Quantum matter and gauge-gravity duality

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Subir Sachdev

PHYSICS
HARVARD

Outline

1. Conformal quantum matter

2. Compressible quantum matter

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The boson Hubbard model

and the superfluid-insulator transition

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Non-Fermi liquid phases map to holographic spaces with logarithmic violation of entanglement entropy

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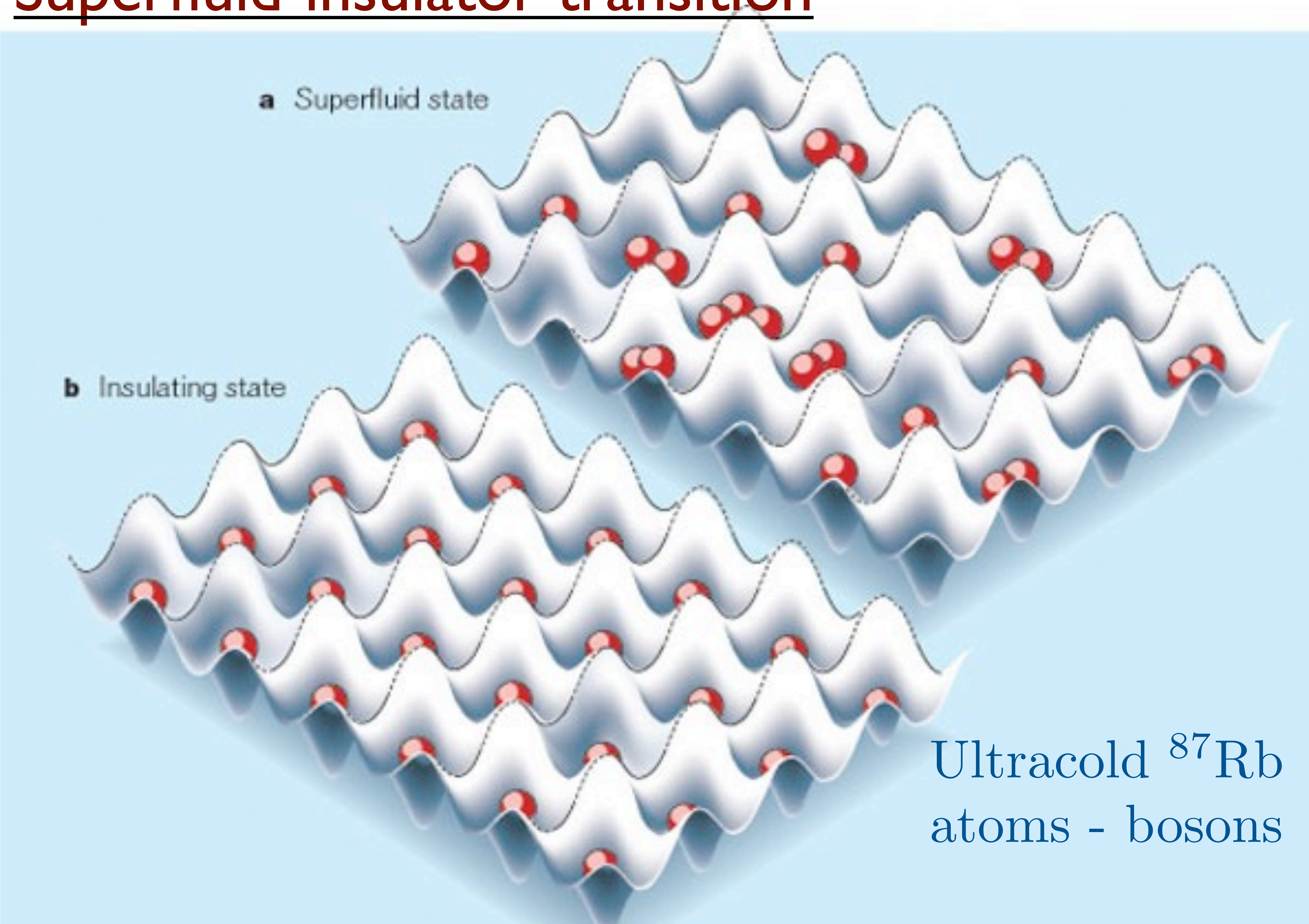
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

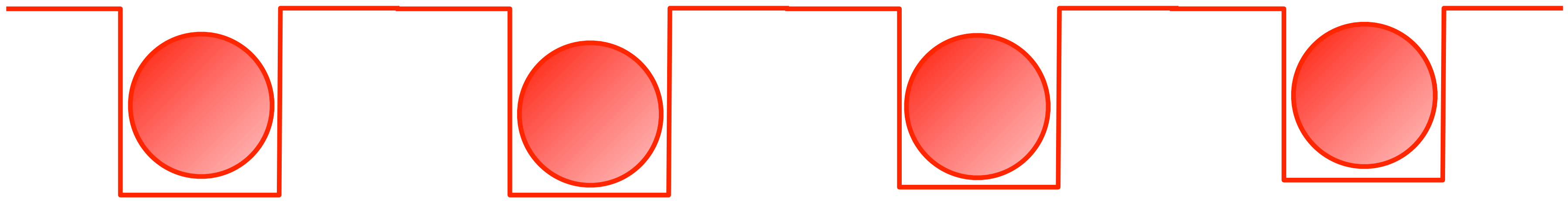
Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

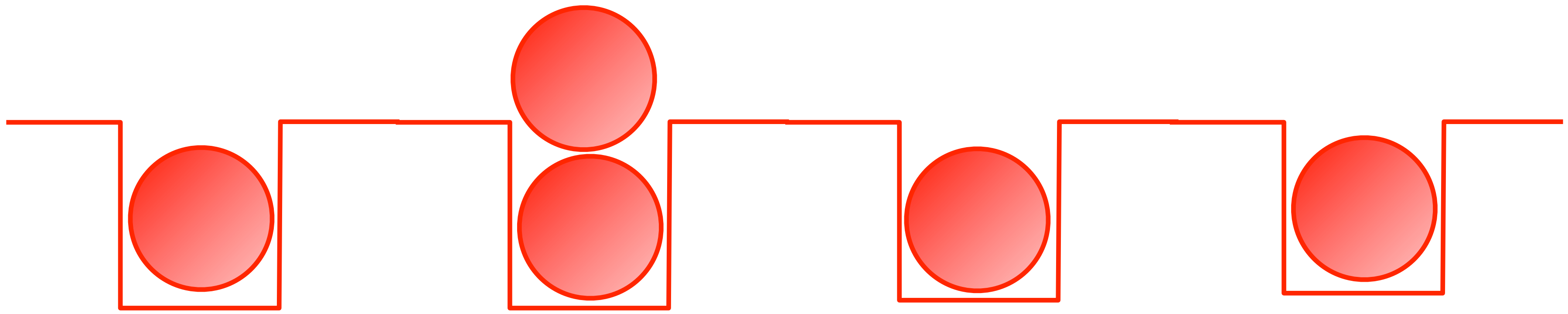
$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).



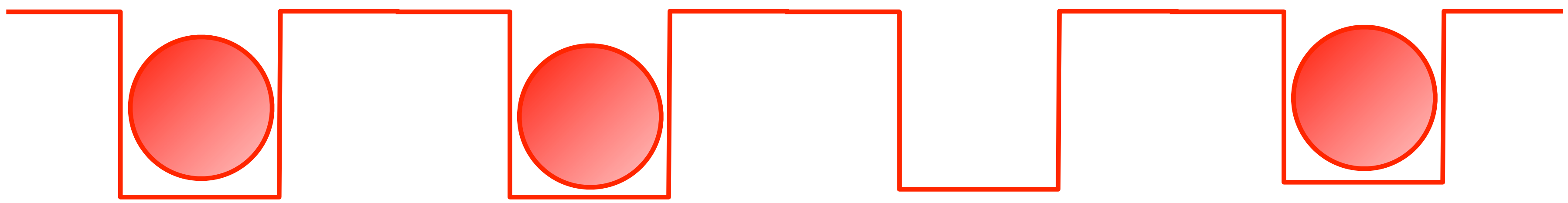
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



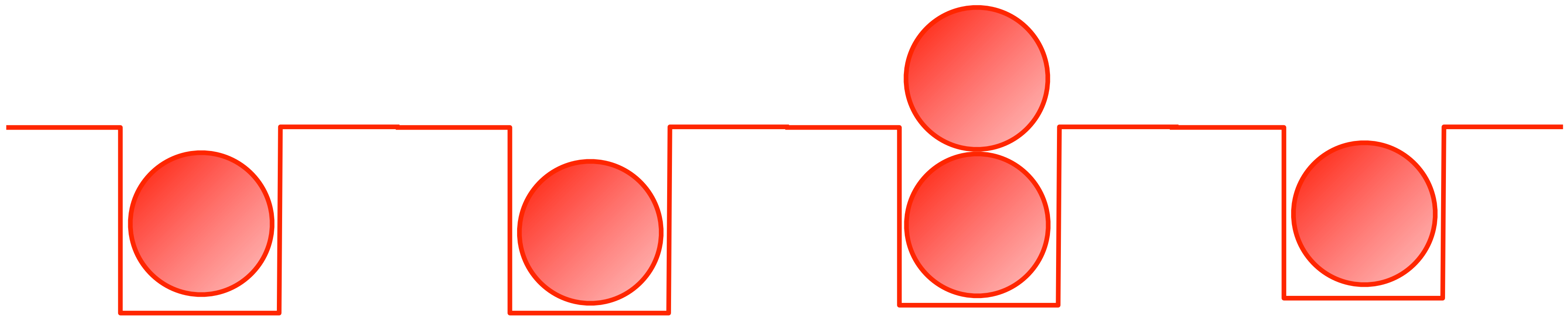
Particles $\sim \psi^\dagger$

Excitations of the insulator:



Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$

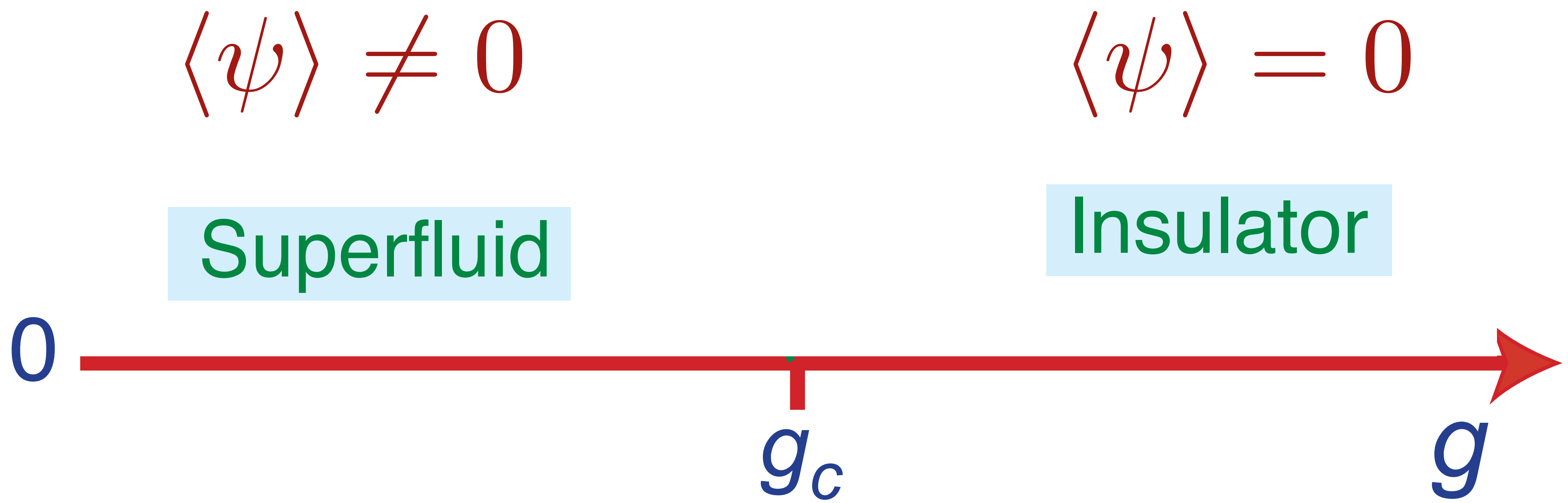
Holes ~ ψ

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

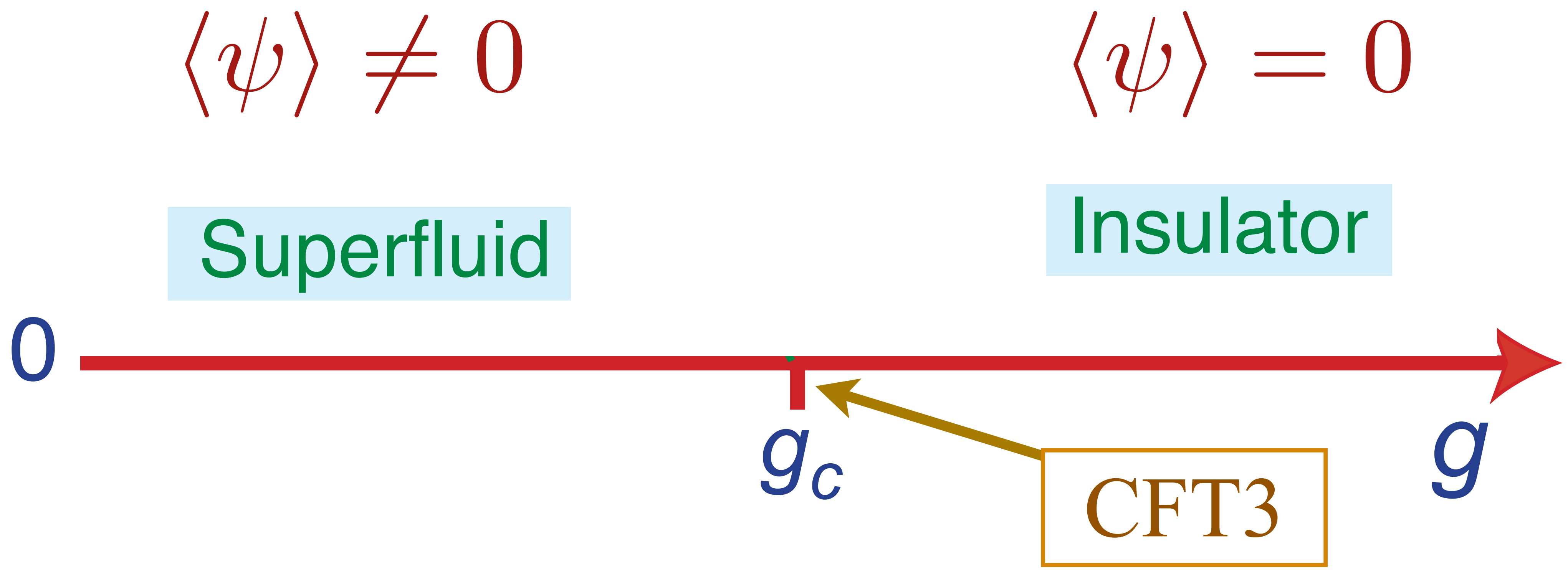
.

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

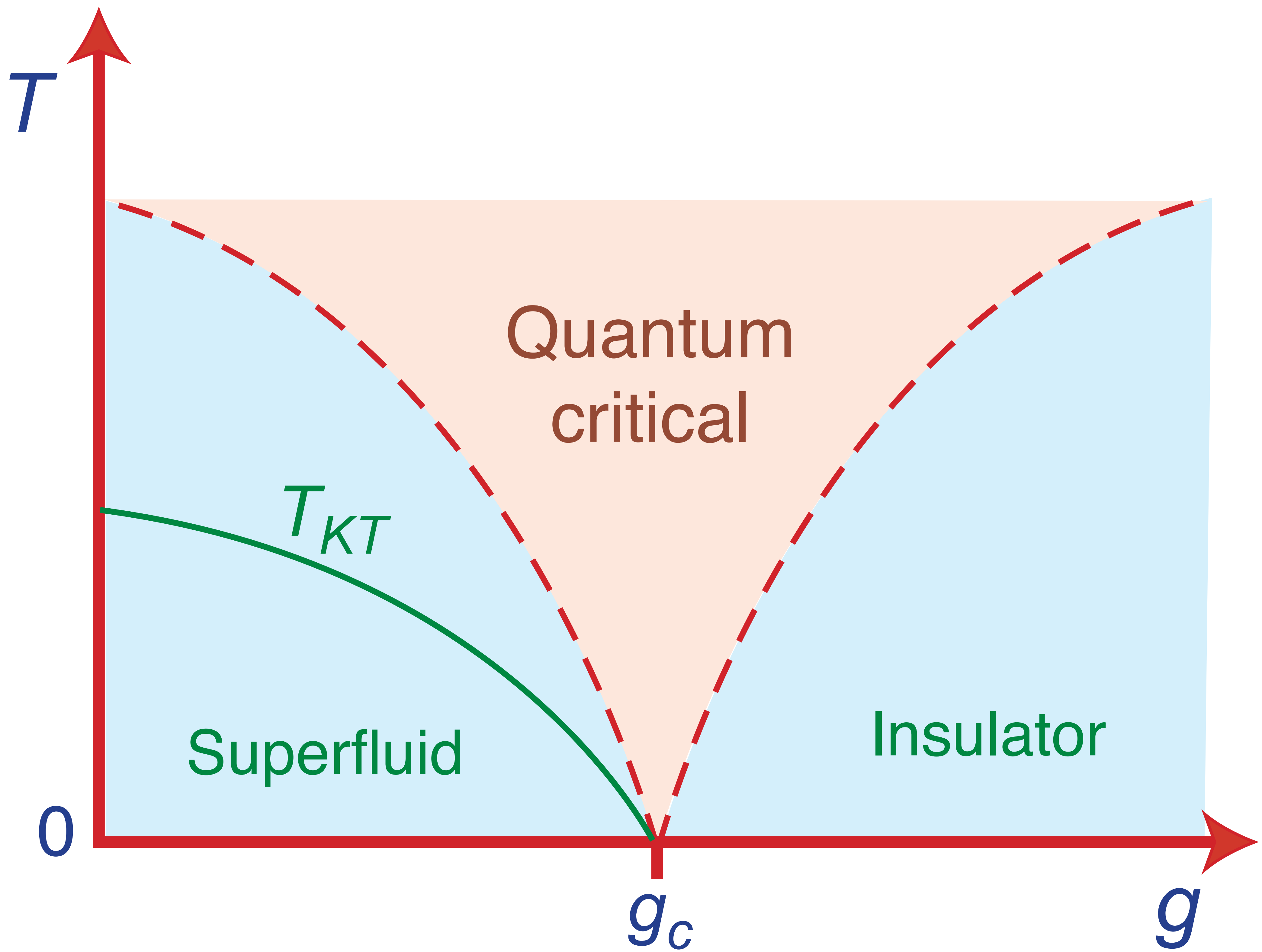
M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).



