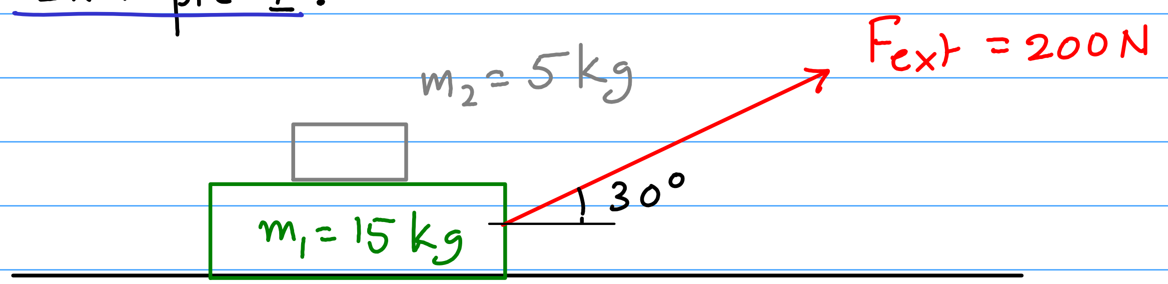
Applications of Newton's Laws with Friction

Now let us consider all of Newton's Laws, including the III Law and combine it with friction.

Example 1:



The coefficient of kinetic friction between m_1 and the ground is $\boxed{\mu_{k1} = 0.2}$ 1. The coefficient of static friction between m_2 and m_1 is

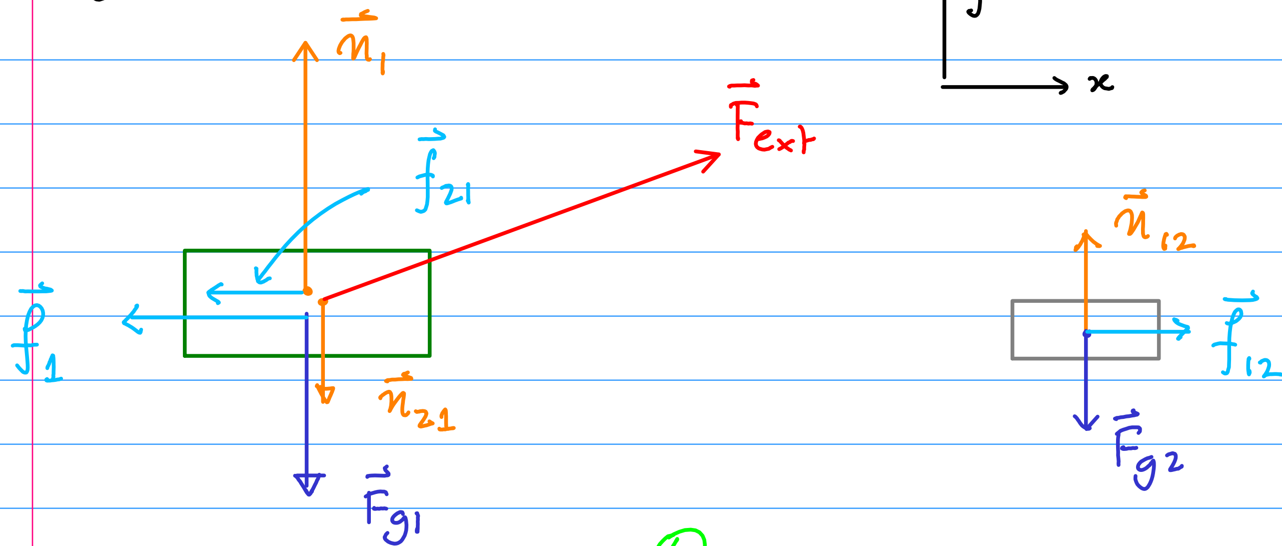
$\boxed{\mu_{s12} = 0.15}$ 2 while the coefficient 3 of kinetic friction between them is $\boxed{\mu_{k12} = 0.1}$

Find the accelerations of the two masses.

The key here is that mass 2 may not accelerate at the same rate as mass 1.

Let us 1st find whether it does. As usual assume that m_1 and m_2 move as a unit. This means there is no relative motion between m_1 and m_2 and thus the friction between m_1, m_2 is static.

