\( \textbf{EQUILIBRIUM} \)

Net Force \( \sum \vec{F} = \sum \vec{F}_n \)

If \( \sum \vec{F} = 0 \) \((N \neq 1)\) then object is in \textbf{EQUILIBRIUM}.

(May be at rest or \( \vec{v} = \text{const} \))

This is a vector equation

So:

\( \Sigma F_x = 0 \)
\( \Sigma F_y = 0 \)
\( (\Sigma F_z = 0) \)

\( \text{In fact the sum of the components along any direction (axis) must be zero!} \)
Let's look at an example where
\[ \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \]

I find it easiest to do:
\[ \Sigma F_x = 0 \quad \text{as} \quad \Sigma F_{\text{left}} = \Sigma F_{\text{right}} \]

And
\[ \Sigma F_y = 0 \quad \text{as} \quad \Sigma F_{\text{up}} = \Sigma F_{\text{down}} \]
\[ \Sigma F_{\text{LEFT}} = F_3 \cos \theta_3 \]
\[ \Sigma F_{\text{RIGHT}} = F_1 \cos \theta_1 + F_2 \cos \theta_2 \]

So: \[ F_3 \cos \theta_3 = F_1 \cos \theta_1 + F_2 \cos \theta_2 \]

\[ \Sigma F_{\text{UP}} = F_1 \sin \theta_1 \]
\[ \Sigma F_{\text{DOWN}} = F_3 \sin \theta_3 + F_2 \sin \theta_2 \]

So \[ F_1 \sin \theta_1 = F_3 \sin \theta_3 + F_2 \sin \theta_2 \]

**NOTE:** LEFT = RIGHT \[ 3 = 1 + 2 \]

UP = DOWN \[ 1 = 3 + 2 \]
A NEW FORCE — THE NORMAL FORCE

DROP AN OBJECT

NOT IN EQUILIBRIUM!

\[ \Sigma F \neq 0 \]

HOLD OUT AN OBJECT

IN EQUILIBRIUM:

\[ \Sigma F = 0 \] \[ \Rightarrow F_{my} = mg \]

OBJECT AT REST ON TABLE

\[ F_c = mg \]

IT'S IN EQUILIBRIUM! SOMETHING'S MISSING!
THE NORMAL FORCE $N$

OBJECT ON TABLE

$N = mg$ (HERE), (EQUILIBRIUM)

ATOMS OF TABLE ACT ON ATOMS OF OBJECT

$N$ IS PERPENDICULAR TO SURFACE (IE NORMAL)

SO

IN OBJECT ON SLOPE

PERP TO SURFACE!
WE WILL EXAMINE TWO TYPES OF "FORCE" PROBLEMS

**FIRST:** Non-Equilibrium Problems

\[ \vec{F} = m \vec{a} \]
\[ \vec{a} \neq \vec{0} \]
\[ \sum \vec{F} \neq \vec{0} \]

**SECOND:** Equilibrium Problems

\[ \vec{F} = m \vec{a} \]
\[ \vec{a} = \vec{0} \]
\[ \sum \vec{F} = \vec{0} \]

NOTE:

1) IF \( \vec{F}_{\text{net}} = \sum \vec{F} = \vec{0} \) THEN \( \vec{a} = \vec{0} \)
2) IF \( \vec{a} = \vec{0} \) THEN \( \sum \vec{F} = \vec{0} \)
3) IF \( \sum \vec{F} \neq \vec{0} \) THEN \( \vec{a} \neq \vec{0} \)
4) IF \( \vec{a} \neq \vec{0} \) THEN \( \sum \vec{F} \neq \vec{0} \)

**NO EXCEPTIONS!!**
HOW TO USE $F = ma$

STANDARD EXAMPLES:

1) **MASSES ON SLOPES**

2) **MASSES + ROPES**

3) **MASSES + ROPES + PULLEYS**

(AND COMBINATIONS OF 1), 2), 3,)
FREE BODY DIAGRAMS (FBD's)

INCREDIBLY USEFUL FOR SOLVING PROBLEMS!

