Introduction, Measurement and Dimensional Analysis

In PHY231 you will learn how to make sense of the mechanical aspects of the natural world.

We will be using a lot of math, including calculus, but physics is much more than math.

The crucial difference is that math explores any set of consistent rules while physics concentrates on the ones that apply to our world.

To test whether a set of natural laws applies, we need to measure quantities.

For example, Newton's II law says

$$\boxed{\vec{F} = m\vec{a}} \quad (1)$$

where $\vec{F}$ is the (vector) force, m is the mass of the object being accelerated, and $\vec{a}$ is the (vector) acceleration.

UNITS

Before we measure physical quantities, we need to agree on the units in which we will measure them. There are many systems in use, but most scientific measurements are carried out in the MKS system (also called the SI units). In this system length is measured in meters, mass in kg and time in seconds. These are fundamental units.

For PHY 231 this will be enough, but for 232 you will need one more fundamental unit for electric charge, the Coulomb.

From the fundamental units, m, kg, s, we can construct derived units for any mechanical physical quantity.

$$\text{Position} = \text{length} \Rightarrow \text{unit} = m$$

$$\text{Velocity} = \frac{\Delta \text{length}}{\Delta \text{time}} \Rightarrow \text{unit} = m/s$$

$$\text{Acceleration} = \frac{\Delta \text{Velocity}}{\Delta \text{time}} \Rightarrow \text{unit} = m/s^2 \qquad (2)$$

$$\text{Area} = (\text{length})^2 \Rightarrow \text{unit} = m^2$$

$$\text{Volume} = (\text{length})^3 \Rightarrow \text{unit} = m^3$$

$$\text{Volume density} = \frac{\text{Mass}}{\text{Volume}} \Rightarrow \text{unit} = \frac{kg}{m^3}$$

$$\text{Force} = \text{mass} \times \text{acceleration} \Rightarrow \text{unit} = kg\,\frac{m}{s^2}$$

Some derived units appear so frequently that they are given names

$$1\ kg\ m/s^2 = 1\ \text{Newton} \equiv 1\,N$$

ERRORS

No measurement except the counting of discrete objects is free of error. The simplest measurement error is the smallest gradation on your measuring instrument, but this is not the only type of error. If your meter stick is smaller than the standard meter that is a systematic error. Typically, one cannot control all the random factors that influence a measurement, and this leads to statistical error.

How well you know a physical quantity is expressed by the number of significant figures. If a length is given as 2.05 m you know that the error is less than a cm, but if it is given as 2.057 m the error is less than a mm.

Since every physical measurement is subject to error, any physical law can be verified only approximately, but can be falsified completely.

DIMENSIONS

While units tell us how to express physical quantities in terms of numbers, dimensions tell us in an abstract algebraic way the kind of physical quantity something is.

Length	is	symbolized	by	L
Mass	"	"	"	M
Time	"	"	"	T

(3)

Putting a set of square brackets around a quantity means asking the question: What are the abstract dimensions of this quantity?

Thus

$$\boxed{\begin{aligned} [\text{Area}] &= L^2 \\ [\text{velocity}] &= L/T = LT^{-1} \\ [\text{Force}] &= MLT^{-2} \end{aligned}}$$

(4)

Most physical laws can be expressed as algebraic equations. If two physical quantities are equal under all possible circumstances, they must have the same dimensions. That is they must be the same kind of quantity.

Again, take $F = ma$ (1 dimension so no vectors)

$[F] = MLT^{-2}$

$[m] = M \qquad [a] = LT^{-2}$

$$\boxed{[F] = [m][a]}$$

(5)

Dimensions match!

This may seem trivial but we can use this to obtain some nontrivial information about the dependence of one physical quantity on another.

DIMENSIONAL ANALYSIS

Consider a simple pendulum. What could the time period depend on? Certainly the length l and the local acceleration due to gravity g. It could also depend on the mass of the bob m_b, and the angular amplitude of the swing θ_0.