Draw the FBDs



Note: $\boxed{\vec{n}_{12} = -\vec{n}_{21}}$ 4 forms an III Law action - reaction pair

$\boxed{\vec{f}_{12} = -\vec{f}_{21}}$ 5 also is a III Law

action-reaction pair.

Body 1: $\boxed{\vec{F}_{g_1} = -m_1 g\,\hat{j}}$ 6 $\boxed{\vec{n}_1 = n_1 \hat{j}}$ 7

$\boxed{\vec{n}_{21} = -n_{12}\,\hat{j}}$ 8 $\boxed{\vec{f}_1 = -f_1\,\hat{i}}$ 9

$\boxed{\vec{f}_{21} = -f_{12}\,\hat{i}}$ 10

$$\boxed{\vec{F}_{ext} = F_{ext}\cos 30^\circ\,\hat{i} + F_{ext}\sin 30^\circ\,\hat{j}}$$ 11

$$\boxed{\vec{F}_{tot,1} = \hat{i}\left(F_{ext}\cos 30^\circ - f_1 - f_{12}\right) + \hat{j}\left(n_1 - n_{12} - m_1 g + F_{ext}\sin 30^\circ\right)}$$ 12

Body 2: $\boxed{\vec{F}_{g2} = -m_2 g}$ (13) $\boxed{\vec{n}_{12} = n_{12}\hat{j}}$ (14)

$\boxed{\vec{f}_{12} = f_{12}\hat{i}}$ (15)

$$\boxed{\vec{F}_{tot,2} = f_{12}\hat{i} + (n_{12} - m_2 g)\hat{j}}$$ (16)

Since the weight of both blocks is

$20\,kg \times 9.8\,\frac{m}{s^2} = 196\,N$ is greater than

the vertical component of F_{ext}, we know that the masses are not accelerating vertically.

$\boxed{a_{1y} = 0}$ (17) $\boxed{a_{2y} = 0}$ (18)

$\Rightarrow$ $\boxed{F_{tot,1y} = 0}$ (19) $\boxed{F_{tot,2y} = 0}$ (20)

or $$\boxed{n_1 - n_{12} - m_1 g + \frac{1}{2} F_{ext} = 0}$$ (21)

$\boxed{n_{12} - m_2 g = 0}$ (22) or $n_{12} = m_2 g = 49\,N$

and $n_1 = n_{12} + m_1 g - \frac{1}{2} F_{ext} = 49 + 157 - 100$
$= 96\,N$

So $\boxed{n_1 = 96\,N}$ (23) $\boxed{n_2 = 49\,N}$ (24)

f_1 results from kinetic friction, so

$$\boxed{f_1 = \mu_{k1} n_1 = 0.2 \times 96\,N = 19.2\,N}$$ (25)

Now consider the x-component.

Body 1: $\boxed{F_{ext} \frac{\sqrt{3}}{2} - f_1 - f_{12} = m_1 a_{1x}}$ 26

Body 2: $\boxed{f_{12} = m_2 a_{2x}}$ 27

Assume that they move together $\Rightarrow a_{1x} = a_{2x}$

So $200\,N \cdot x \frac{\sqrt{3}}{2} - 19.2\,N = f_{12} + m_1 a_{1x}$

$= (m_1 + m_2)\, a_{1x}$

or $154\,N = 20\,kg \cdot a_{1x}$

$\boxed{a_{1x} = 7.7\,m/s^2}$ 28 IF they move together

This means the frictional force required to accelerate m_2 is

$\boxed{f_{12} = m_1 a_{1x} = 38.5\,N}$ 29 required if they move together

Now, what is the maximum value of $f_{12,s}$?

$$\boxed{\begin{aligned} f_{12,s,max} = \mu_{s,12}\, n_{12} &= 0.15 \times 49\,N \\ &= 7.35\,N \end{aligned}}$$ 30

Since the required f_{12} exceeds $f_{12,s,max}$

THE TWO MASSES CANNOT ACCELERATE TOGETHER.