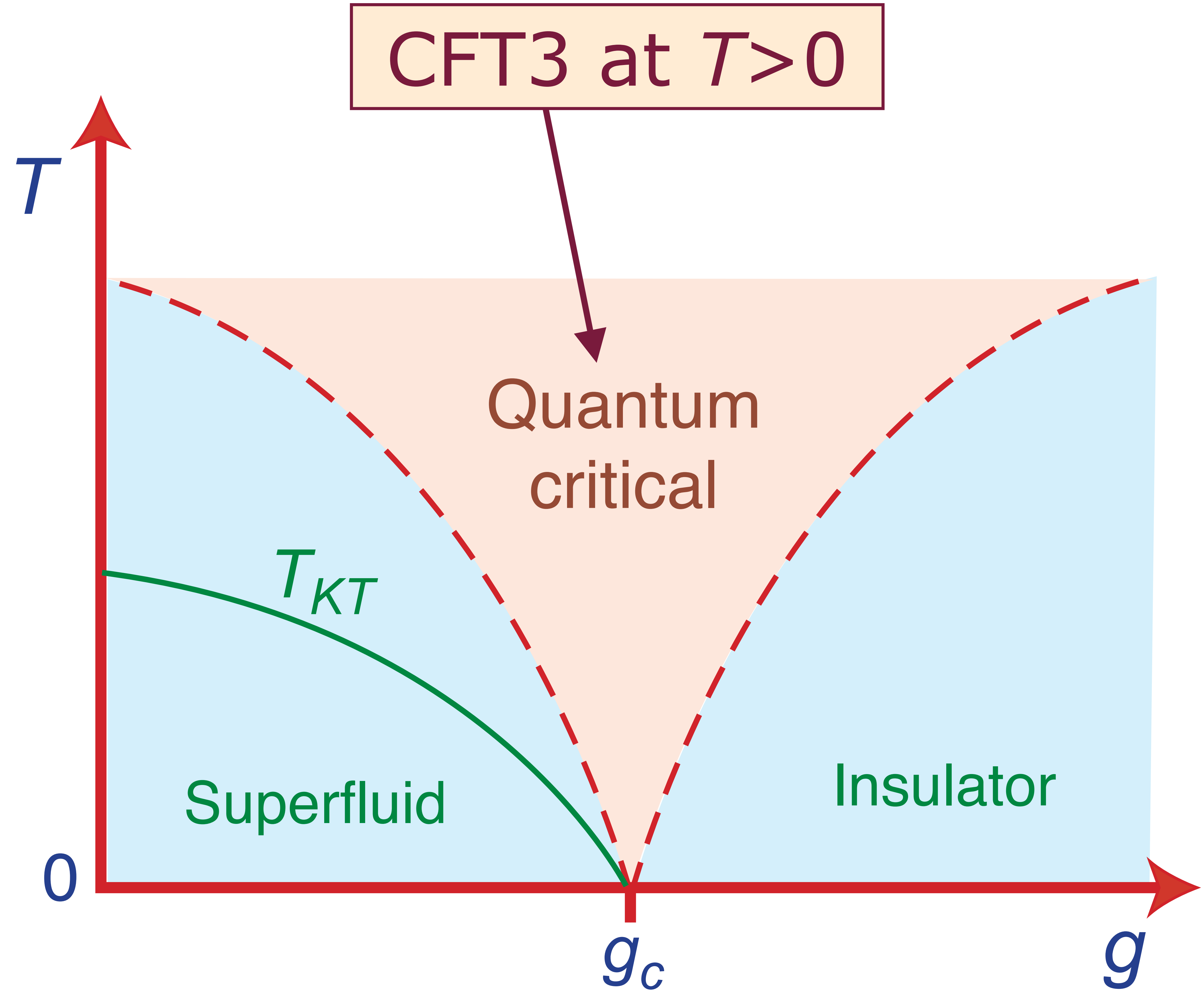
$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g



$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
CFT3
g



T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"* with shortest possible equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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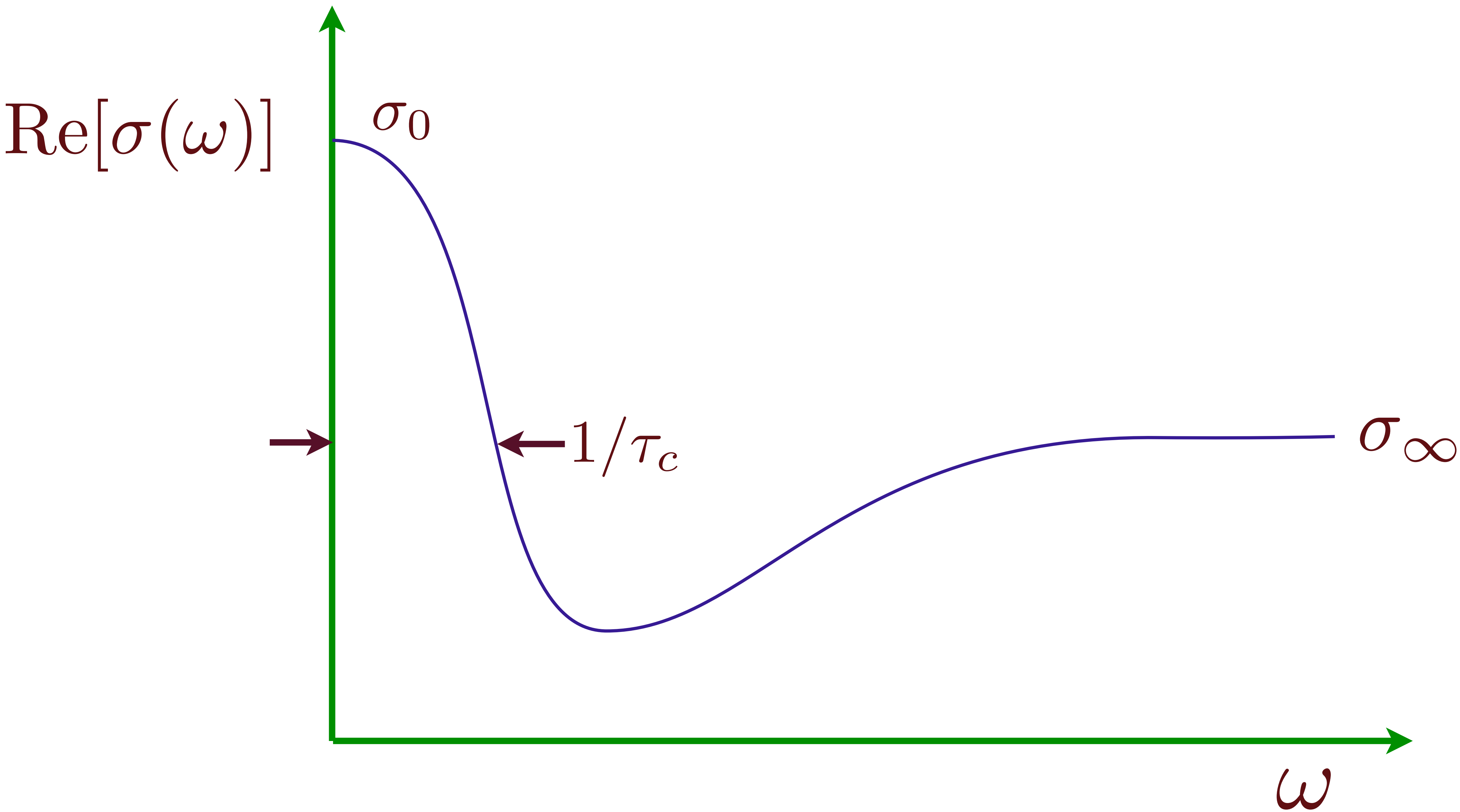
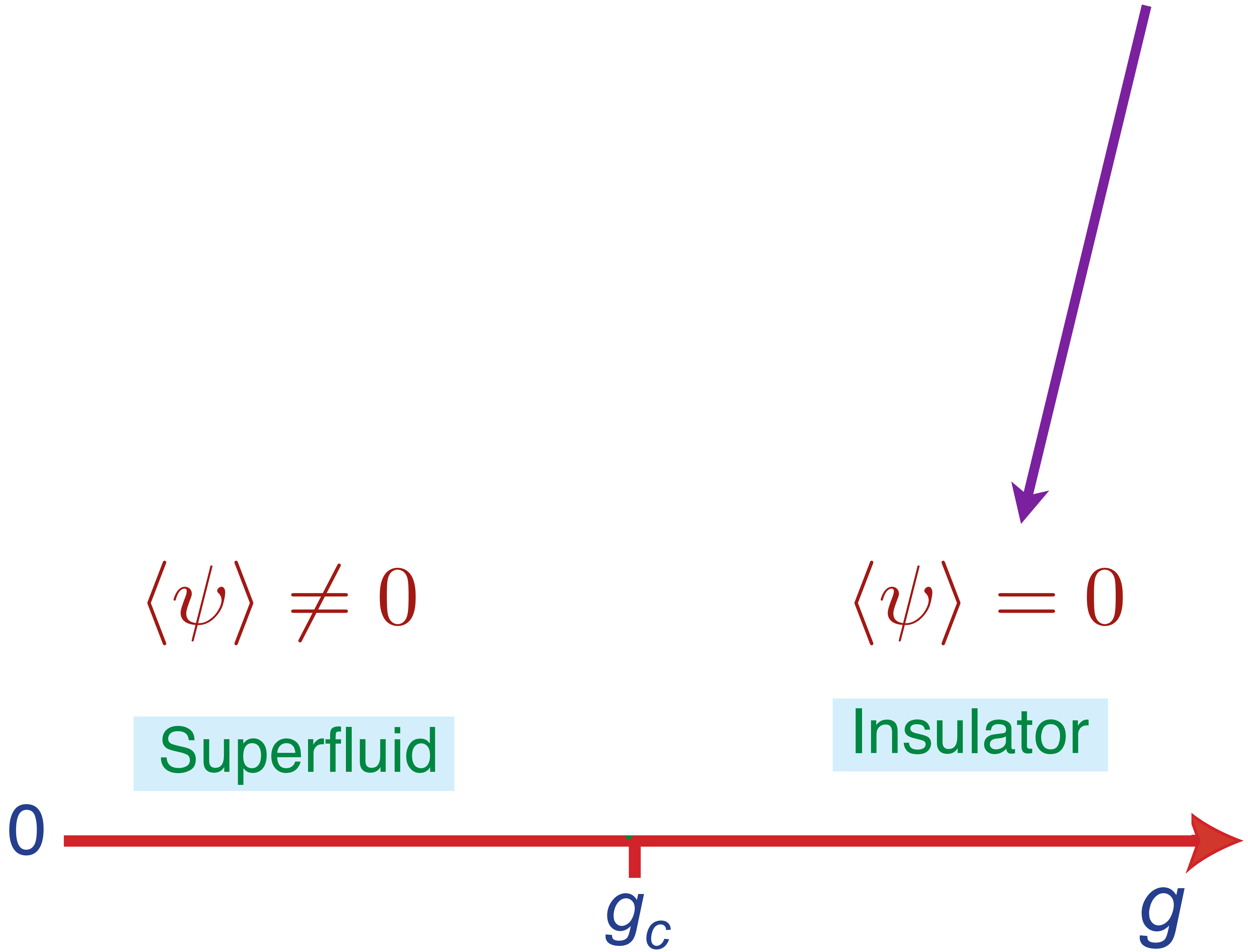
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

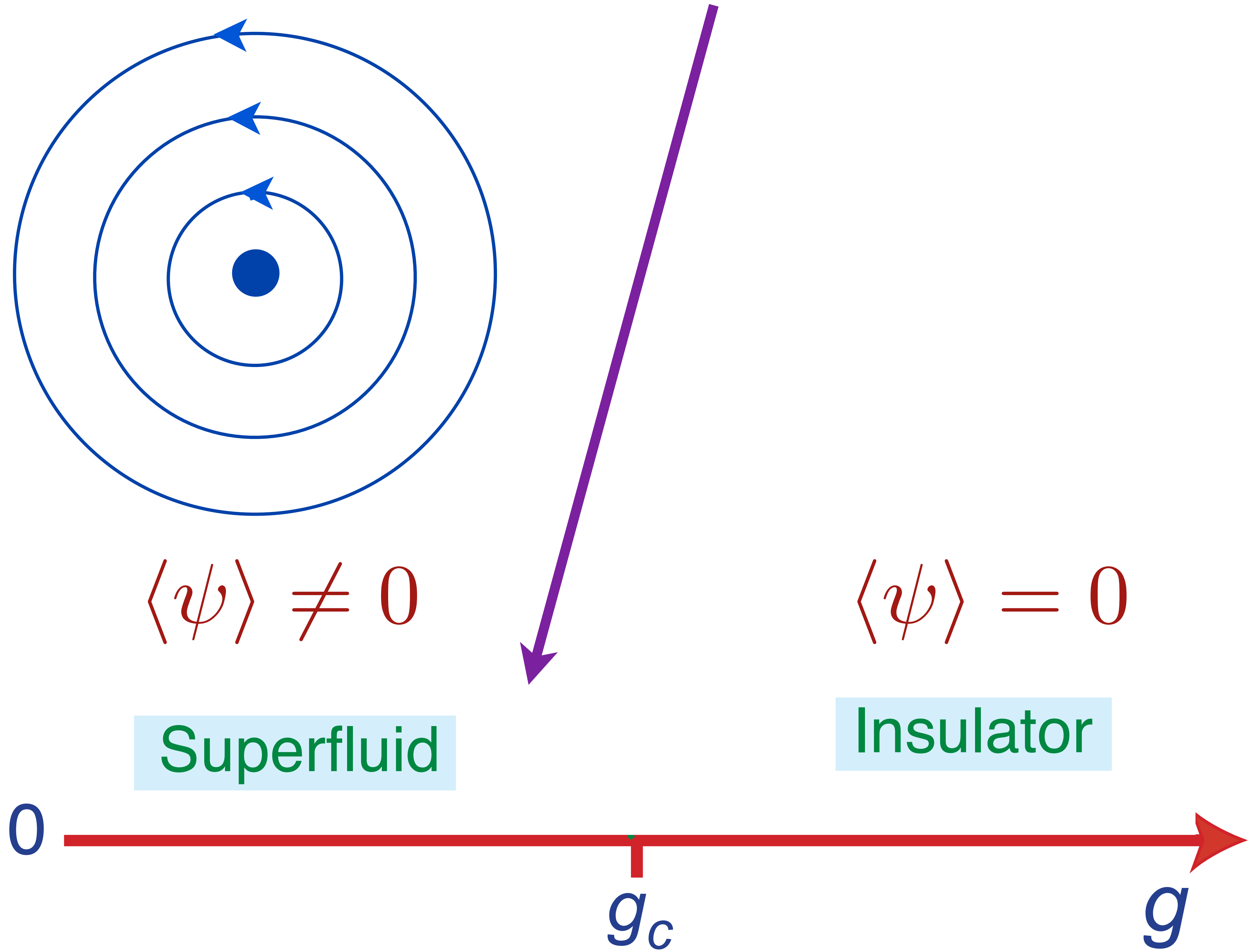
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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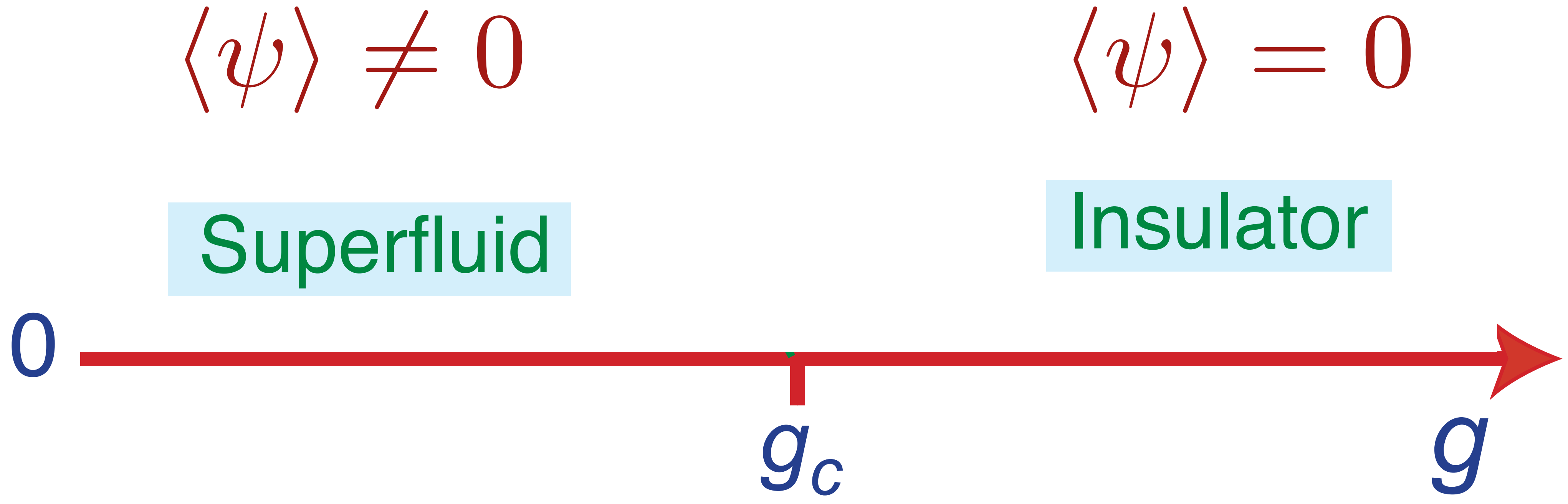
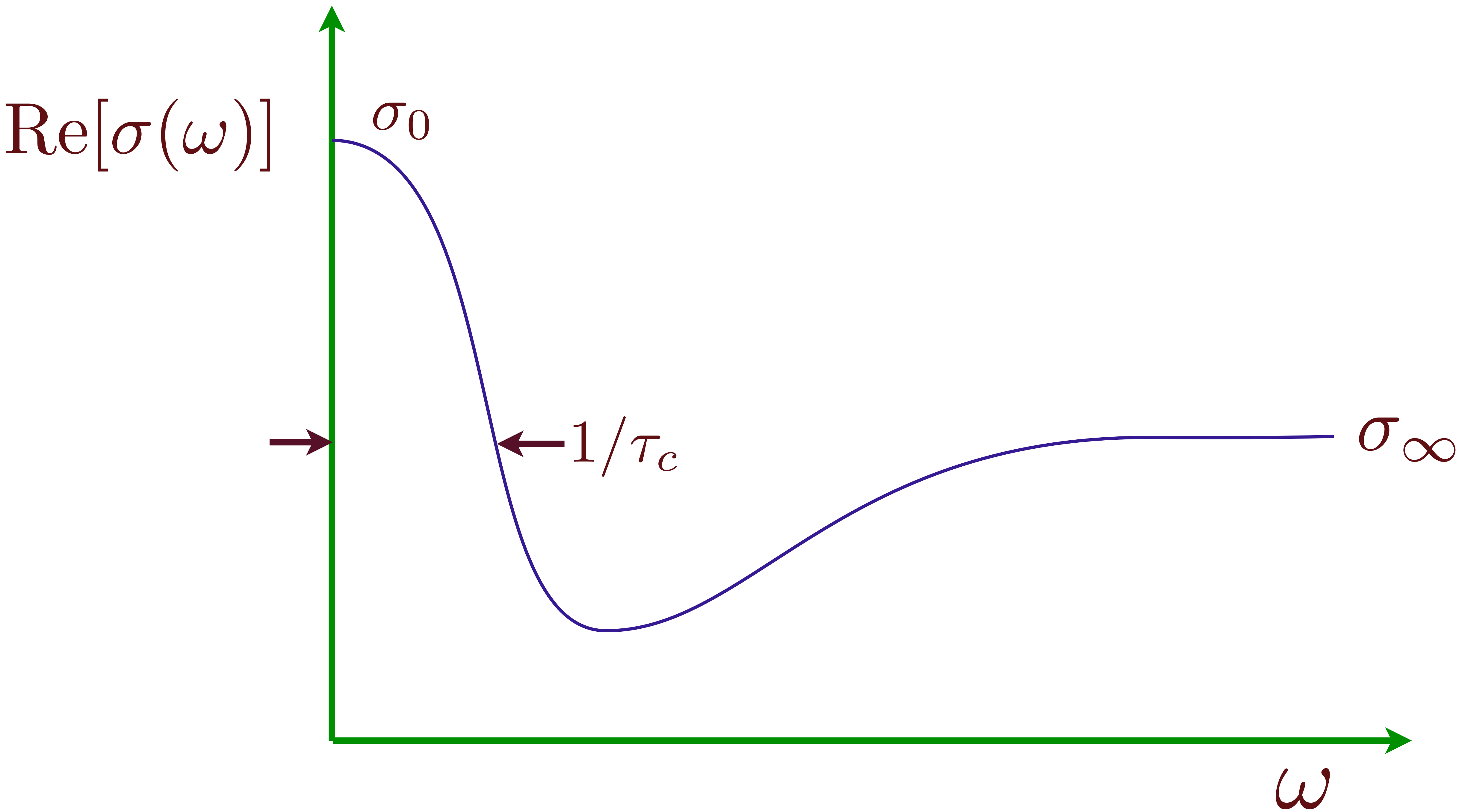
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) "dual" CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