DRAW OBJECT AS A "FREE BODY" AND LABEL ALL FORCES

SIMPLEST EXAMPLE:

MASS ON TABLE:

FBD: (OF MASS)

NO TABLE!
Now do $F = ma$

Here object is at rest

$a = 0$ so $F = 0$ \( \uparrow \text{NET} \)

$F = mg - n = 0$

so $n = mg$
ANOTHER EXAMPLE:

A BODY IN FREE FALL:

\[ \downarrow \]

\[ mg \]

ONLY ONE FORCE!

Here \( a \neq 0 \)

\[ a = g \] (down wards)

So \( F = ma \)

\[ mg = ma \]

\[ a = g \]

(OK ... A BIT SILLY, GOING AROUND IN CIRCLES)
MASS ACCELERATING DOWN A SLOPE

A CLASSIC EXAMPLE OF HOW TO USE FBD’S!

(NO FRICTION)

FIND $a$

$mg$

$\theta$

SLOPE: $\theta$

FBD:

SLOPE ANGLE: HOW COME?
TIME OUT FOR GEOMETRY:

\[ 90^\circ - \theta \]

\[ \theta \]

\[ 90^\circ - \theta \]

\[ \text{mg} \]

\[ 90^\circ - \theta \]

SO FBD:

\[ \text{mg} \]

\[ \theta \]

\[ n \]
WE WANT a DOWN THE SLOPE

WE NEED COMPONENTS OF FORCES DOWN (PARALLEL) AND PERPENDICULAR TO THE SLOPE.

I DRAW A SECOND FBD:
WE CAN LEARN TWO THINGS:

1) **PERP TO SLOPE:**
   \[ a_{\perp} = 0 \therefore F_{\perp} = 0 \]
   \[ \therefore mg \cos \theta - n = 0 \]
   \[ \therefore n = mg \cos \theta \]
   TRUE, BUT NOT USEFUL HERE!

2) **PARALLEL TO SLOPE:**
   \[ a_{\parallel} = a \] USE \[ F_{\parallel} = ma \]
   \[ F_{\parallel} = mg \sin \theta = ma \]
   \[ \alpha = g \sin \theta \]
   NO \( m \).
So:

1) Draw a diagram

2) Draw and label an FBD

3) Draw a second FBD with components in the appropriate directions

4) Apply \( F = ma \)
SO NOW WE CAN ASK VARIOUS QUESTIONS:

E.G. HOW LONG TO SLIDE DOWN A LENGTH \( L \)? (FROM REST)

IT'S A 1-D PROBLEM:-

\[
x = v_0 t + \frac{1}{2} at^2
\]

\[
L = 0 + \frac{1}{2} g \sin \theta t^2
\]

\[
t = \sqrt{\frac{2L}{g \sin \theta}}
\]

NOTE \( a \) is \( \oplus \) HERE (DIRECTION OF POSITIVE \( y \))
We need to discuss tension $T$.

We will use massless ropes.

Object hanging from ceiling.

FBD: of mass

$mg - T = ma$

$a = 0$

$T = mg$
TENSION + ROPEs

(FBD of rope)

Every tiny piece of rope has equal and opposite forces:

\[ \begin{align*}
\text{Net force} & = m \text{a} \\
\text{Net force} & = 0 \times \text{a} = 0
\end{align*} \]

So \( T \longleftrightarrow T' \) \( \Rightarrow T' = T \)

But wait - there's more!

