Now we come to:

**CENTRIPETAL ACCELERATION**

(***CENTER-SEEKING** **ACCE**

_to do with "point objects" executing circular motion*

E.g.: **Earth around Sun**

**Tennis ball fixed onto string**

**Car going over "hump backed" bridge**
CIRCULAR MOTION AT CONSTANT SPEED:

\[ \vec{v} = \text{const} \]

But \( \vec{v} \neq \text{const} \! \uparrow \)

\[ \Delta \vec{v} = \vec{v}_B - \vec{v}_A \]

\[ \Delta \vec{v} : \]

MOVE B CLOSE TO A:

\[ \overrightarrow{\Delta \vec{v}} \]

\[ \overrightarrow{\Delta \vec{a}} \]

\( \overrightarrow{\vec{a}} \text{ TOWARDS CENTER} \)
LETS GO BACK TO SCALARS.

WE WILL FIND THE ACC. TOWARDS THE CENTER - CENTRIPETAL ACC $a_c$:

$$a_c = \frac{v^2}{r}$$

AND FROM NEWTON # 2 THE FORCE REQUIRED TO RESULT IN CIRCULAR MOTION IS

$$F_c = ma_c = \frac{mv^2}{r}$$

(Book uses Vectors)
MY SIMPLIFIED SCALAR PROOF USING COMPONENTS

CONSIDER CIRCULAR MOTION IN HORIZONTAL PLANE (x-y) WITH CONSTANT SPEED U

STEP 1

CONSIDER THE INSTANT THAT THE OBJECT CROSSES THE X AXIS:

\[ \begin{align*}
F_y & \uparrow \\
\rightarrow F_x \\
& \rightarrow z
\end{align*} \]

SINCE \( U = \text{const} \) WE MUST HAVE \( F_y = 0 \) AT THIS INSTANT (IE \( a_y = 0 \)).

SO THE ONLY FORCE POSSIBLE IS \( F_x \neq 0 \) AT THIS INSTANT.

IE TOWARDS THE CENTER OF THE CIRCLE.
**Step 2.** A short time \( t \) later the object has swept out angle \( \Delta \theta \):

\[
\begin{align*}
U_1 & \rightarrow U_2 \\
{\Delta x} & \rightarrow {\Delta x} \\
U_{x1} & = 0
\end{align*}
\]

\[
\begin{align*}
U_{x2} & = U \cos (90 - \Delta \theta) \\
& = U \sin \Delta \theta
\end{align*}
\]

\[
\begin{align*}
\overline{a_x} & = \frac{\Delta u_x}{\Delta t} \\
& = \frac{U_{x2} - U_{x1}}{\Delta t} \\
& = \frac{U \sin \Delta \theta}{\Delta t}
\end{align*}
\]

As \( \Delta \theta \to \text{smaller and smaller} \)

\[
\sin \Delta \theta \to \Delta \theta \quad [\text{in radians}]
\]

**Check:**

<table>
<thead>
<tr>
<th>( \frac{\theta}{1} )</th>
<th>( \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8415</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4794</td>
</tr>
<tr>
<td>0.25</td>
<td>0.2474</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0998</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0099998</td>
</tr>
</tbody>
</table>

\( \sqrt{\circ} \)
so \( a_x = u \frac{\Delta \theta}{\Delta t} = u \omega \)

use \( \omega = \frac{u}{r} \)

\[ a_x = u \frac{u}{r} = \frac{u^2}{r} \]

But there is nothing special about our point.

So:

\[ a_c = \frac{u^2}{r} \]

Or

\[ a_c = \omega^2 r \]

Since \( u = \omega r \); \( \frac{u^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r \)
NEWTON #2 is always TRUE

So if

\[ \alpha_c = \frac{v^2}{r} = \omega^2 + \]

there must exist a net force

\[ F_c = \frac{mv^2}{r} = mC_0^2 r \]

that is causing the acceleration.
Let's examine our three examples:

1) Earth around Sun

2) Ball whirled around on end of string

3) Car on hump back bridge

Important:
Use free body diagrams!

\[ F_c = \frac{mv^2}{r} \] is required magnitude of force due to 1) gravitational force, 2) tension, 3) wait and see ....
1) Earth around Sun.

Due to gravitational attraction

\[ F = \frac{G M_e M_s}{r^2} \]

\[ F_c = \frac{M_e u^2}{r} \]

So

\[ F_c = F_c \]

\[ G \frac{M_e M_s}{r^2} = \frac{M_e u^2}{r} \]

or

\[ u^2 = \frac{G M_s}{r} \]

So all planets have

\[ u^2 r = G M_s \]

(circular motion ...)

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2) **Ball on End of String**

FBD:

\[ \text{THE ONLY FORCE!} \]

So \( T = F_c \)

\[ T = \frac{m u^2}{r} \]

*(Note: Horizontal Plane and Ignore Gravity)*
(3) CAR ON BRIDGE RADIUS \( r \)

Assume \( v \neq \text{const!} \)

**Two Physical Forces:**
- Gravity
- Normal Force

FBD:

\[ mg \]

\[ \begin{align*}
\text{normal force} & \quad \Rightarrow \quad mg \sin \theta \quad \downarrow \\
& \quad \Rightarrow \quad mg \cos \theta
\end{align*} \]

So \( F_c = \frac{mv^2}{r} = mg \cos \theta - n \)

At any instant

(We will do a worked problem later)
COMMUNICATIONS SATELLITES IN GEO STATIONARY ORBITS

For such an orbit the satellite is always directly above the same point of the Earth.

$\omega' = \omega$

Also the orbital period is 1 day!

$T = 24$ hours

We can find the conditions on $r$ (orbital radius) and $v$ (speed)
MASS OF EARTH

EARTH ATTRACTION SAT.

\[ F_c = G \frac{mM}{r^2} \]

CIRCULAR MOTION:

\[ F_c = m\frac{v^2}{r} \]

\[ \frac{F_c}{F_c} : \frac{m \frac{v^2}{r}}{m} = G \frac{M}{r^2} \]

\[ v^2 r = GM \]

ORBITAL PERIOD

\[ T = \text{ORBIT \over SPEED} = \frac{2\pi r}{v} \]

\[ v = \frac{2\pi r}{T} \]

PUT (2) IN (1):

\[ \frac{(2\pi)^2 r^3}{T^2} = GM \]
Solve for $r$: -

$$r = \left( \frac{GMT^2}{4\pi^2} \right)^{\frac{1}{3}}$$

And speed: -

$$v = \frac{2\pi r}{T}$$

$T = 24$ hrs

Putting in numbers: A geostationary satellite orbits at $r = 42,300$ km at $v = 3.08$ km/s.

[Note: $r = 22,000$ mi above surface of Earth]

[SiriusXM Radio = Free AD]
So to summarize:

**Circular Motion Requires a Center-Seeking Force of Magnitude**

\[ F_c = \frac{m v^2}{r} \]

This equation **does not** tell you the **origin** of the force — it merely tells you what magnitude is required for a given \( m \), \( v \), \( r \).

The **origin** of the force depends on the **physical situation**.

It might be gravity, tension, or even the normal force — (see last worked example on schedule).
Finally:

WE DO NOT DO KEPLER'S LAWS (CH 7)

(WE DID THE GEO-STATIONARY CASE WITHOUT THEM)