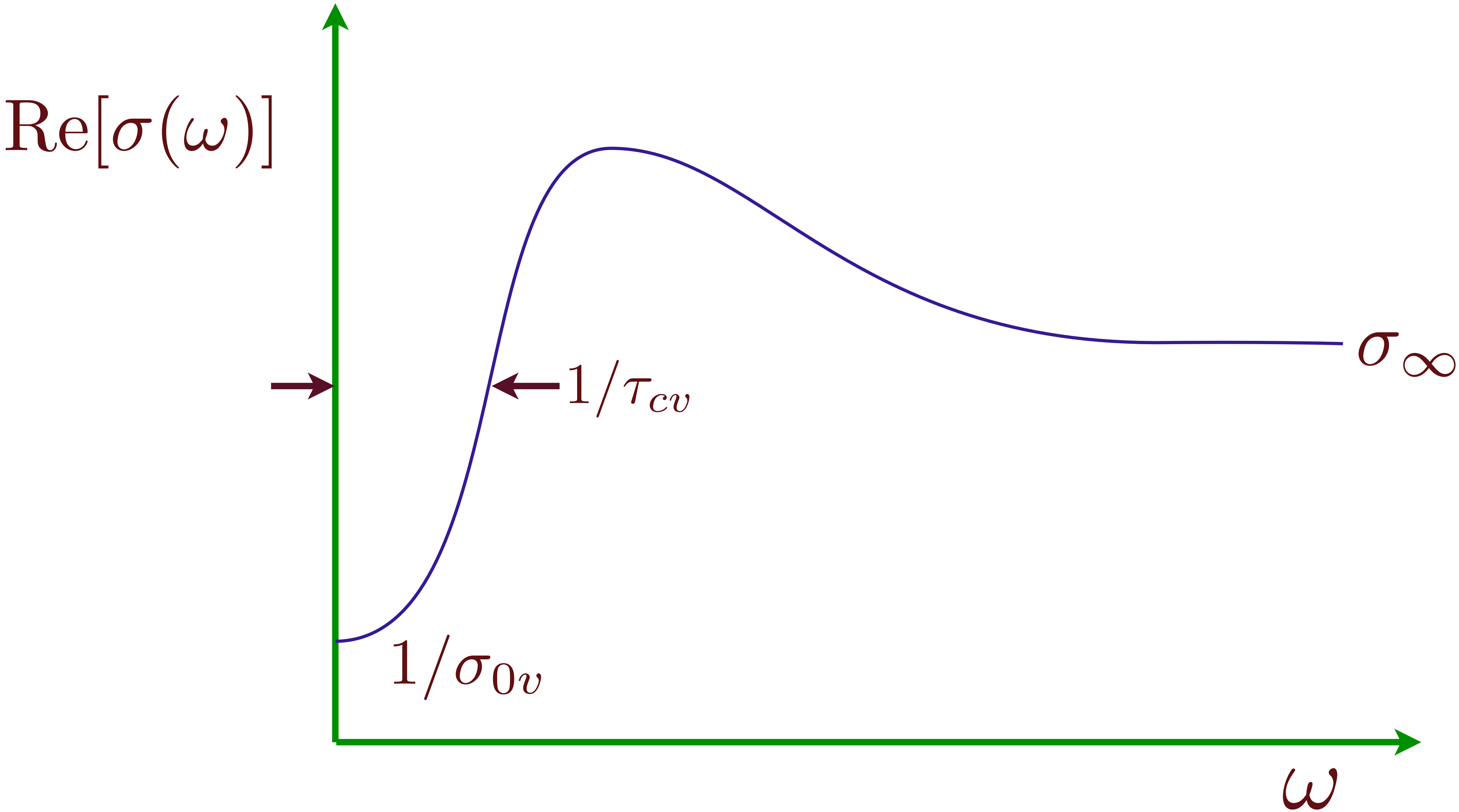
$$\text{Conductivity} = \text{Resistivity of vortices}$$