Ok, now we know they have different accelerations

$$\boxed{a_{1x} \neq a_{2x}} \quad (31)$$

We also know f_{12}, because it comes from kinetic friction.

$$\boxed{f_{12} = \mu_{k12}\, n_{12} = 0.1 \times 49N = 4.9\,N} \quad (32)$$

Thus, immediately from (27)

$$4.9\,N = 5\,kg \cdot a_{2x}$$

$$\Rightarrow \quad \boxed{a_{2x} = 0.98\ m/s^2} \quad (33)$$

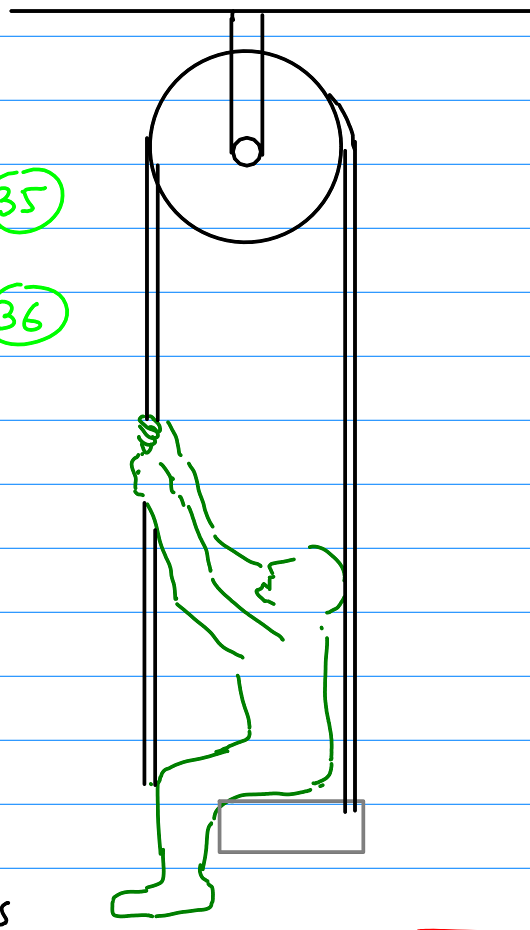
And from (26)

$$200\,\frac{\sqrt{3}}{2} - 19.2 - 4.9 = 15\,a_{1x} \quad (34)$$

$$149.1\,N = 15\,a_{1x} \qquad \boxed{a_{1x} = 9.94\ m/s^2}$$

<u>Example 2</u>

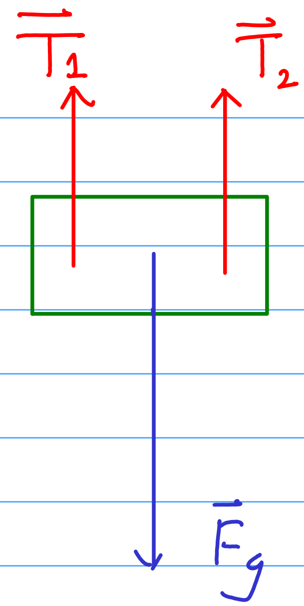
A child of mass $\boxed{m_1 = 40 kg}$ 35 sits in a seat of mass $\boxed{m_2 = 10 kg}$. 36 The seat is attached to a rope which goes over the pulley (massless and frictionless) with the child holding the other end.



Now the child pulls down on the rope with a force of $\boxed{300 N.}$ 37 What is the force of the seat on the child? What is their acceleration?

The key point is when the child pulls down on the rope, the III Law reaction force is the rope pulling the child up.

First, to determine the acceleration, consider the child and seat as one unit. The FBD is on the next page.



The rope is attached in two places to the child + seat, so applies both $\vec{T}_1$ and $\vec{T}_2$

Since the rope is massless and inextensible

$$\boxed{\vec{T}_1 = \vec{T}_2 = T\hat{j} = 300\,N\,\hat{j}} \quad (38)$$

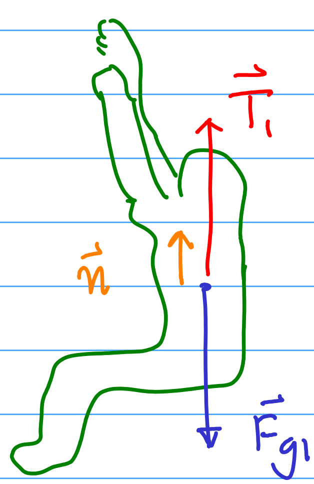
$$\boxed{\vec{F}_g = -(10\,kg + 40\,kg) \times 9.8\,m/s^2 = 490\,N} \quad (39)$$

$$\boxed{\vec{F}_{tot} = (2T - mg)\,\hat{j} = 110\,N\,\hat{j} = m\vec{a}} \quad (40)$$

