GRAVITY

Let's work out some examples.

Example 1
Drop something over a cliff of height $h$.

\[ y = 0 \]

\[ y = + \]

\[ y = - \]

CLIFF

\[ y = -h \]

Note sign?

How long does it take to reach the sea?

\[ y = v_0 t - \frac{1}{2} g t^2 \]

Drop $v_0 = 0$.

\[ y = -\frac{1}{2} g t^2 \quad \text{AND} \quad y = -h \]

\[ -h = -\frac{1}{2} g t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}} \]
How fast is it going when it lands?

\[ v^2 = u_0^2 - 2gy \]

\[ y = -h \]
\[ u_0 = 0 \]

\[ v^2 = -2g(-h) = +2gh \]

\[ v = \pm \sqrt{2gh} \]

\[ \text{CAREFUL! \pm FOR \sqrt{}} \]

The physics says \( v \) is downwards \( \downarrow \)

So \( v \) is \( \leq \)

\[ v = -\sqrt{2gh} \]
EXAMPLE 2

2) THROW SOMETHING VERTICALLY UPWARDS WITH $u_0$

$y = +h$

$y = 0$

----

a) HOW HIGH DOES IT GO?

WE KNOW $u_0$, $u = 0$

$a = -9$

so: $u^2 = u_0^2 - 2gh$

$0 = u_0^2 - 2gh$

$v_0^2 = 2gh$

$h = \frac{v_0^2}{2g}$
H) What is $u$ when it comes back down to the ground?

Let's do it two ways:

**First Way**

It starts from $y = h$ with initial velocity $v_0 = 0$.

Same as being dropped off a cliff of height $h$.

So $v = -\sqrt{2gh}$.

But $h = \frac{v_0^2}{2g}$.

So $v = -\sqrt{2g\left(\frac{v_0^2}{2g}\right)} = -\sqrt{v_0^2}$

$u = -u_0$ same speed as it started with!!
So the first way did it in two separate steps: **UP** then **DOWN**

**UP** gave \( h \).

**DOWN** used \( h \).

**SECOND WAY**

**SINGLE STEP: THE MAGIC OF OUR EQUATIONS.**

\[
\begin{align*}
\begin{aligned}
\text{IT STARTS AT} & \quad y = 0. \\
\text{IT ENDS AT} & \quad y = 0.
\end{aligned}
\end{align*}
\]

\[
\begin{align*}
50 \quad v^2 &= v_0^2 - 2gy \\
50 \quad v^2 &= v_0^2 \\
50 \quad v &= \pm \sqrt{v_0^2} \\
50 \quad v = -v_0
\end{align*}
\]

"Physics"

**SIMPLE!"
HOW LONG IS IT IN THE AIR?

SINGLE STEP:

\[ y = v_0 t - \frac{1}{2} g t^2 \]

\[ y = 0 \]

\[ (v_0 - \frac{1}{2} g t) t = 0 \]

So \( t = 0 \) [ie it's at \( y = 0 \) at \( t = 0 \) - true.]

Or \[ v_0 - \frac{1}{2} g t = 0 \]

\[ v_0 = \frac{1}{2} g t \]

\[ t = \frac{2v_0}{g} \]

which is when it lands.
USE UNITS TO CHECK YOUR ANSWER!

WE HAD \( h = \frac{u_0^2}{2g} \)

UNITs: \( u_0 \equiv \frac{m}{s} \)  
\( g \equiv \frac{m}{s^2} \)

\( h = \frac{u_0^2}{g} \equiv \frac{(\frac{m}{s})^2}{\frac{m}{s^2}} \equiv \frac{m^2}{s^2} \cdot \frac{s^2}{m} \)

\( h \equiv m \checkmark \)
Example 3

3) Let's combine examples 1 + 2

\[ y = 0 \]

\[ y = v_0 t \]

\[ y = \frac{1}{2} gt^2 \]

**WHAT IS \( v \) WHEN IT REACHES THE BOTTOM OF THE CLIFF?**

The motion seems complicated:

(a) Goes up: \( y = 0 \) \( \Rightarrow \) \( y = y_{\text{max}} \)

(b) Stops

(c) Comes down \( y = y_{\text{max}} \) \( \Rightarrow \) \( y = -h \)

In fact it isn't!

One equation does it all in a single step.
We know \( v_0, y, a = -g \) but need \( v \).

One of the four equations does the trick:

\[
\begin{align*}
v^2 &= v_0^2 - 2gy \\
&= v_0^2 + 2(-g)(-h) \\
&= v_0^2 + 2gh
\end{align*}
\]

\[
v = -\sqrt{v_0^2 + 2gh}
\]

This works because \( a = -g \) all the time it is airborne.
**NOTE:**

**Throwing an object up with** $v_0$,  

**AND:**  

**Throwing an object down with** $v_0$,  

**GIVE THE SAME FINAL $v$!**

**BECAUSE**

$u^2 = v_o^2 - 2gy$  
$v^2 = (-v_0)^2 - 2gy$  
ARE THE SAME