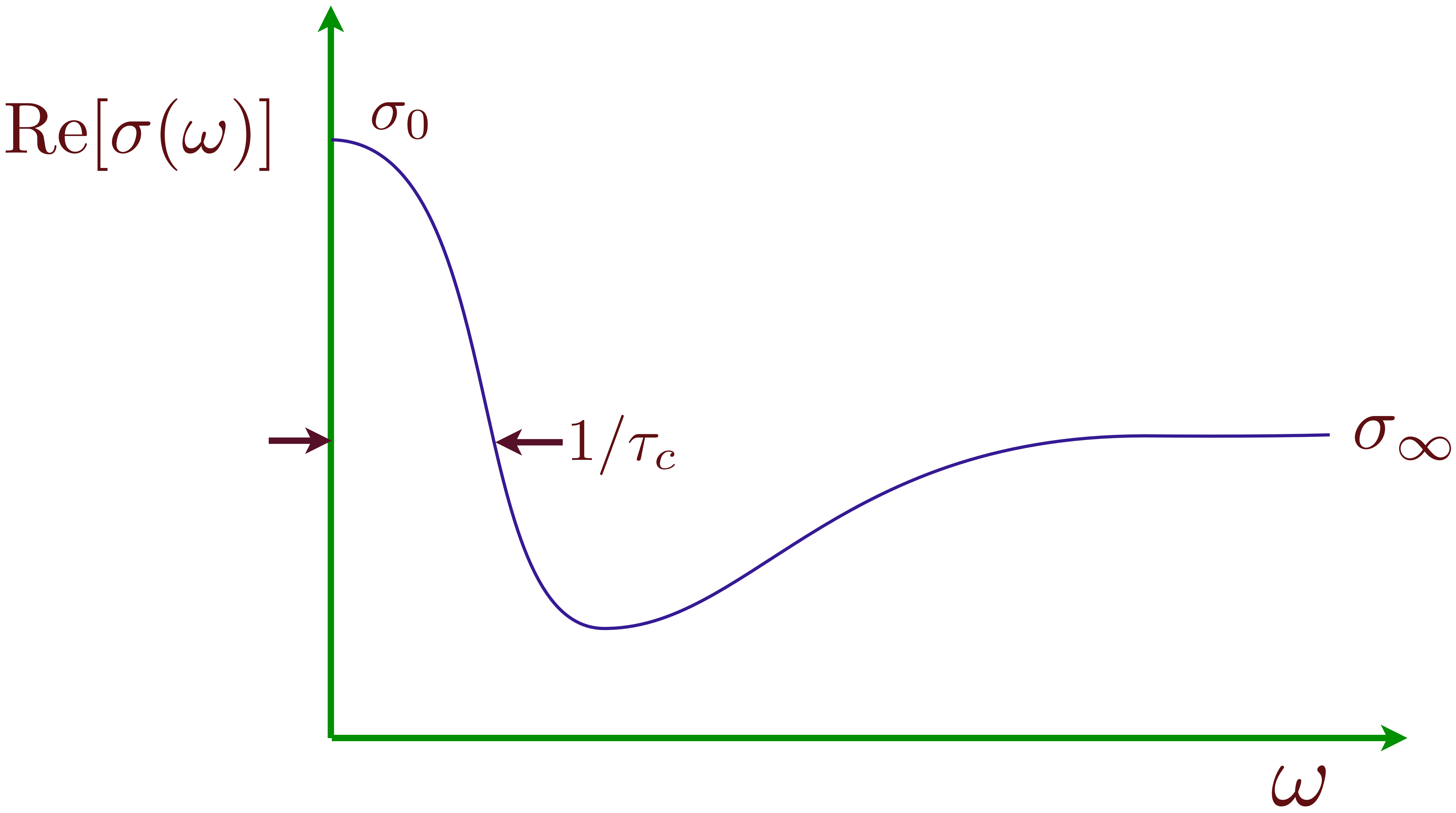
Boltzmann theory of bosons



Boltzmann theory of vortices

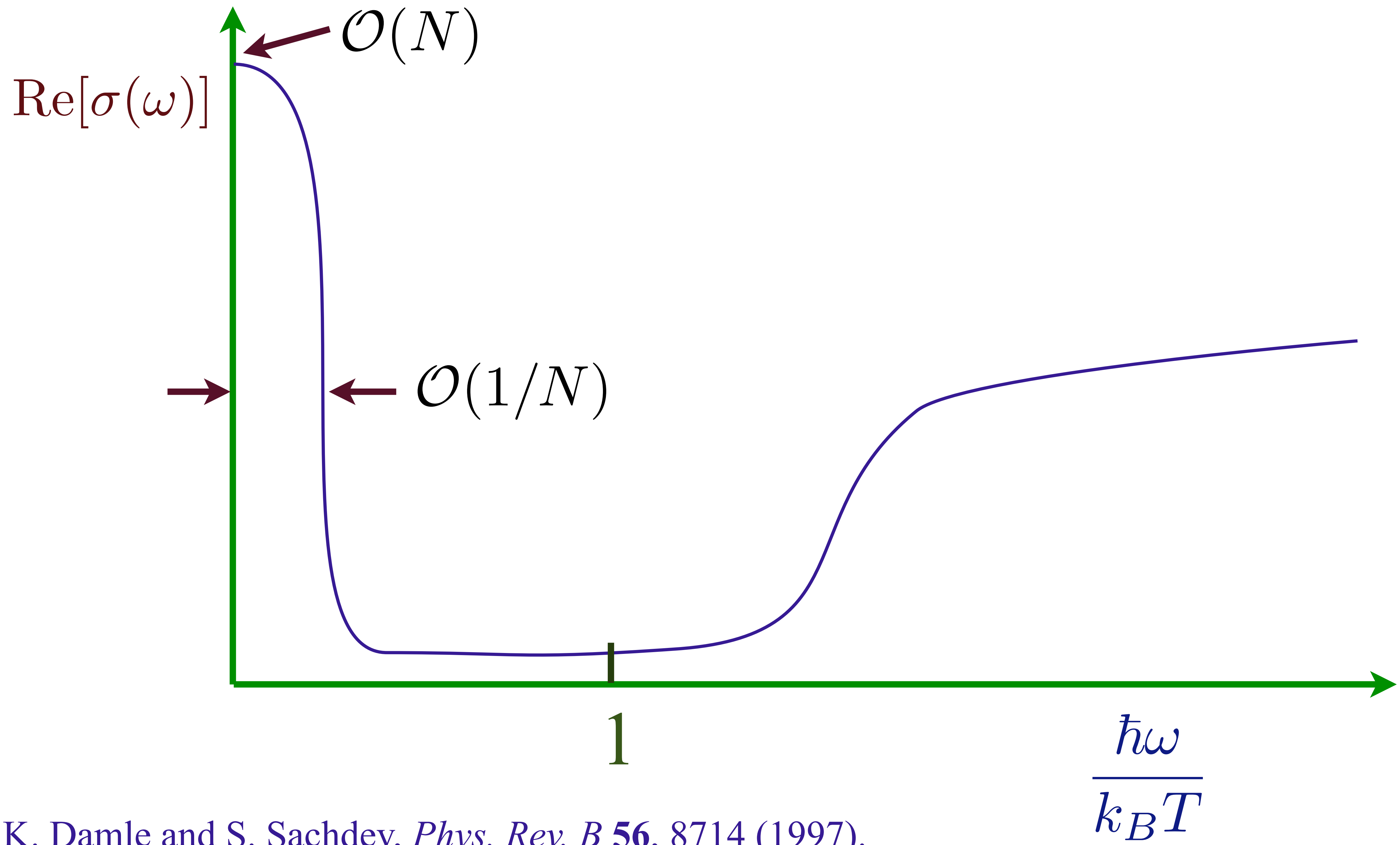


Boltzmann theory of bosons



Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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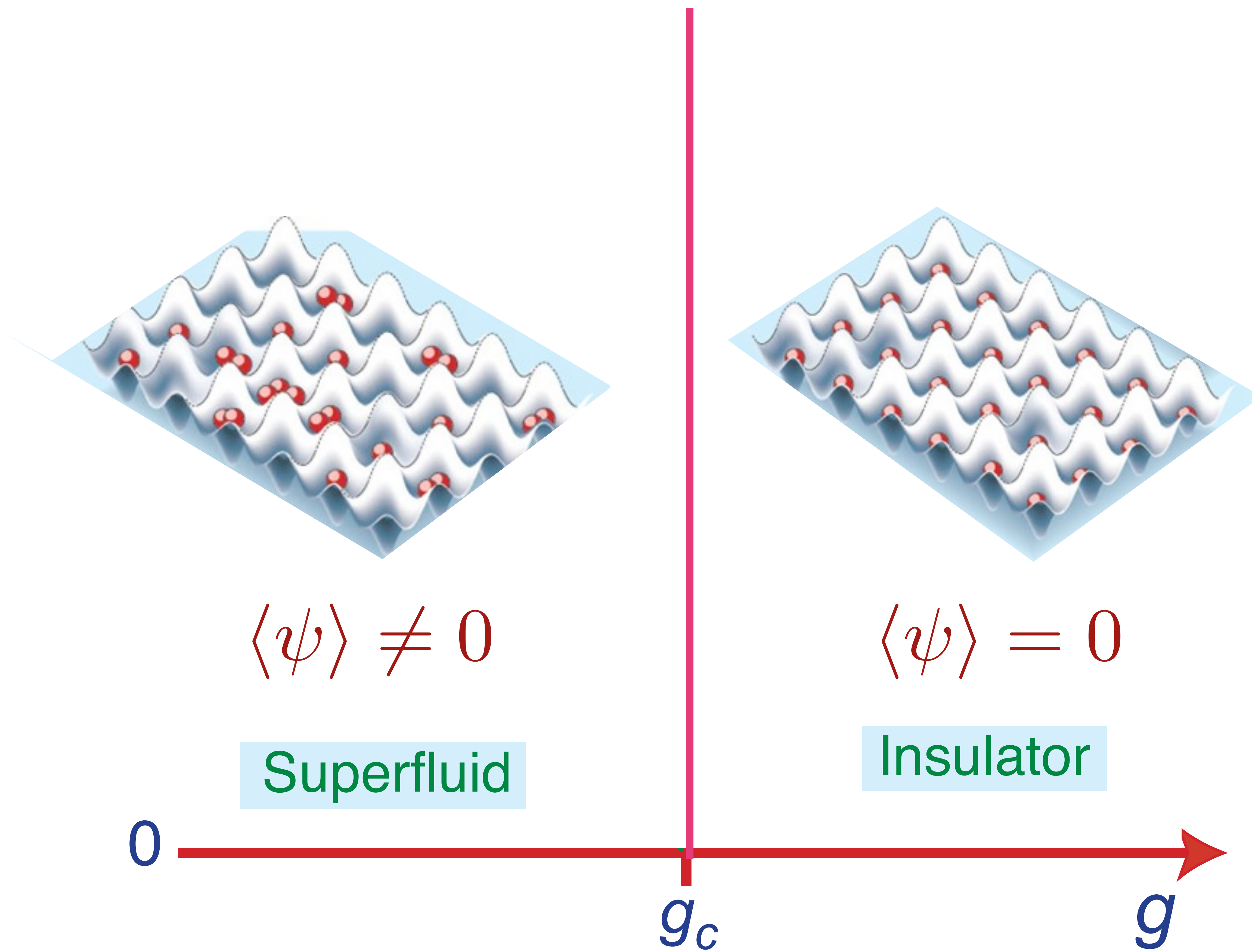
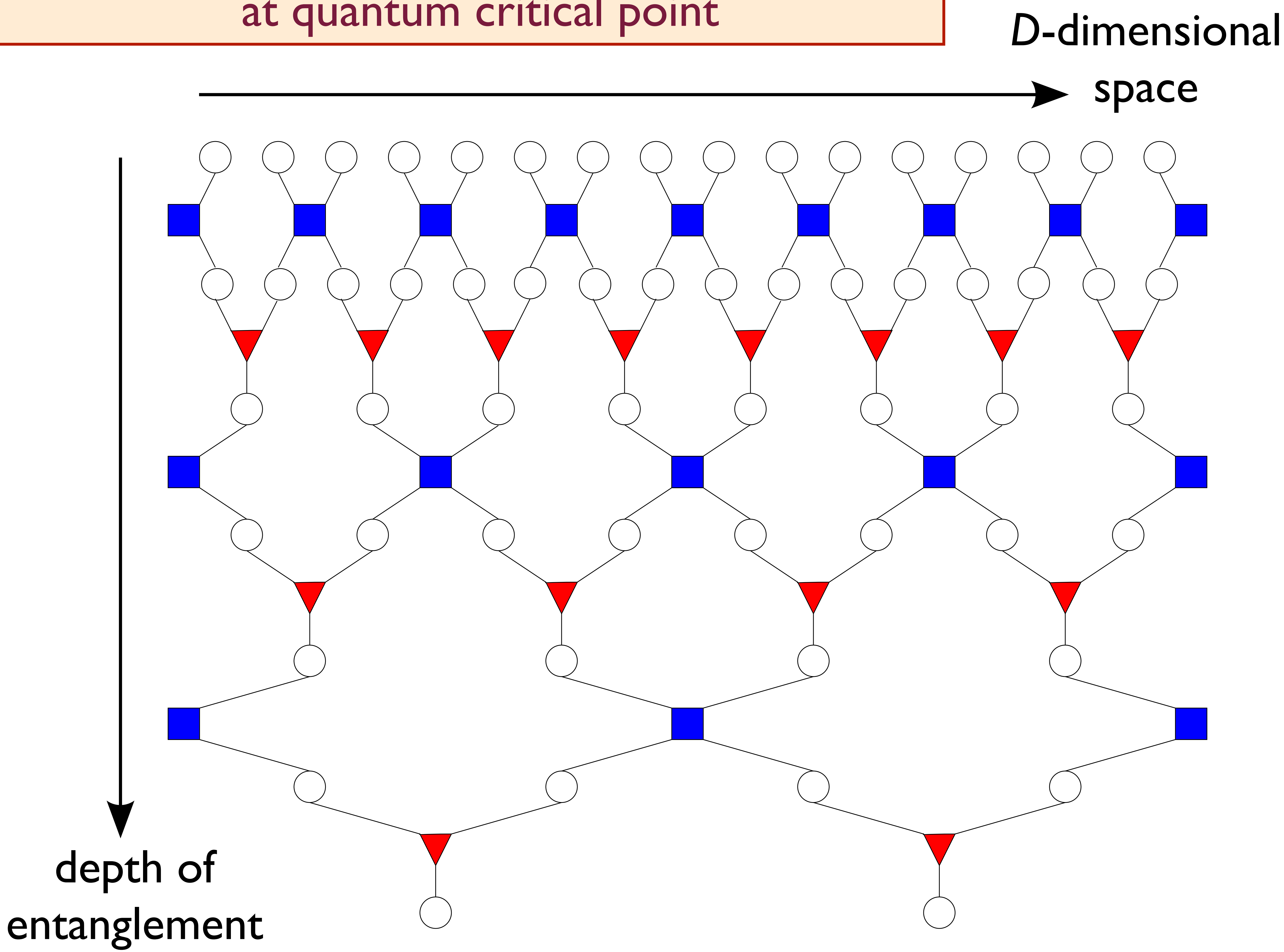

$\langle \psi \rangle \neq 0$
Superfluid
$\langle \psi \rangle = 0$
Insulator
0
g_c
g

Characteristics of quantum critical point

- Long-range entanglement

Tensor network representation of entanglement at quantum critical point



Characteristics of quantum critical point

- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

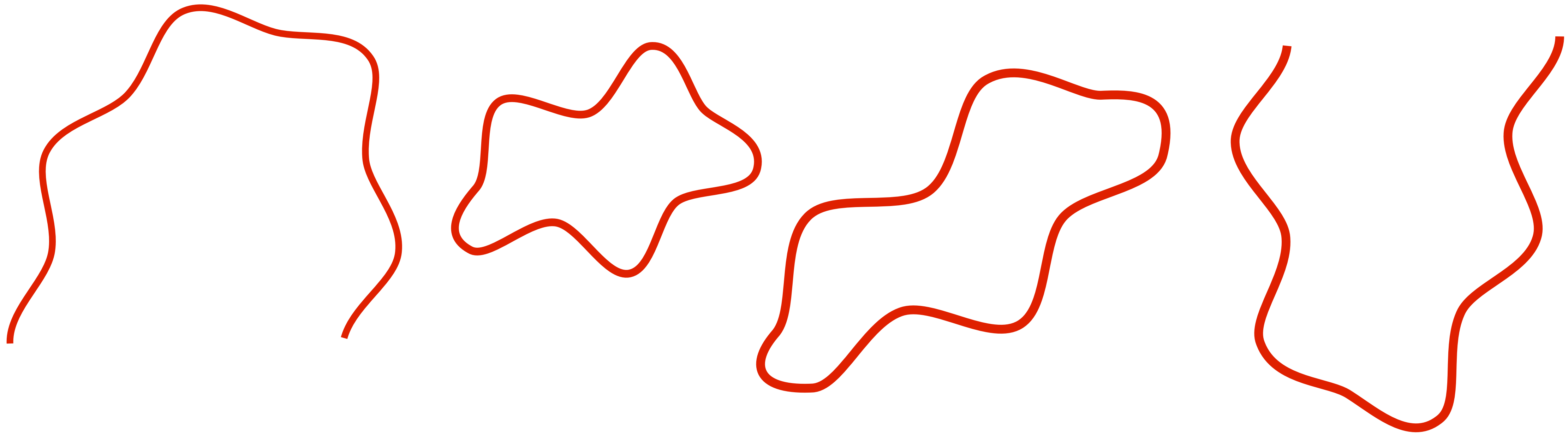
Characteristics of quantum critical point

- Long-range entanglement

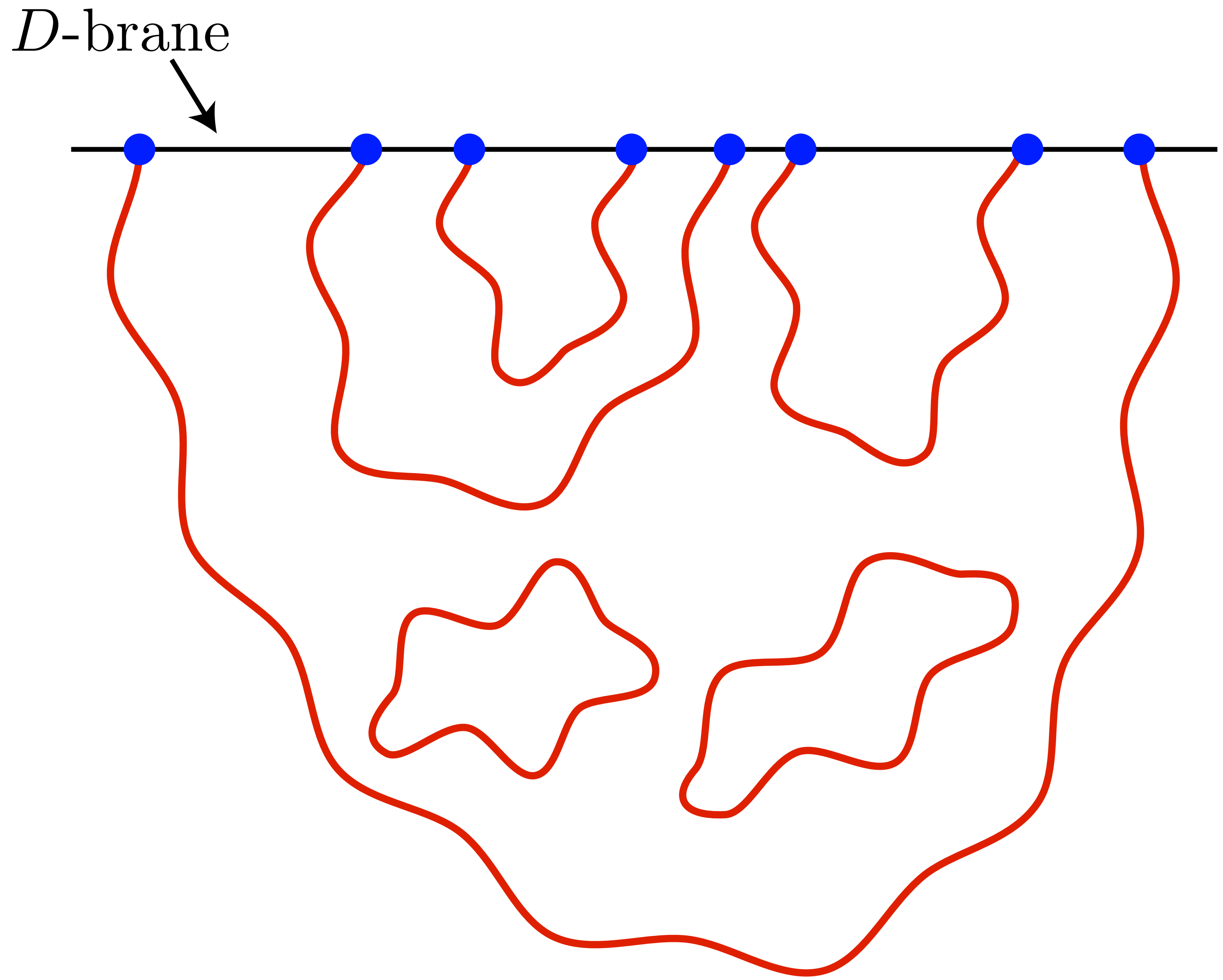
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

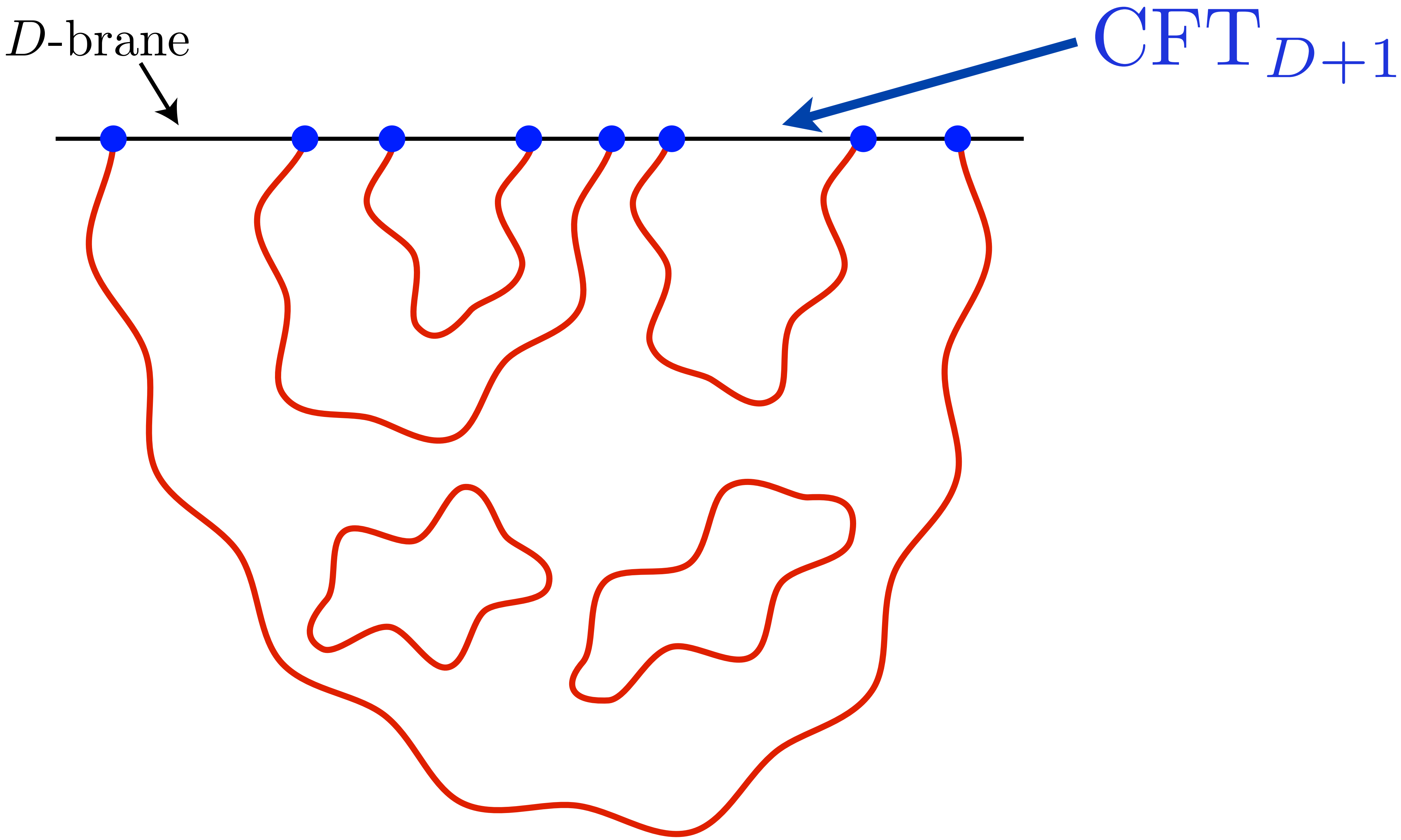
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...

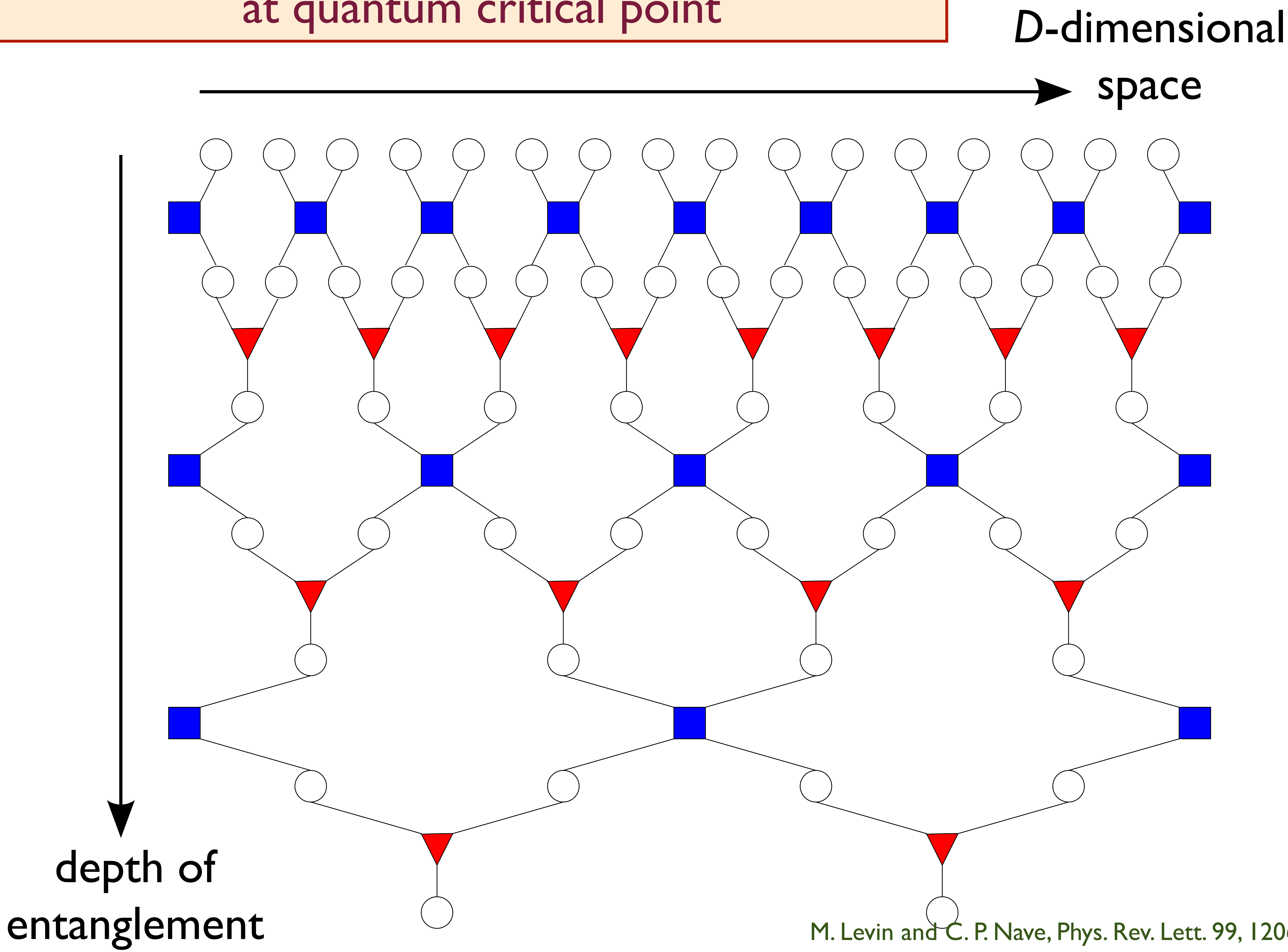


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.