$$\Rightarrow \boxed{\vec{a} = \frac{110}{50}\,m/s^2 = 2.2\,m/s^2} \quad (41)$$

upwards

Now to find the force of the seat on the child draw the FBD for the child



$\vec{n}$ = normal reaction of seat on child

$$\boxed{\vec{T}_1 = 300\ N\hat{j}}$$ 42

$$\boxed{\vec{F}_{g1} = -40kg \times 9.8\frac{m}{s^2}\hat{j} = -392\ N\hat{j}}$$ 43

$$\boxed{\vec{n} = n\hat{j}}$$ 44

45

$$\Rightarrow \boxed{\vec{F}_{tot,1} = (300 - 392 + n)\ N\hat{j} = 40kg \cdot \vec{a}}$$

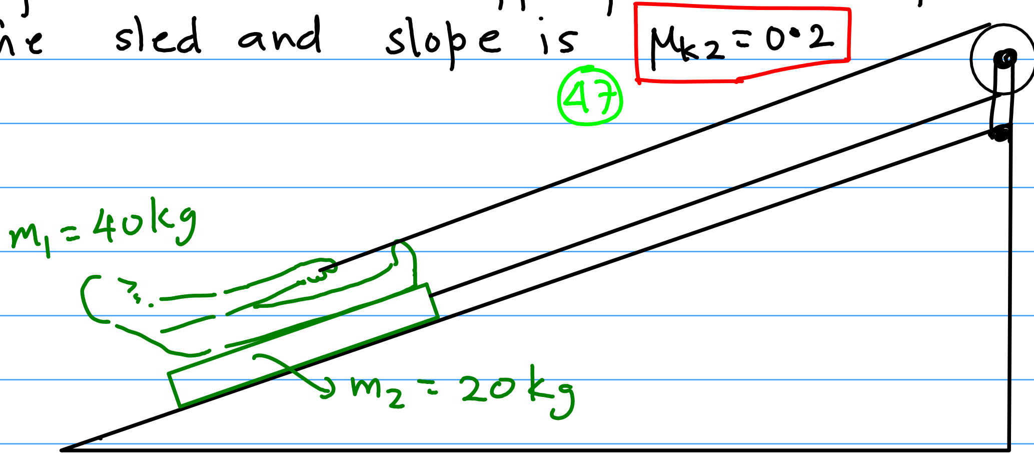
But we know $\vec{a} = 2.2\frac{m}{s^2}\hat{j}$

So $-92 + n = 88$

$$\boxed{n = 180\ N}$$ 46

The seat applies a normal force of 180 N on the child.

Example 3: Let us complicate Example 2 by putting it on a slope and introducing friction. The coeff of kinetic friction between the sled and slope is $\boxed{\mu_{k2} = 0.2}$ 47

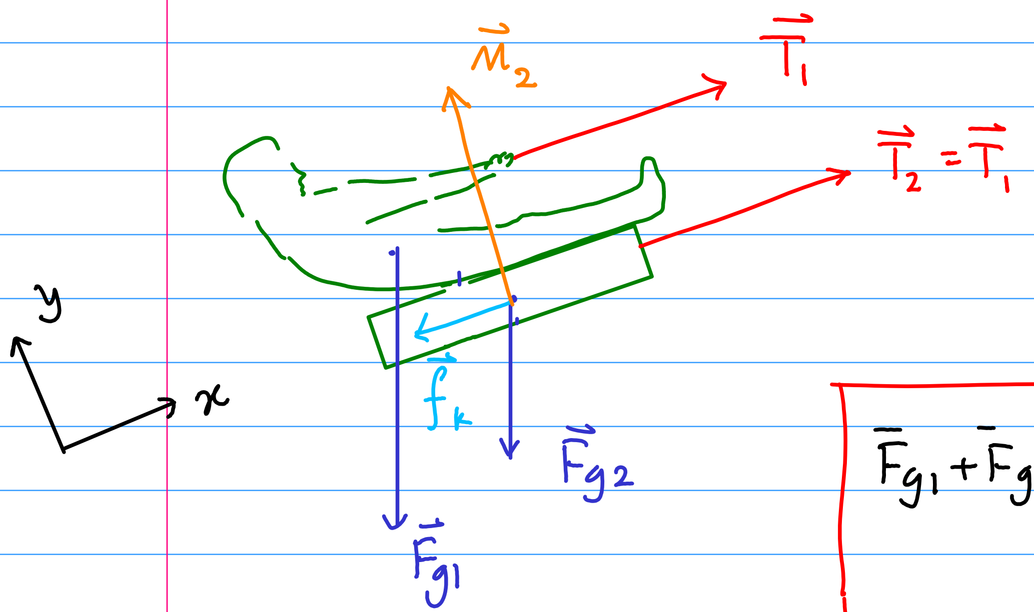


The coeff of static friction between the child and the sled is $\boxed{\mu_{s12} = 0.3.}$ 48

What is the maximum force with which the child can pull on the rope such that he does not slide with respect to the sled?

