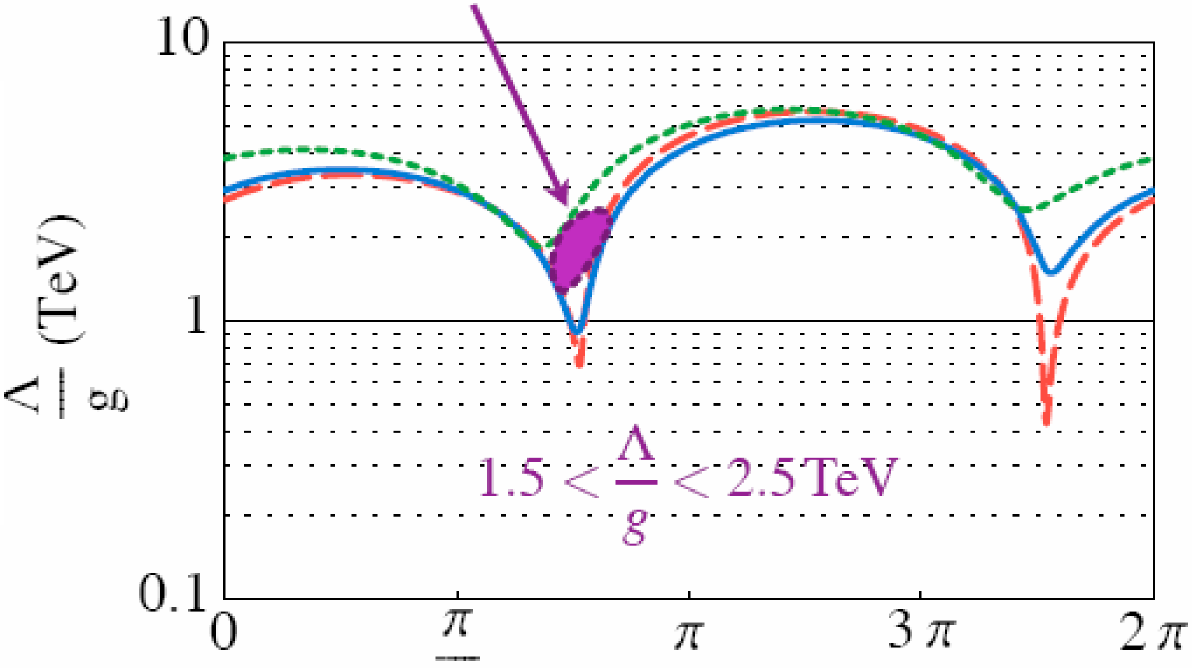
Qweak constrains new physics to beyond 2 TeV

Young et al. (Dec 2006)

But: Q_{weak} has real discovery potential!

Assume Qweak takes central value of current measurements





$$1.5 < \frac{\Lambda}{g} < 2.5\,\text{TeV}$$

If LHC finds new Z', Qweak will help determine nature of interaction

Young et al. (Dec 2006)

Conclusions

- Wonderful synergy between experimental advances at Jlab and progress using Lattice QCD to solve QCD

- Study of hadron properties as function of m_q using data from lattice QCD is extremely valuable…..
  (major qualitative advance in understanding)
  + TEST BEYOND STANDARD MODEL

- Inclusion of model independent constraints of χ PT to get to physical quark mass is essential
  FRR χPT resolves problem of convergence
- Insight enables: accurate, controlled extrapolation of all hadronic observables….
  (e.g. m_H , μ_H , $G_{E,M}^s$, $\langle r^2\rangle_{ch}$, G_E,G_M, $\langle x^n\rangle$……..)

Conclusions….$_2$

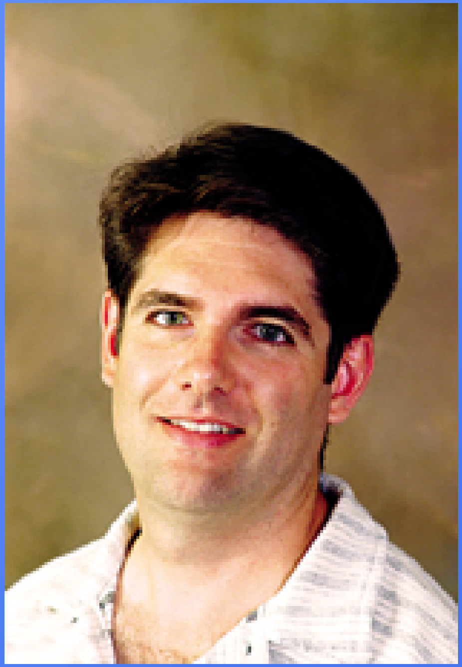
- In case where chiral coefficients are known, FRR enables accurate extrapolation to physical point

- Without chiral coefficients (e.g. spectroscopy of baryons and mesons) need data at very low pion mass (several points below ~ 0.25 GeV)

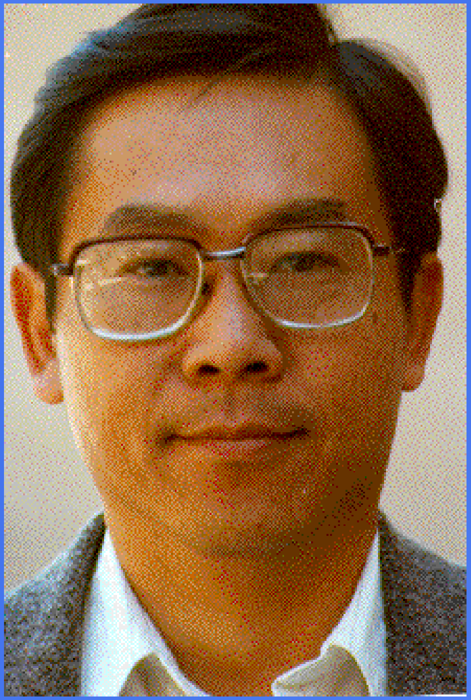
- It is a major challenge to obtain a reliable signal for "disconnected" loops <u>directly</u> in lattice QCD
  — this is a very important challenge

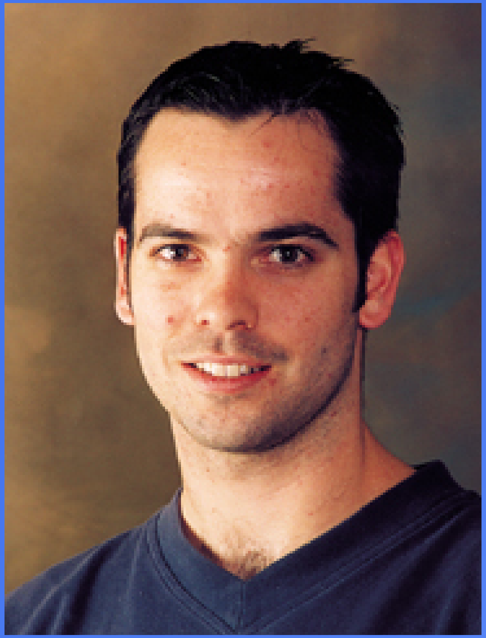
- For future there is a wonderful synergy with 12 GeV program at JLab and work on GPDs, form factors at high Q^2 , and higher moments of PDFs just beginning…..

Special Mentions……



Derek Leinweber





Ross Young

Congratulations and Best Wishes Keh-Fei!!