So t_p = time period of a simple pendulum

$$\boxed{t_p = f(l, g, m_b, \theta_0)} \quad (6)$$

What function could this be? Can one add l and g

$$l + g = \,?$$

NO!!

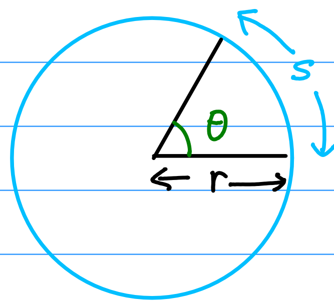
This doesn't make sense! You can only add two quantities if they have the same dimensions.

However, you can multiply or divide any power of one with respect to the other to get a different kind of quantity.

Now $[l] = L$ $\quad [g] = LT^{-2}$ $\quad [m_b] = M$

How about $[\theta_0]$? Angles in physics are measured in radians, but a radian is a ratio of two lengths

$$\boxed{\theta = \frac{s}{r}} \quad (7)$$



$$\Rightarrow \quad \boxed{[\theta] = 1} \quad (8)$$ Dimensionless!

If (and only if) a quantity is dimensionless, any power of it is dimensionless, and one can add different powers together

$$\boxed{f(\theta) = f(0) + \theta f'(0) + \frac{1}{2}\theta^2 f''(0) + \cdots} \quad (9)$$

Makes sense iff $[\theta] = 1$

So the most general possible formula for the time period is

$$\boxed{t_p = \ell^a g^b m_b^c f(\theta_0)} \quad (10)$$

where a, b, c are fixed numbers. Now take dimensions on both sides

$$\boxed{[t_p] = T = [\ell^a g^b m^c f(\theta_0)]} \quad (11)$$

$$\boxed{T = L^a (LT^{-2})^b M^c = L^{a+b} T^{-2b} M^c} \quad (12)$$

Match dimensions on both sides to get

$\boxed{a+b=0}$ (13) $\quad \boxed{-2b=1}$ (14) $\quad \boxed{c=0}$ (15)

$$\Rightarrow \quad \boxed{b = -\frac{1}{2} \qquad a = \frac{1}{2} \qquad c = 0} \quad (16)$$

$$\boxed{t_p = f(\theta_0)\sqrt{\frac{l}{g}}} \quad (17)$$

For $\theta_0 \ll 1$ it turns out that

$$f(\theta_0) \to 2\pi$$

Another example: Consider a nuclear explosion. Its total energy output is E. It generates a "fireball" with a radius r which expands with time t.

What could r depend on? Certainly E and t, but also it could depend on the density of air ρ_{air}

So $$\boxed{r = t^a E^b \rho_{air}^c} \quad (18)$$

Take dimensions, noting $\boxed{[E] = ML^2T^{-2}}$ (19)

$$\boxed{[r] = L = [t^a E^b \rho_{air}^c] = T^a (ML^2T^{-2})^b (ML^{-3})^c} \quad (20)$$

$$\boxed{L = L^{2b-3c} \, T^{a-2b} \, M^{b+c}} \quad (21)$$

Match dimensions on both sides

(22) $\boxed{b+c=0}$ (23) $\boxed{2b-3c=1}$ (24) $\boxed{a-2b=0}$

$$c = -b \qquad 2b - 3(-b) = 1 \qquad a = 2b$$

so $$\boxed{b = 1/5 \qquad c = -1/5 \qquad a = 2/5} \quad (25)$$

$$\Rightarrow \quad \boxed{r(t, E, \rho_{air}) = t^{2/5} \left(\frac{E}{\rho_{air}} \right)^{\frac{1}{5}}} \quad (26)$$

So just by studying time-lapse satellite pictures of a nuclear explosion, one can tell the total energy output.

More mundanely, if someone presents you with a potential physical law, the first thing you should check is if the dimensions on the two sides of the equation match.