In this situation what is the acceleration of child + sled?

We know that the child and the sled move together. So treat them as a unit.



Since the rope contacts the unit twice it applies twice the force

49

$$\boxed{\vec{F}_{g1} + \vec{F}_{g2} = -(m_1+m_2) g \sin 30° \hat{i} - (m_1+m_2) g \cos 30° \hat{j}}$$

50

$$\boxed{\vec{F}_{tot} = \hat{i}\left(2T - (m_1+m_2) g \sin 30° - f_k\right) + \hat{j}\left(n_2 - (m_1+m_2) g \cos 30°\right) = (m_1+m_2)\vec{a}}$$

$a_y = 0 \Rightarrow \quad n_2 = (m_1+m_2) g \cos 30° = 60\,kg \times 9.8 \frac{m}{s^2} \times \frac{\sqrt{3}}{2}$

51

$$\boxed{n_2 = 509.2\ N}$$

52

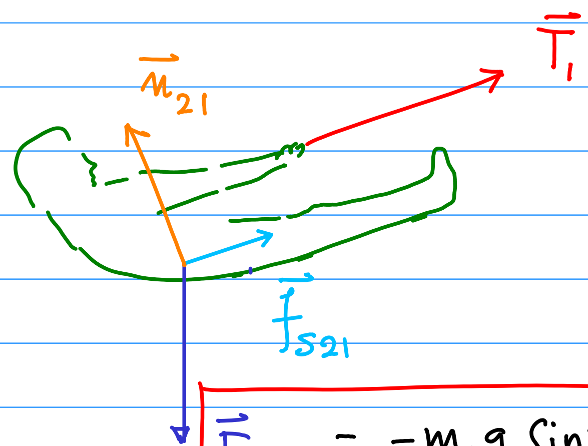
$$\boxed{f_k = \mu_k n_2 = 0.2 \times 509.2\ N = 101.8\ N}$$

So $\boxed{F_{tot,x} = 2T - 60kg \times 9.8\frac{m}{s^2} \times \frac{1}{2} - 101.8\,N = 2T - 395.8\,N = (m_1+m_2)\,a_x}$ 53

So $\boxed{a_x = \frac{2T - 395.8\,N}{60\,kg}}$ 54

Now that we know a_x let us draw the FBD of the child

$\vec{n}_{21}$ = Normal force of the sled on the child



$\vec{f}_{s21}$ = Static force of friction of the sled on the child

55

$\boxed{\vec{F}_{g1} = -m_1 g \sin 30° \hat{i} - m_1 g \cos 30° \hat{j}}$

56

$\boxed{\vec{F}_{tot,1} = \hat{i}\left(T + f_{s21} - m_1 g \sin 30°\right) + \hat{j}\left(n_{21} - m_1 g \cos 30°\right)}$

We know $a_y = 0 \Rightarrow n_{21} = m_1 g \cos 30° = 40kg \times 9.8\frac{m}{s^2}\frac{\sqrt{3}}{2}$

$\boxed{n_{21} = 339.5\,N}$ 57

We also know that f_{s21} must be at its maximum value

58

$\boxed{f_{s,21,max} = \mu_{s21}\, n_{21} = 0.3 \times 339.5\,N = 101.8\,N}$

So looking at the x-component (59)

$$\boxed{T + 101.8\,N - 196\,N = m_1 a_x = 40\left(\frac{2T - 395.8}{60}\right)}$$

$$T - 94.2\,N = \frac{4T}{3} - 263.9\ N$$ (60)

or $\frac{T}{3} = 169.7\,N \Rightarrow \boxed{T_{max} = 509\,N}$

The acceleration for this T_{max} is

$$\boxed{a_{x,max} = \frac{2T_{max} - 395.8}{60} = 10.4\ m/s^2}$$ (61)