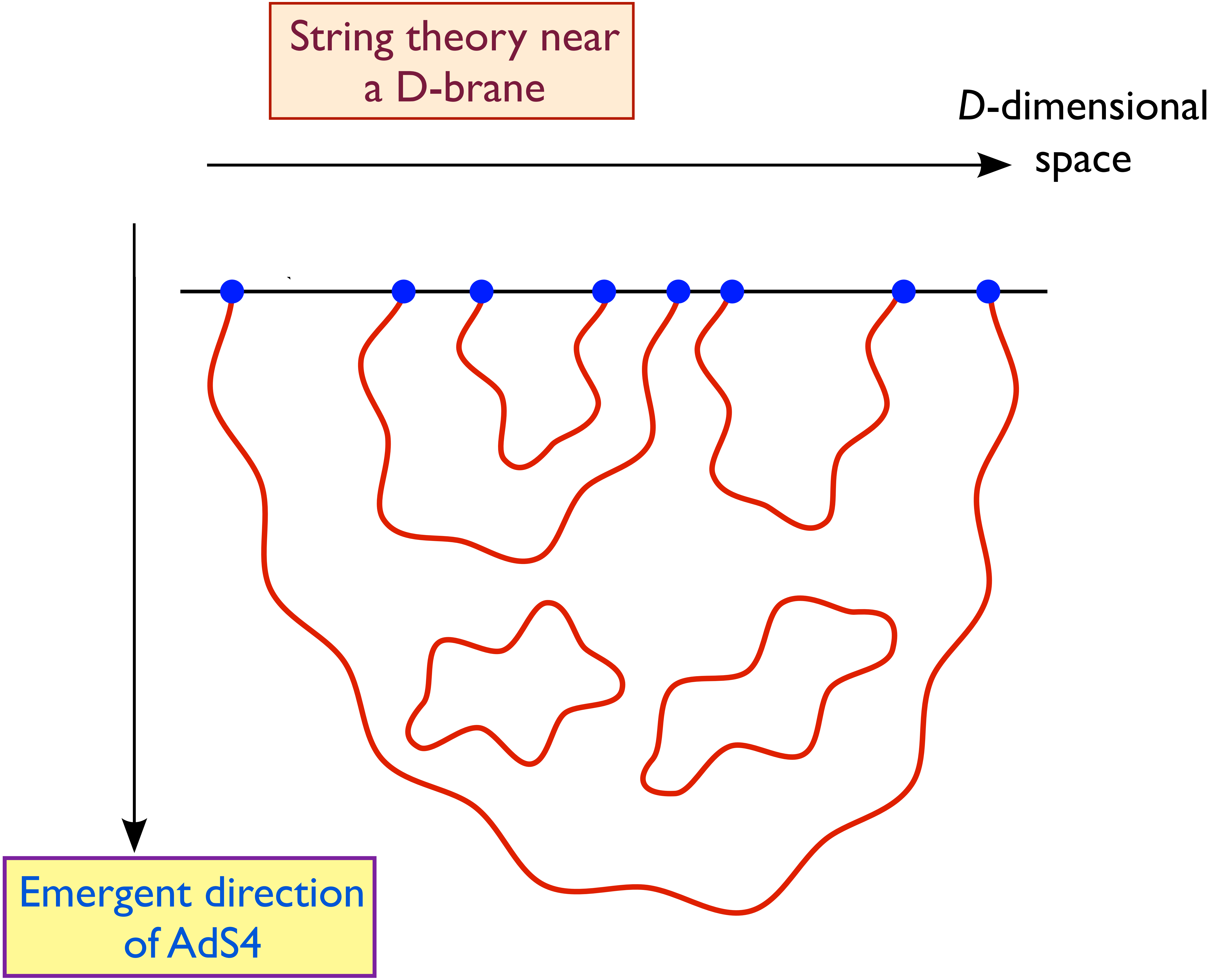


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.
- In $D = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

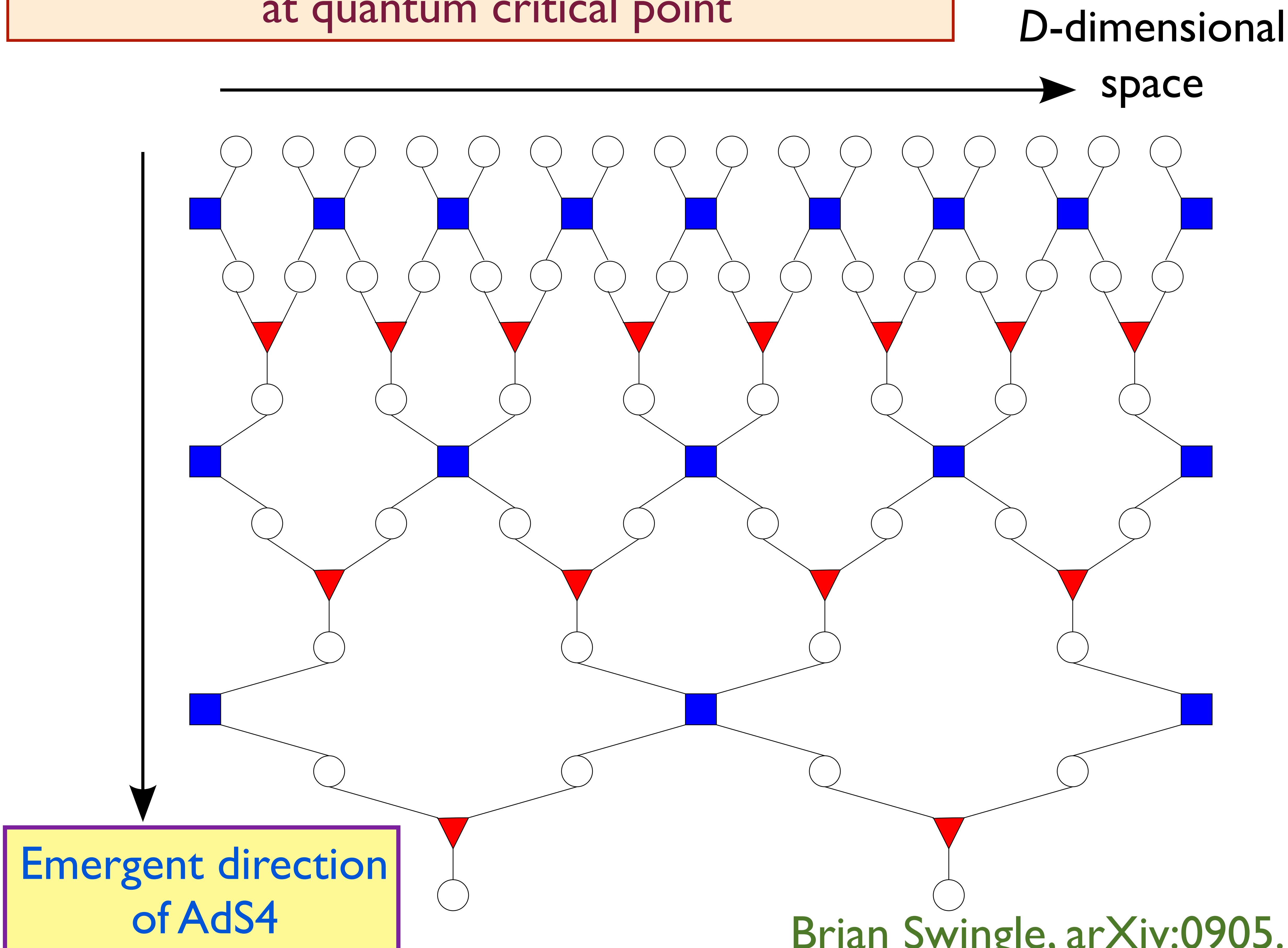
Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

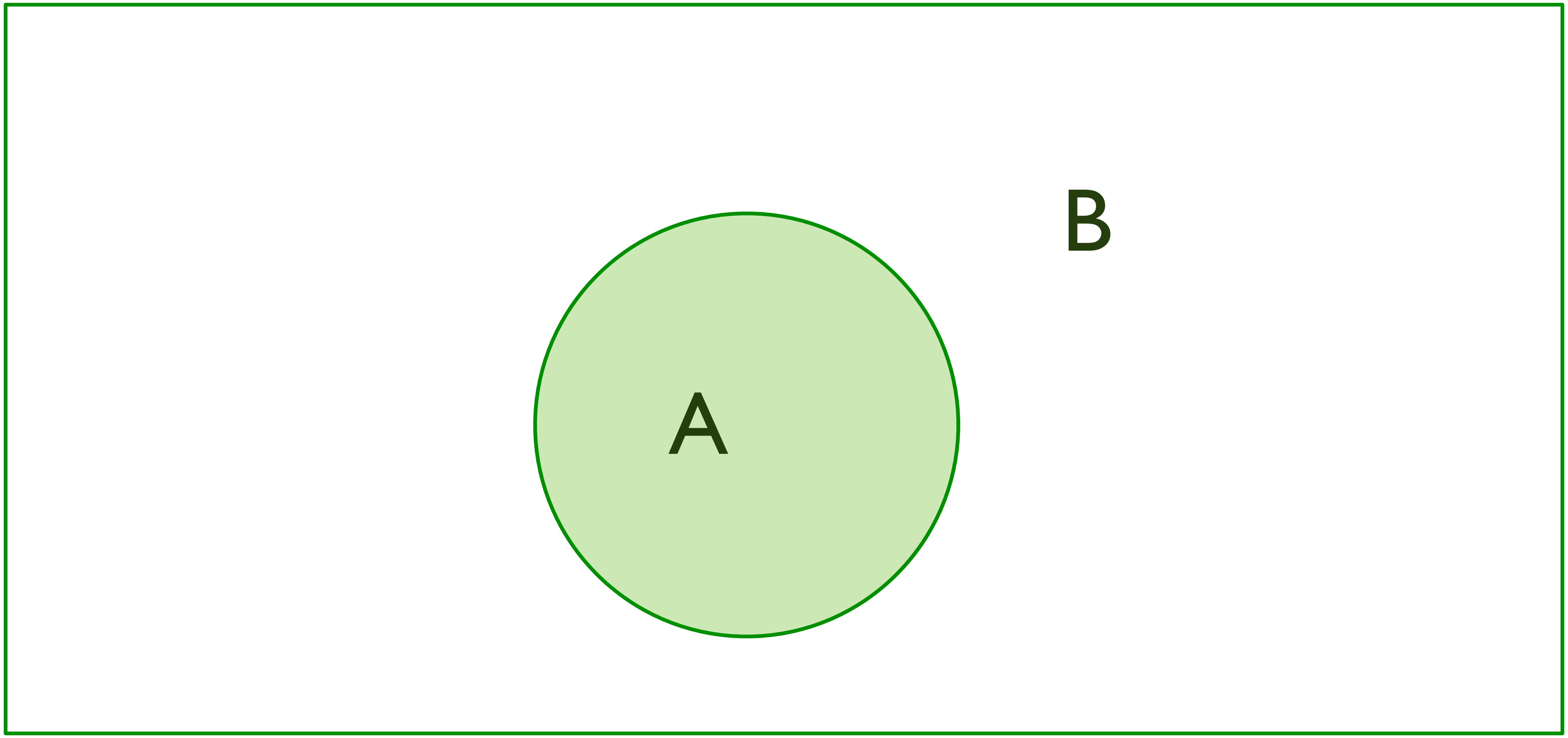

String theory near a D-brane
D-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

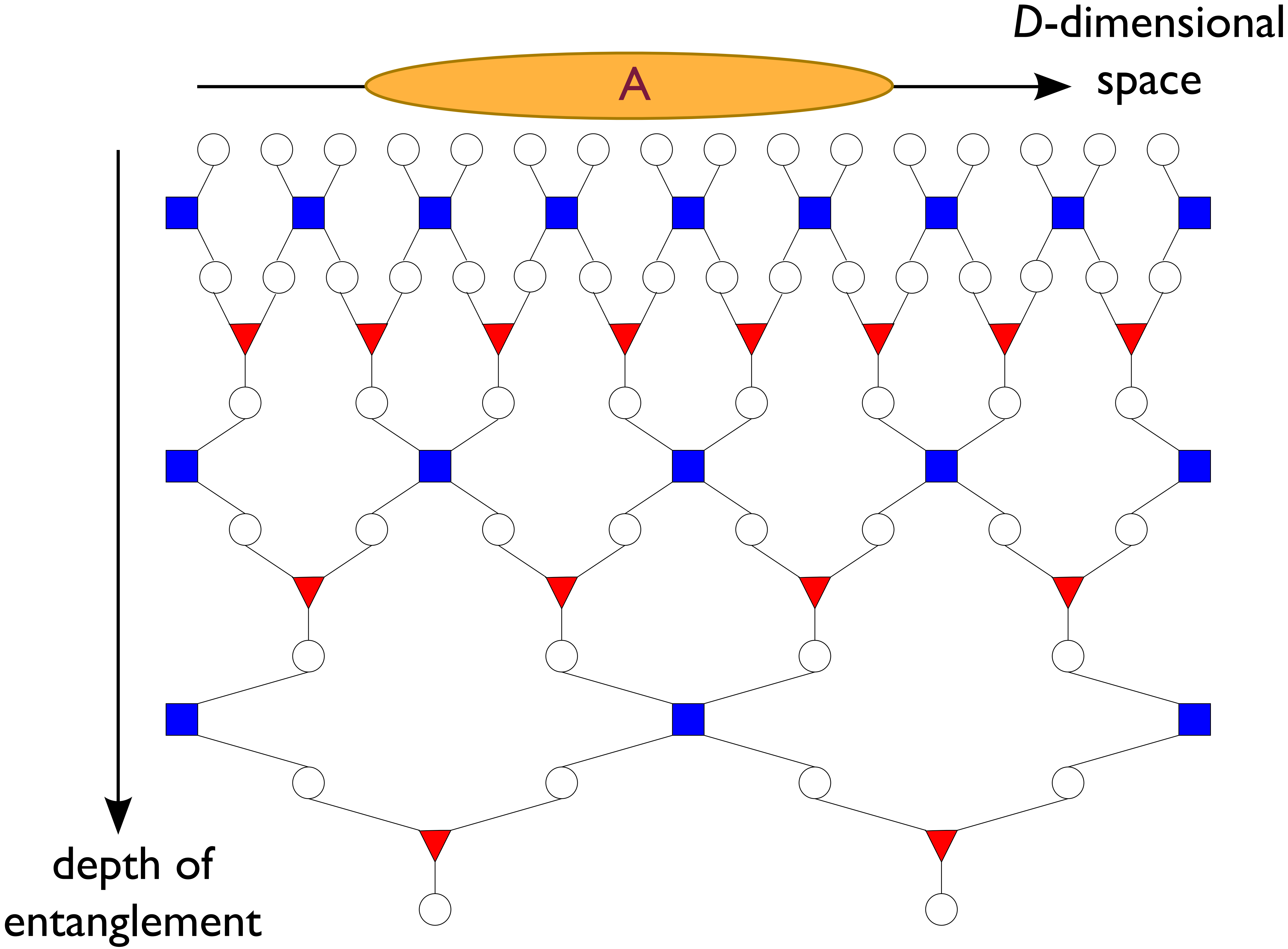
Entanglement entropy



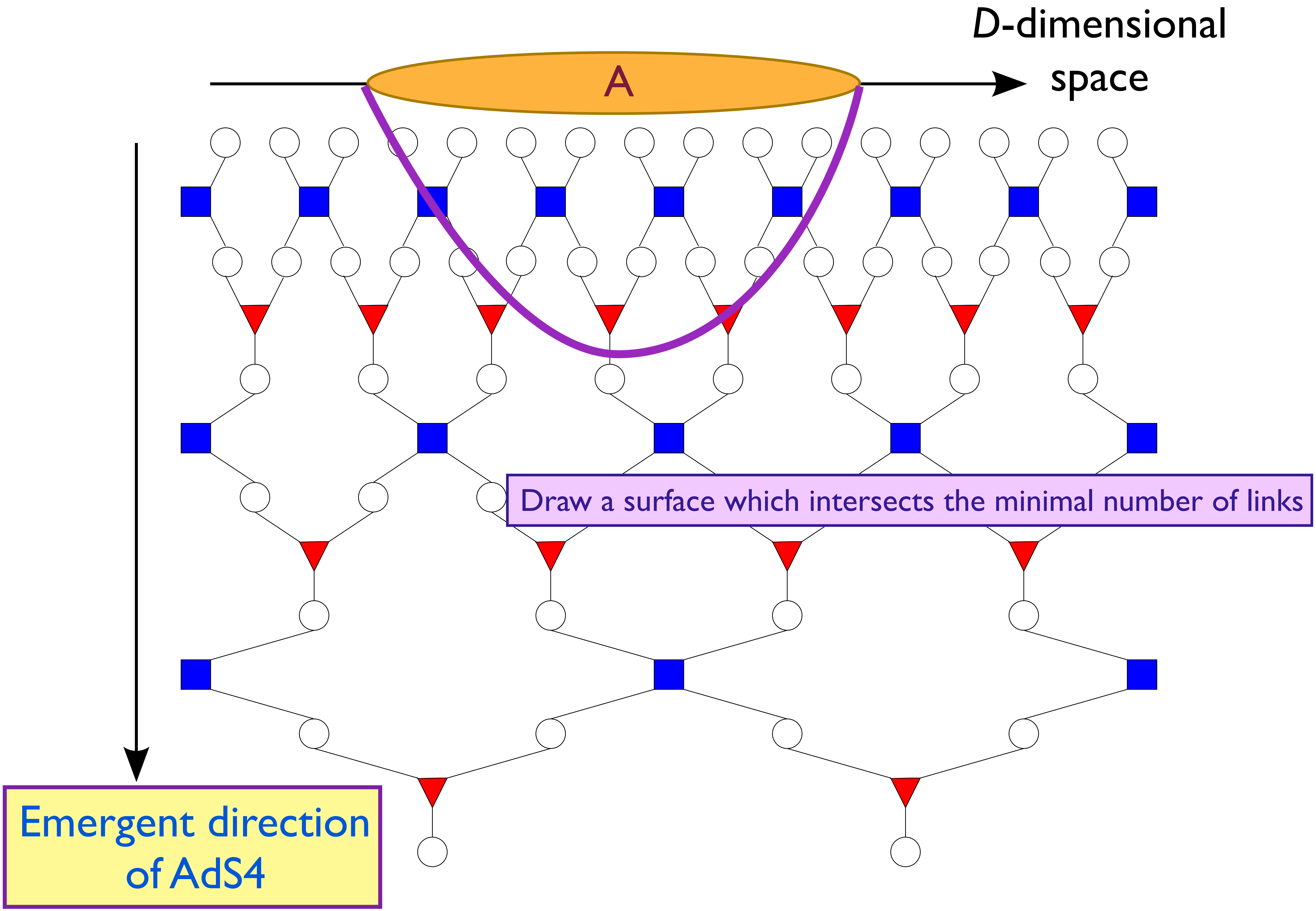
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy



Entanglement entropy



Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

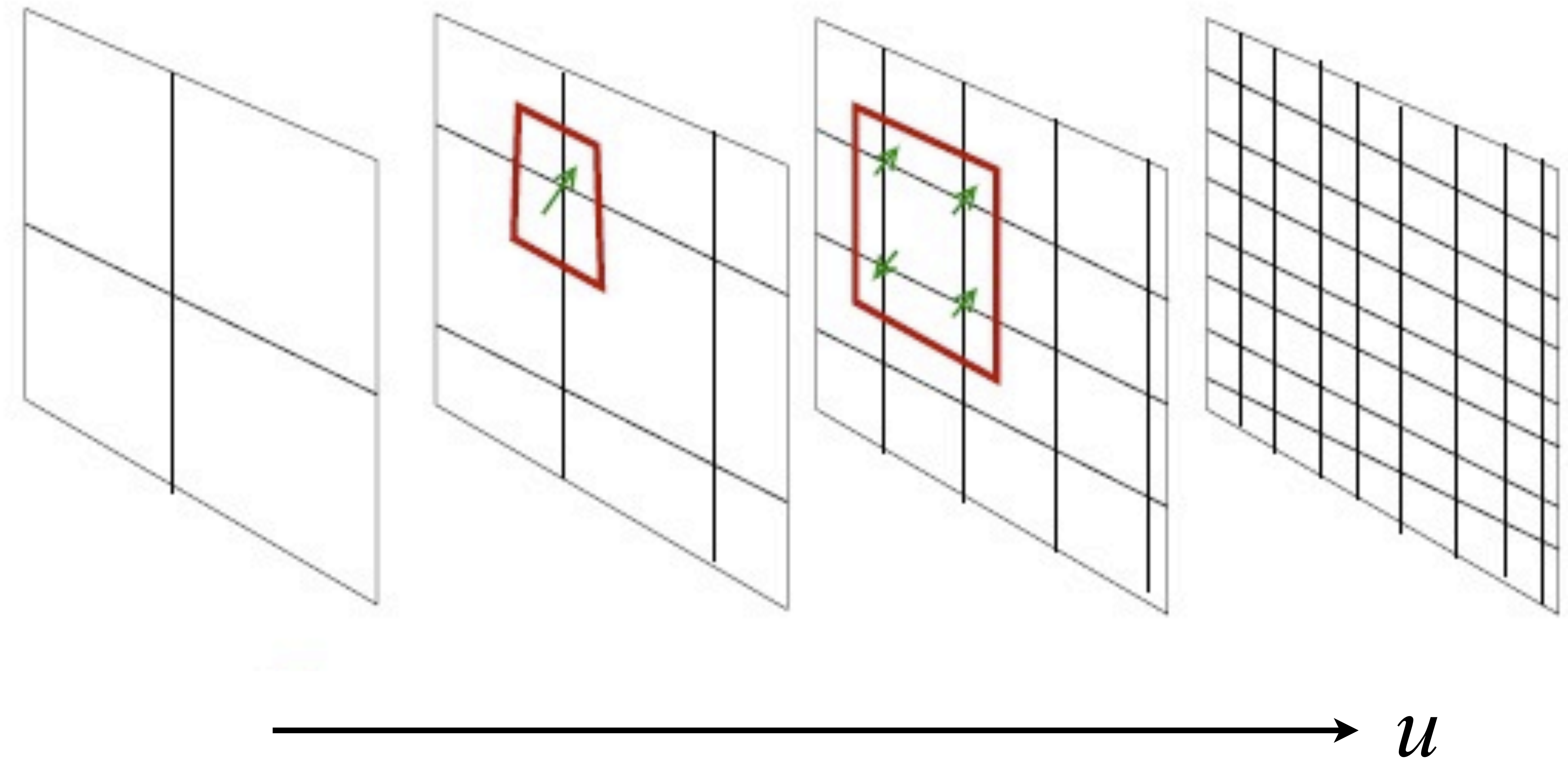
This can be seen both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317

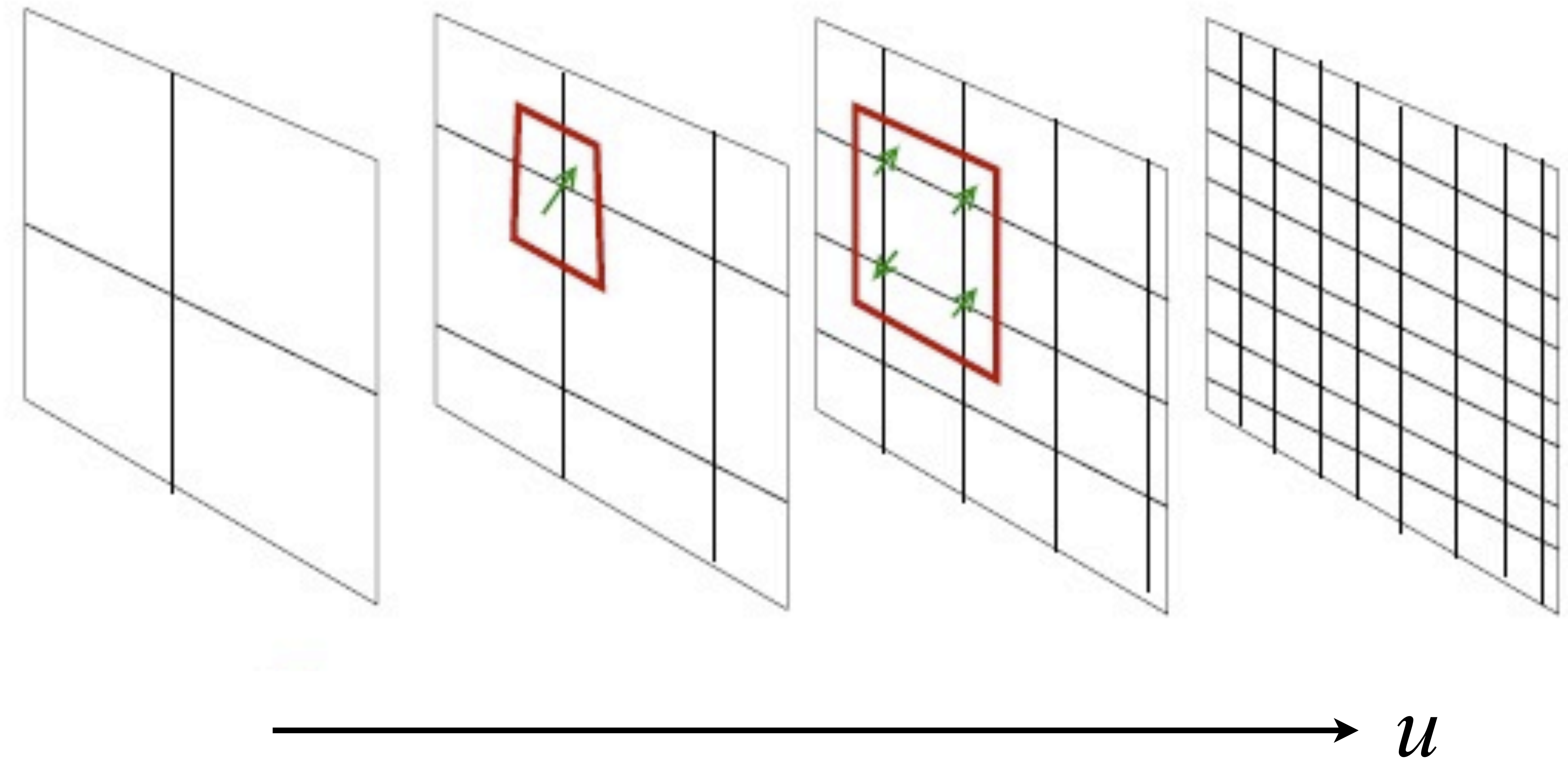
Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.



u



Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

At the RG fixed point, $\beta(g) = 0$, the $(d+1)$-dimensional "relativistic" field theory is invariant under the scale transformation $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad u \to u/\zeta$$

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$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad u \to u/\zeta$$

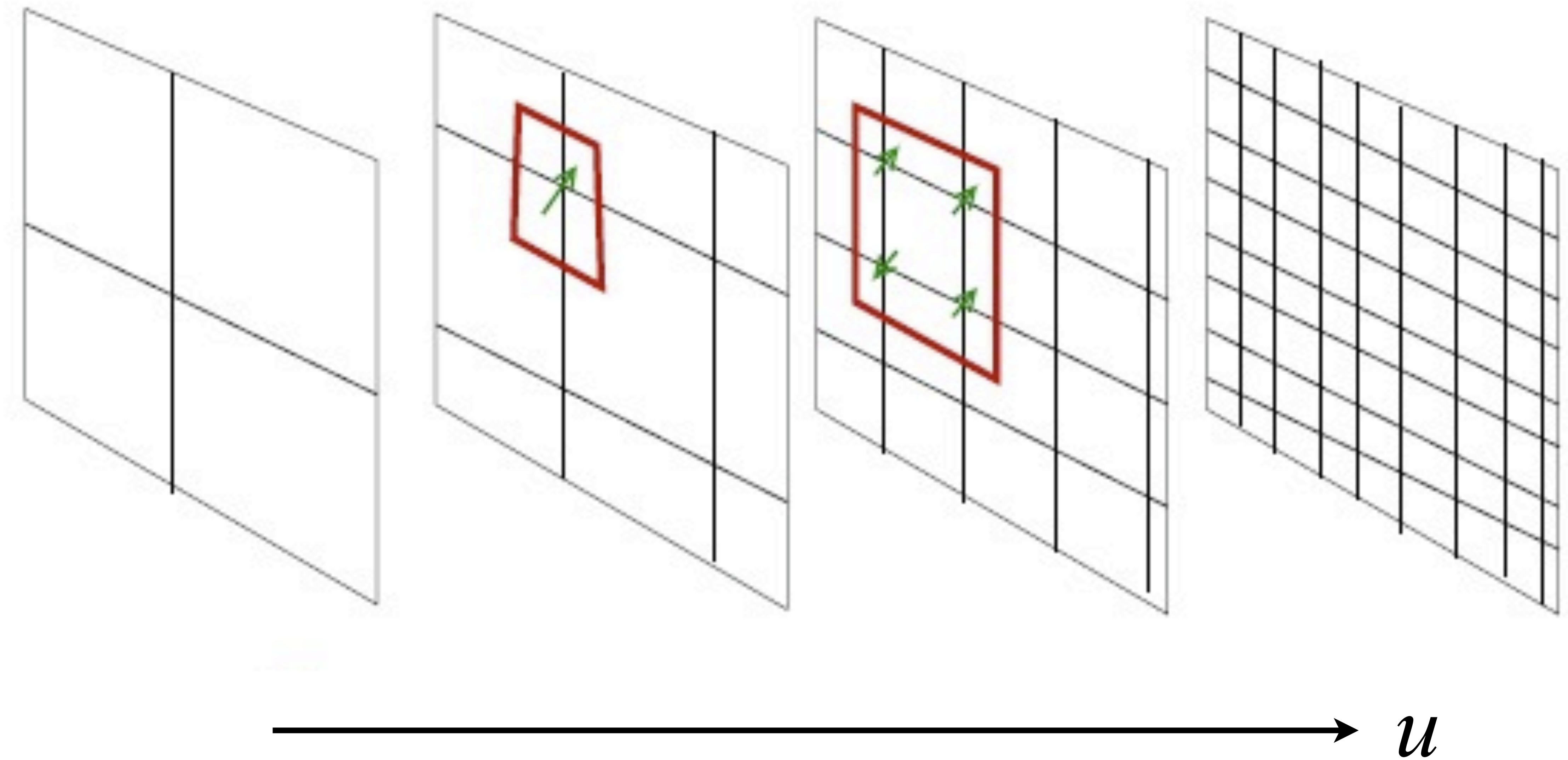
This is assumed to be an invariance of the *metric* of the theory in $d+2$ dimensions. The unique solution is

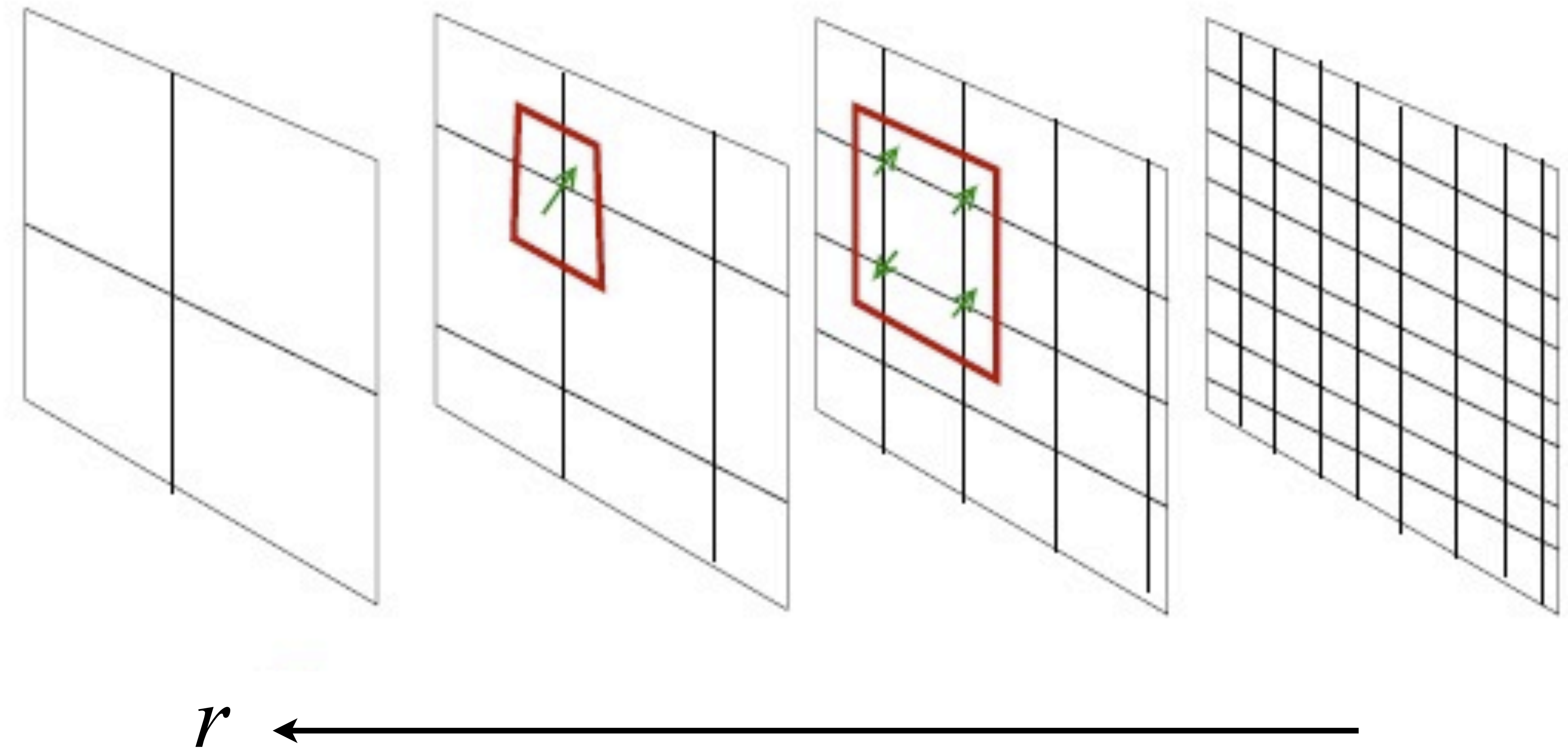
$$ds^2 = \left(\frac{u}{L}\right)^2 \left(-dt^2 + dx_i^2\right) + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $r = L^2/u$

