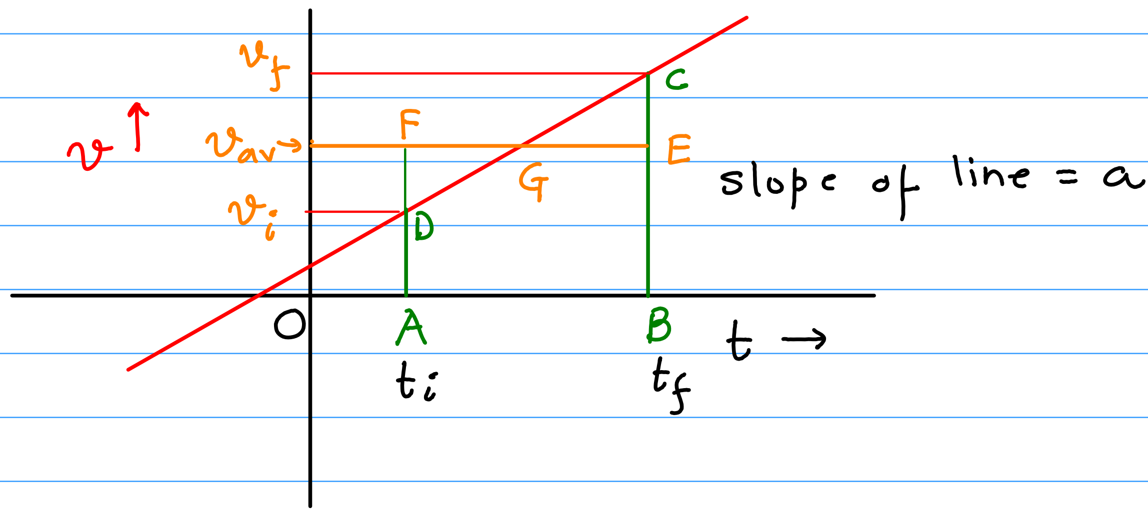
Flowchart for motion with constant acceleration in 1D

Constant acceleration is an important special case of general motion in 1D.

Since $\boxed{a = \text{constant} = a_0}$ ①

$\boxed{v(t) = v_0 + at}$ ②

Every second the velocity changes by a



③

Now for the displacement $\boxed{\Delta x = x(t_f) - x(t_i)}$

④

The key idea is $\boxed{\Delta x = v_{av}(t_i; t_f)(t_f - t_i)}$

For constant acceleration $\boxed{v_{av} = \frac{v_i + v_f}{2}}$ ⑤

Beware, this is guaranteed to be true only for constant acceleration, so don't use it indiscriminately.

Geometrically $\boxed{\Delta x = \text{area of quadrilateral } ABCD}$ (6)

Because the areas of ΔCEG and ΔDFG are identical

$$\boxed{\begin{aligned}\text{area of } ABCD &= \text{area of } ABEF \\ &= v_{av} \otimes (t_f - t_i)\end{aligned}} \quad (7)$$

times symbol so as not to confuse with x

$$\boxed{\Delta x = \frac{1}{2}(v_f + v_i)(t_f - t_i)} \quad (8)$$

Now we can go two ways. If v_i, a, and $t_f - t_i$ are known

$$\boxed{v_f = v_i + a(t_f - t_i)} \quad (9)$$

$$\Rightarrow \quad v_{av} = \frac{1}{2}\left[v_i + v_i + a(t_f - t_i)\right]$$

$$\boxed{v_{av} = v_i + \frac{1}{2}a(t_f - t_i) = v_i + \frac{1}{2}a\Delta t} \quad (10)$$

and $$\boxed{\Delta x = v_i \Delta t + \frac{1}{2}a(\Delta t)^2} \quad (11)$$

Suppose Δt is not known, but instead v_f is known. Then take

$$v_f = v_i + a\Delta t \longrightarrow \boxed{\Delta t = \frac{v_f - v_i}{a}} \quad (12)$$

$$\Rightarrow \quad \boxed{\Delta x = \frac{1}{2}\,\frac{(v_f + v_i)(v_f - v_i)}{a} = \frac{v_f^2 - v_i^2}{2a}} \quad (13)$$

$$\text{or} \quad \boxed{v_f^2 - v_i^2 = 2a\,\Delta x} \quad (14)$$

Here is the flowchart.