If \( a \neq 0 \); rope has \( m = 0 \)
TOWING LOCO + CAR

CAR

LOCO

\[ M_c \quad \text{ROPE} \quad M_L \quad \rightarrow a \]

LOCO CAN PROVIDE \( F_{TOT} \) \( \rightarrow \)

EVERYTHING TOGETHER. \( \alpha = ? \) \( T = ? \)

TWO FBD's:

CAR: \[ M_c \rightarrow T \]

\[ F = m_a \]

\[ 0 \ T = M_c a \]

LOCO:

\[ T \leftarrow M_L \rightarrow F_{TOT} \]

\( F_{TOT} - T = M_L a \)

SAME \( a \)

ADD \( 1 + 2 \):

\[ T = M_c a \]

\[ F_{TOT} - T = M_L a \]

\[ F_{TOT} = M_c a + M_L a \]

\[ = (M_c + M_L) a \]

\[ a = \frac{F_{TOT}}{(M_c + M_L)} \]
WHAT ABOUT $T$?

USE CAR FBD:

$[M_c] \rightarrow T$

$T = M_c \cdot \alpha$

$= M_c \left( \frac{F_{tot}}{M_c + M_L} \right)$

$\bar{T} = \frac{M_c}{M_c + M_L} \cdot F_{tot}$

NOTE: CAN GET $\alpha$ DIRECTLY; WITHOUT FBD'S:

PUT CAR + LOCO IN "BROWN BAG"

$\Rightarrow \quad \boxed{\square \square} \Rightarrow F_{tot}$

$M = M_c + M_L$

$\Rightarrow \quad \boxed{\square \square} \Rightarrow F_{tot}$

$so \quad F = ma$

$F_{tot} = (M_c + M_L) \alpha$

etc.
THAT WAS A PRETTY FEEBLE TRAIN!

LET'S ADD A CAR:

![Diagram showing car 2, car 1, and locomotive connected by forces with an arrow indicating acceleration.]

(I'VE CHANGED NOTATION!!)

FBD's:

**CAR 2**
- \[ T_2 = M_3 a \]
- \[ T_1 - T_2 = M_3 a \]

**CAR 1**
- \[ T_2 \]
- \[ T_1 \]

**LOCO**
- \[ F_T \]
- \[ F_T - T_1 = M_1 a \]

ADD:

\[ F_T - T_1 + T_1 - T_2 + T_2 = (M_1 + M_2 + M_3) a \]

SO

\[ a = \frac{F_T}{M_1 + M_2 + M_3} \]

(IE \( a = \frac{F_{TOT}}{M_{TOT}} \) AS BEFORE)
THEN:

\[ T_2 = M_3 a = \frac{M_3 F_T}{M_1 + M_2 + M_3} \]

To get \( T_1 \) can use either:

\[ T_1 - T_2 = M_2 a \quad \text{(so } T_1 = M_2 a + T_2) \]

or:

\[ F_T - T_1 = M_1 a \quad (T_1 = F_T - M_1 a) \]

Both give:

\[ T_1 = \frac{M_2 + M_3}{M_1 + M_2 + M_3} F_T \]

(check it out)

(exercise for student)

\[ \text{[we can add more cars if we wish...]} \]
Masses + Ropes + Pulleys

A pulley is a device that redirects the tension in the rope.

Massless pulley! No friction!

T same before and after pulley.
**FRICTION NOW ANYWHERE**

- **a = ?**
- **T = ?**

**TWO EBD's**

- **M₁**
  - **n → a**
  - **T**
  - **M₁g**
  - **Horiz:** **T = M₁a**

- **M₂**
  - **T**
  - **M₂g**
  - **Vert:** **M₂g - T = M₂a**

**Vert:**
- **N = M₁g** (not used here)
- **F = m₁a**

**ADD:**
- **M₂g + T**
- **= M₁a + M₂a**

**SO:**
- **M₂g = (M₁ + M₂)a**

**a = \frac{M₂g}{M₁ + M₂}**

**AND:**
- **T = M₁a**
- **T = \left(\frac{M₂}{M₁ + M₂}\right)g**
SO NOW WE COULD TACKLE ANYTHING!

E.G.

SLOPES + ROPES + PULLEYS

BUT WE WON'T DO THIS ONE! — NOT WITH SO MANY MASSES!