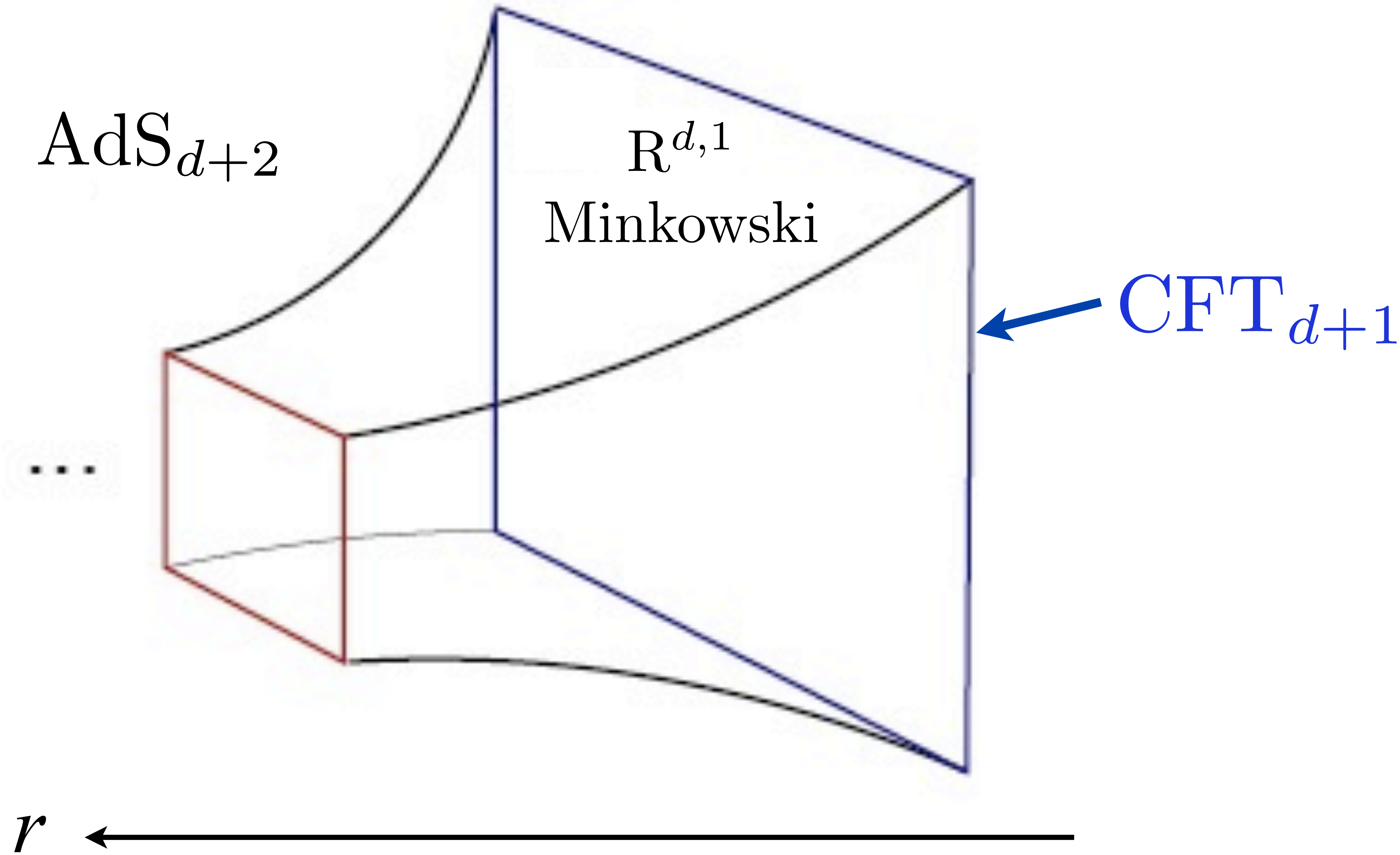
$$ds^2 = L^2 \frac{\left(-dt^2 + dx_i^2 + dr^2\right)}{r^2}.$$

This is the space AdS$_{d+2}$, and L is the AdS radius.


u

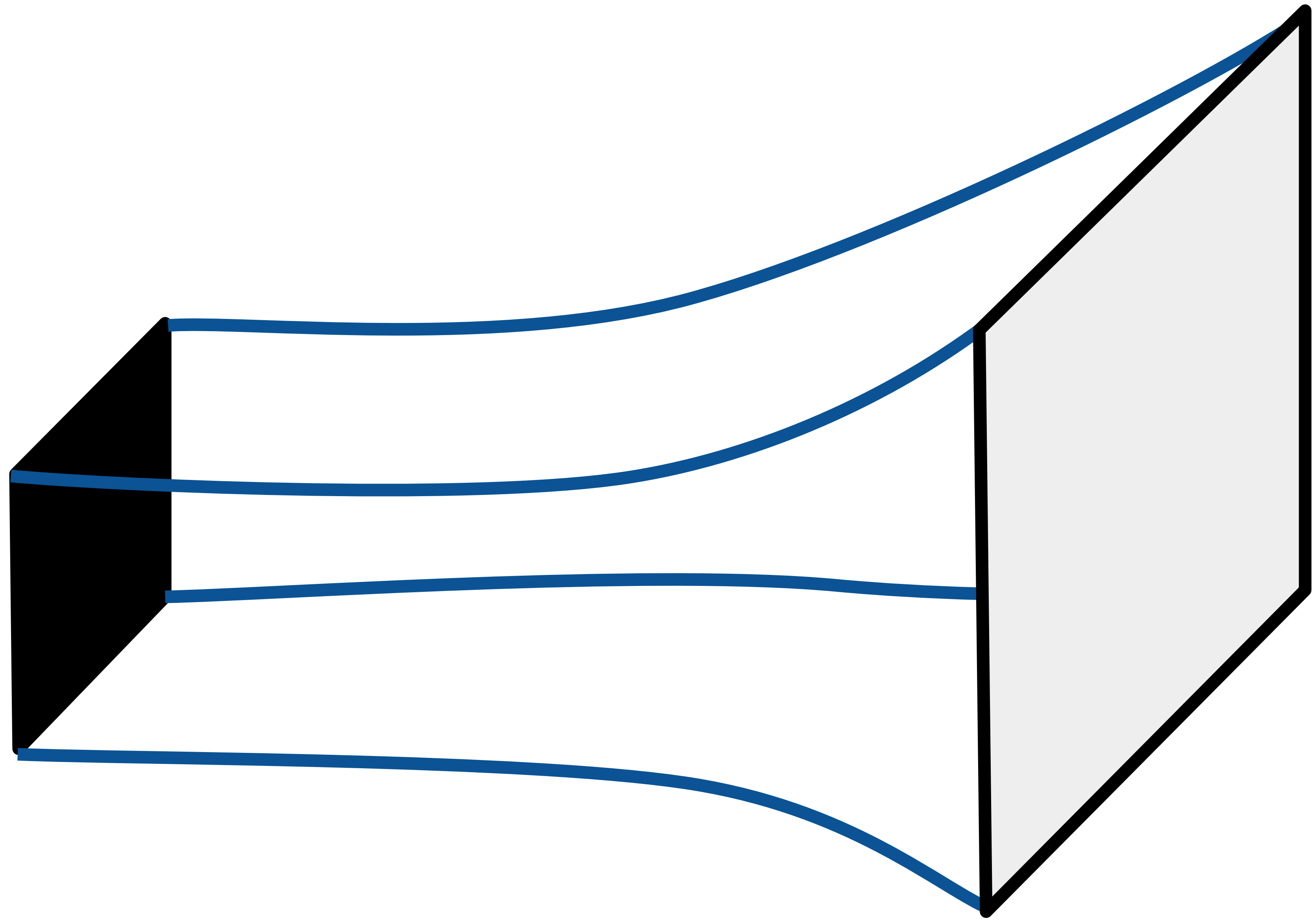


J. McGreevy, arXiv0909.0518



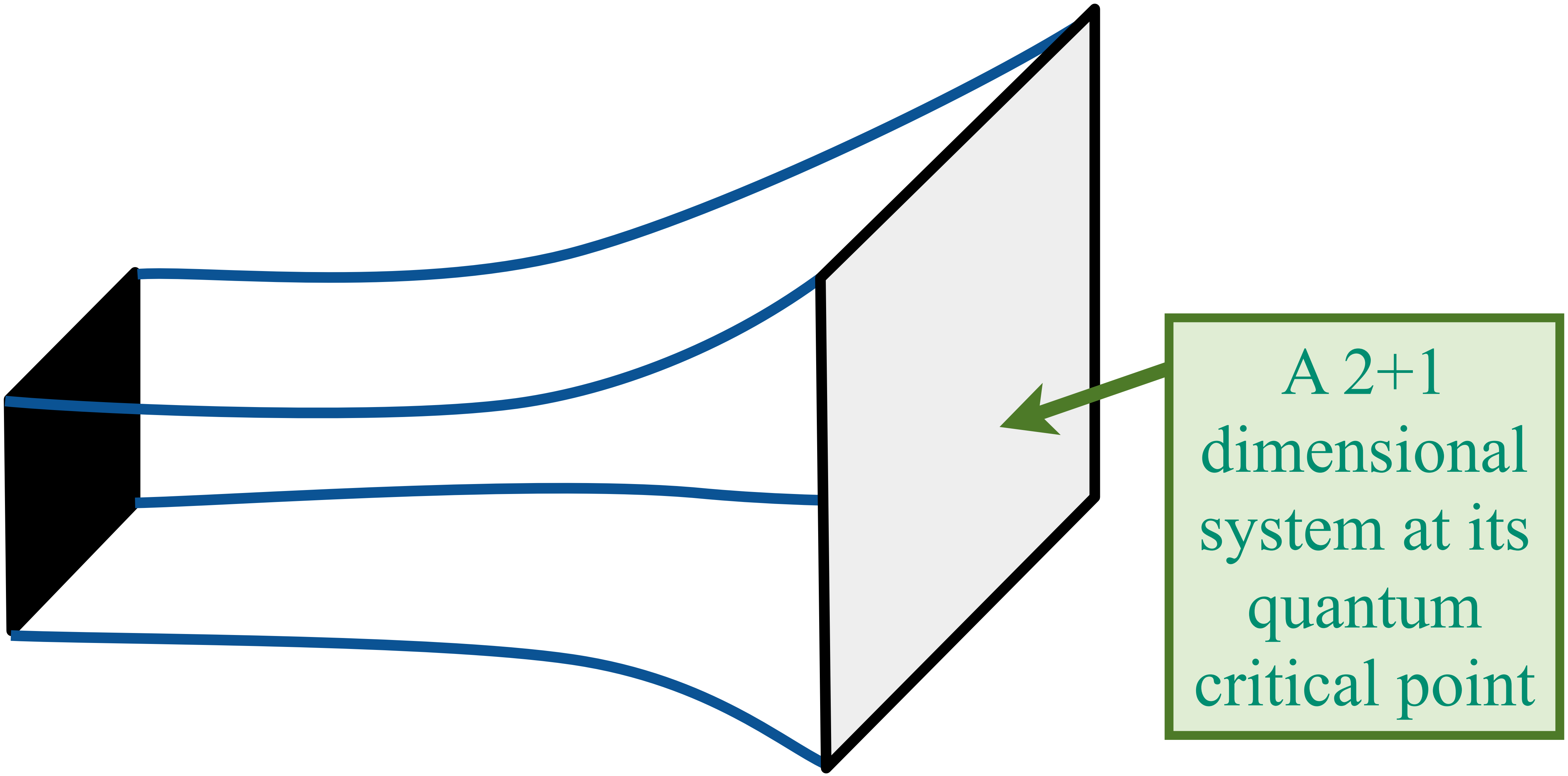
J. McGreevy, arXiv0909.0518

AdS_4-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS_4-Schwarzschild black-brane



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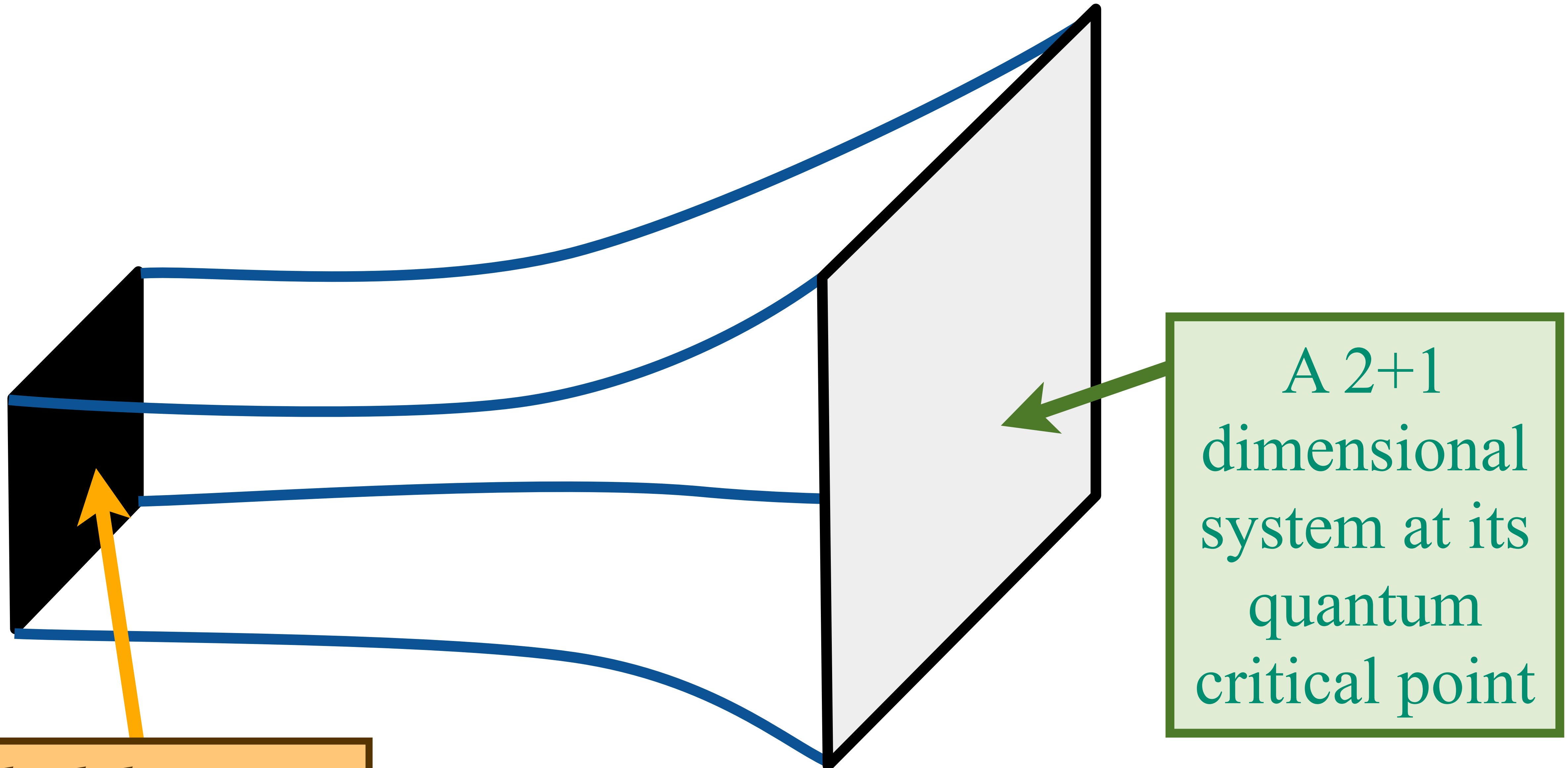
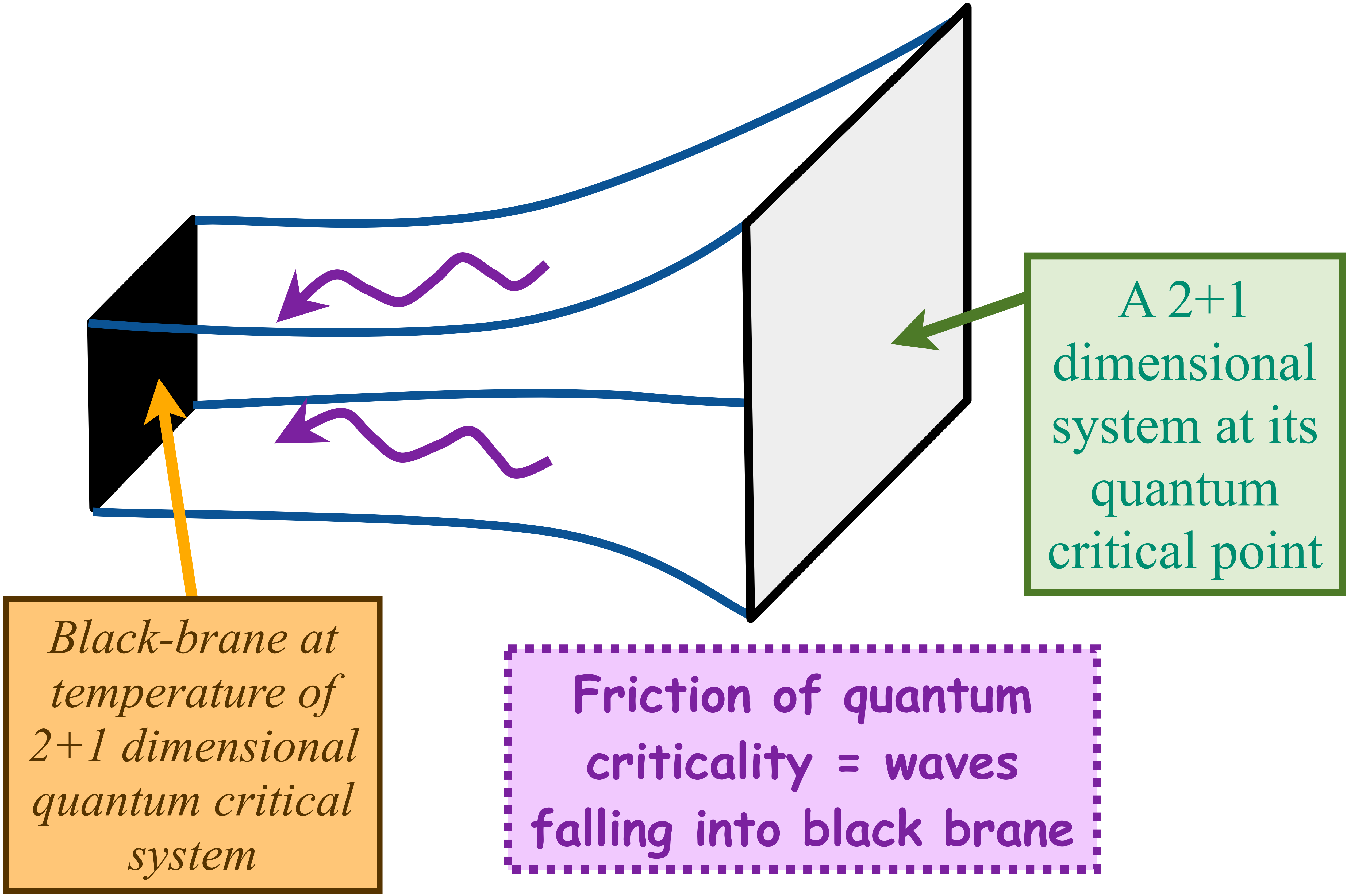


Black-brane at temperature of 2+1 dimensional quantum critical system

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence

AdS_4-Schwarzschild black-brane



AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

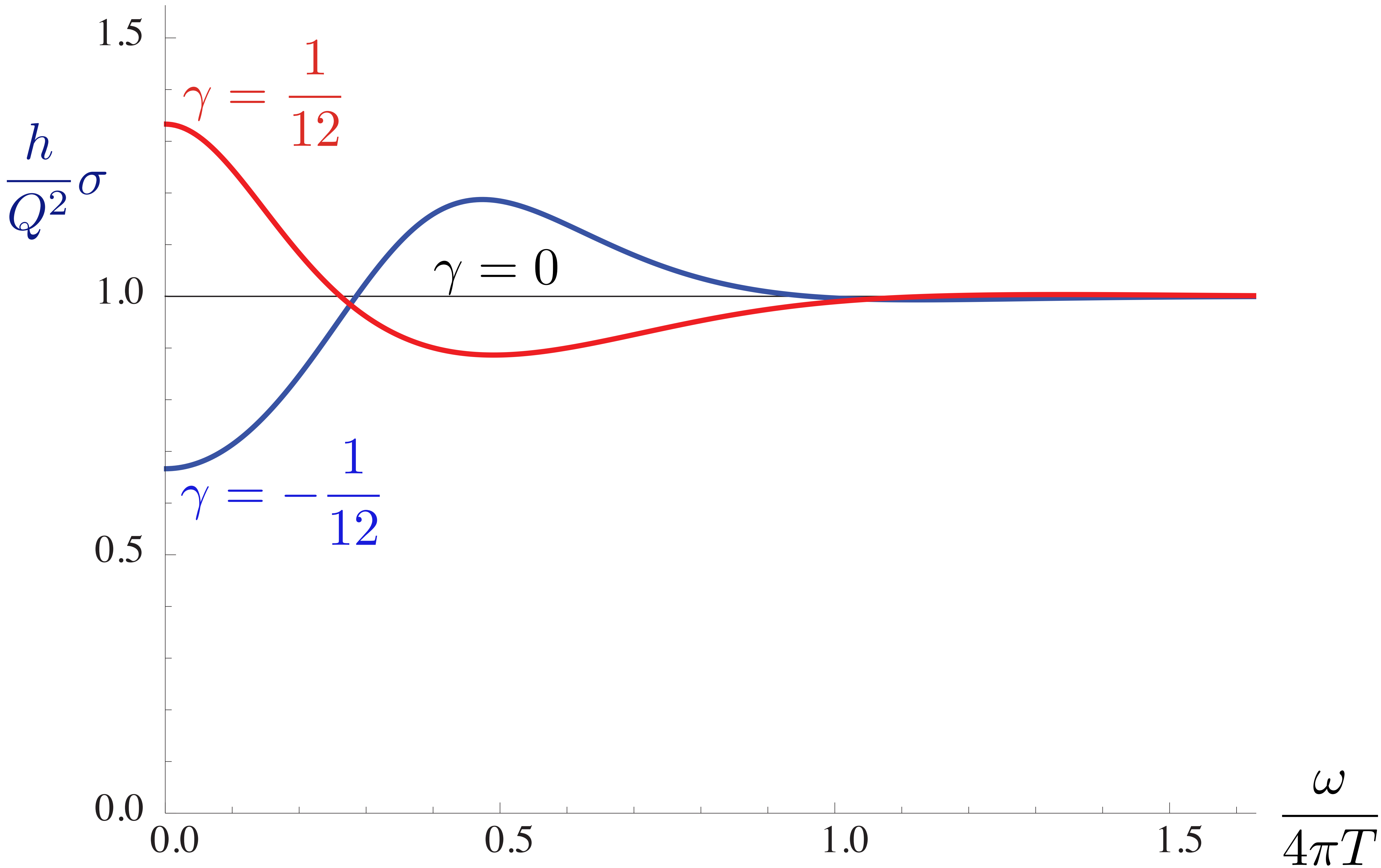
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS$_4$):

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS_4 theory of strongly interacting "perfect fluids"



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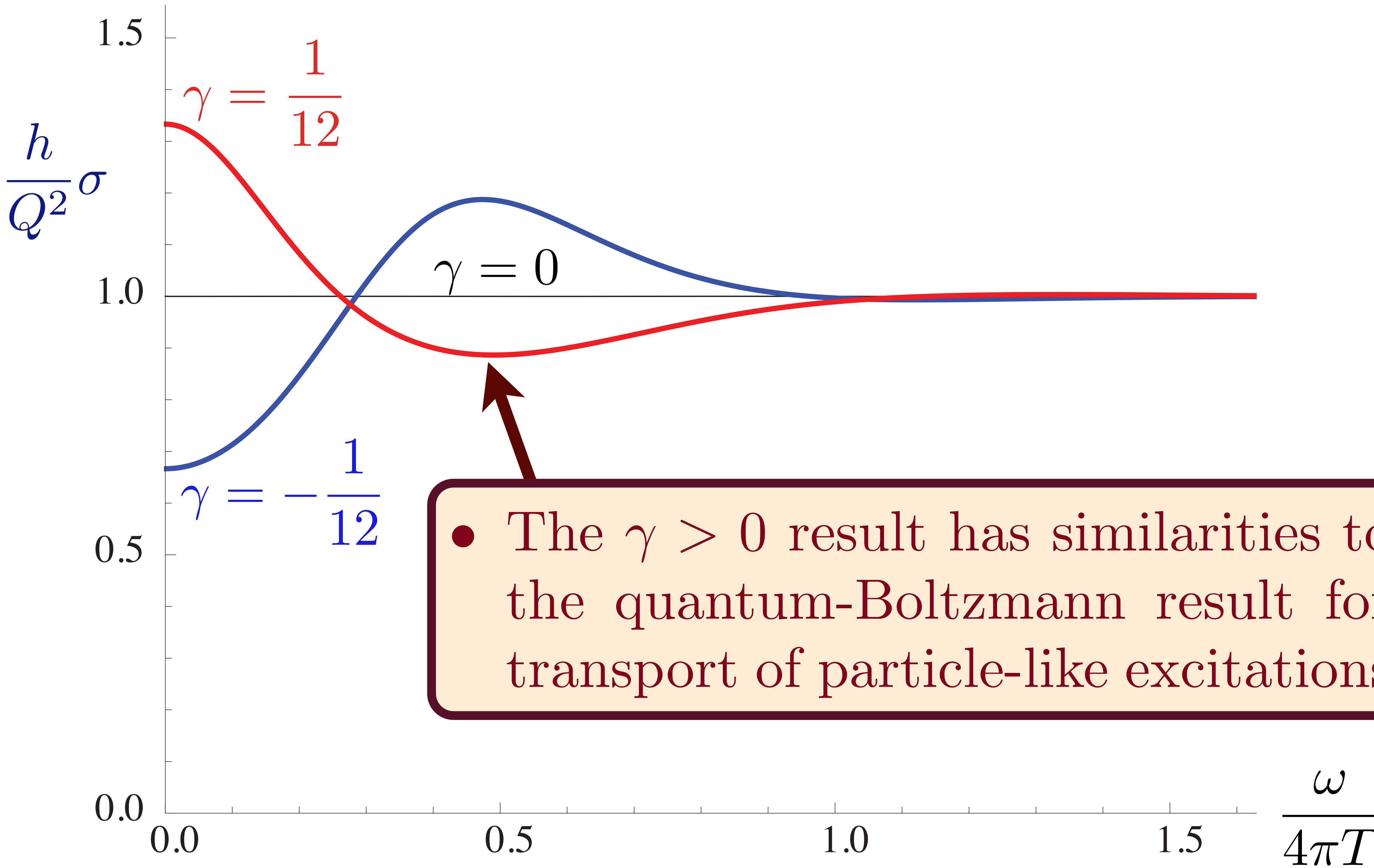

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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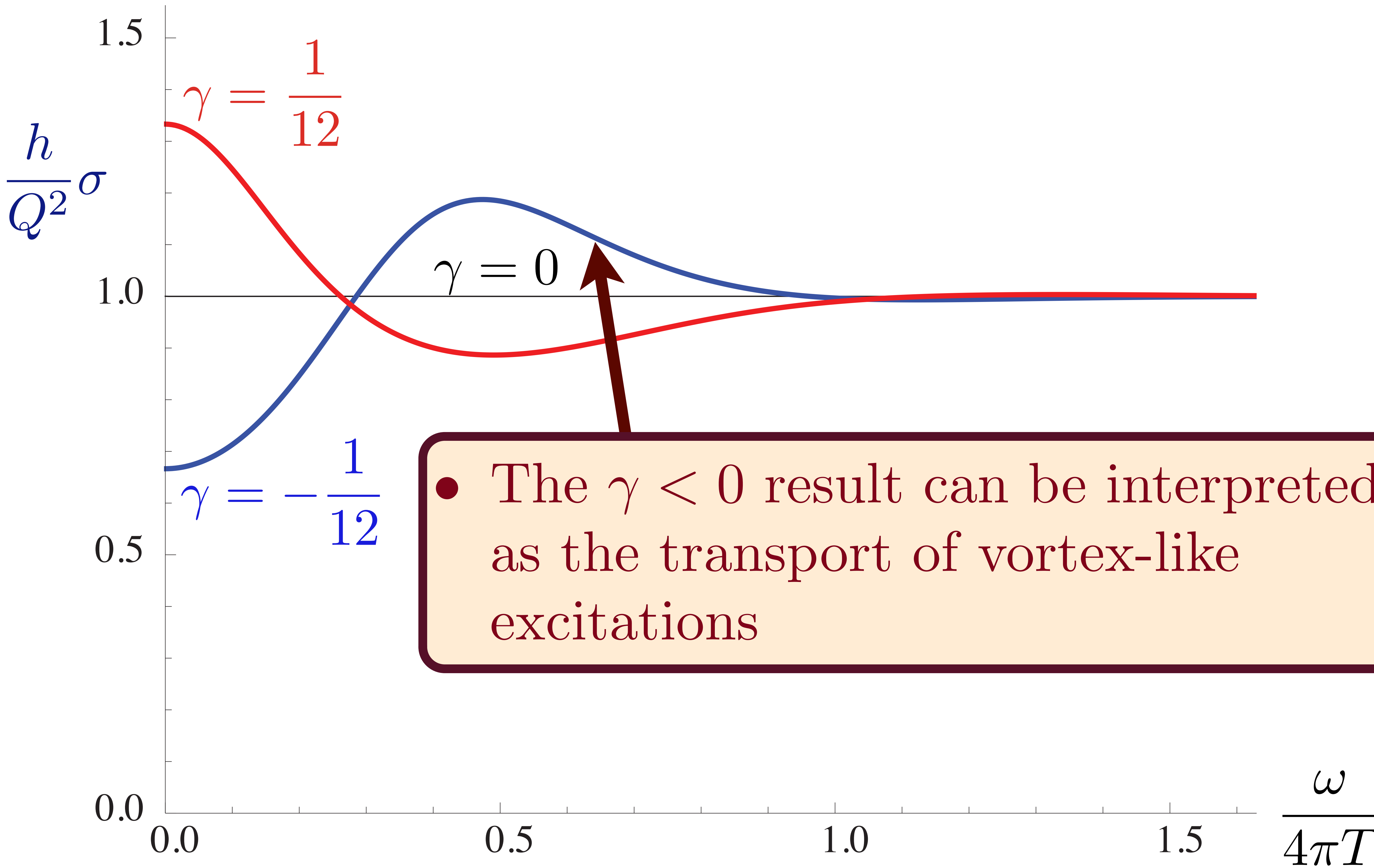

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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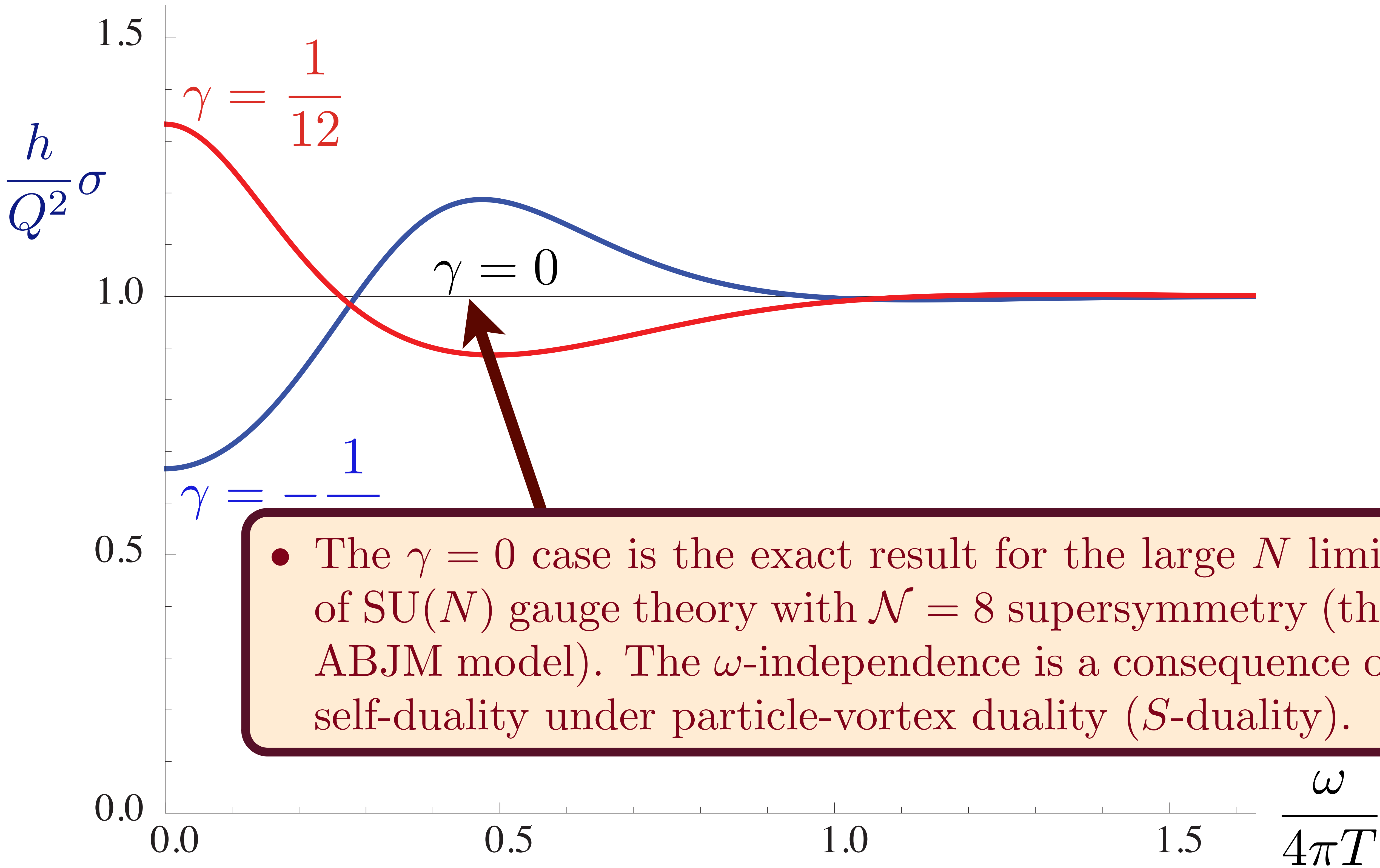
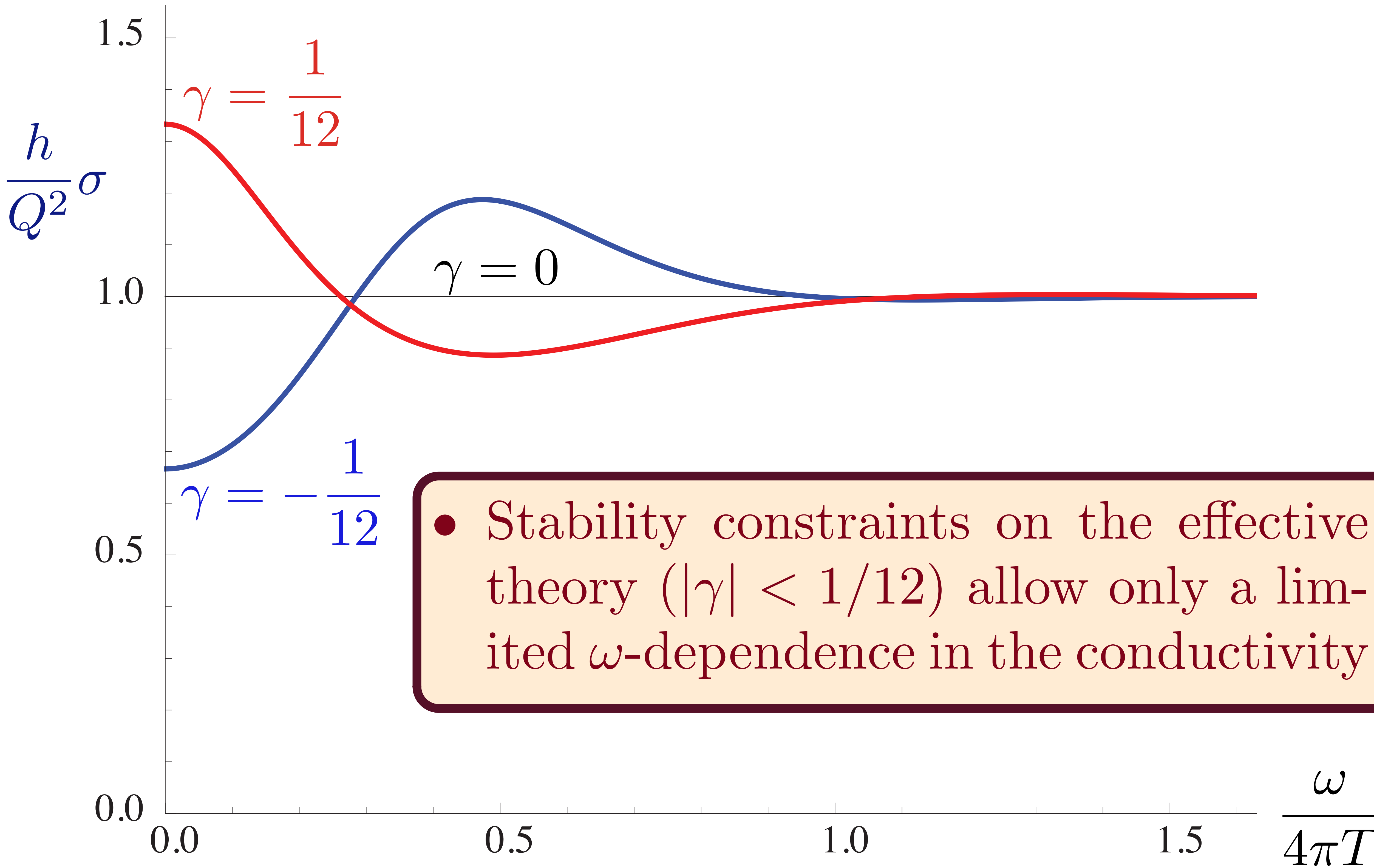


- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)